SIMON FRASER UNIVERSITY
Department of Economics

Econ 305 Prof. Kasa
Intermediate Macroeconomic Theory Fall 2013

PROBLEM SET 2
(Due October 30)

1. (20 points). Consider the 2-period Fisherian model discussed in class. Suppose Yin and Yang have
the same preferences. Also suppose Yin earns $100 in period-1 and $100 in period-2, while Yang earns
nothing in period-1, but earns $210 in period-2. Finally, suppose we observe that Yin and Yang both
consume $100 in each period. Given this information, what is the interest rate? Suppose interest rates
increase. What happens to Yin’s consumption? What happens to Yang’s consumption? Are Yin and
Yang better off worse off after the interest rate increase? Illustrate your answers with a graph.

2. (30 points). This question asks you to work through the complete (2-period) dynamic intertempo-
ral model, for a particular specification of preferences and technology. Suppose the representative
household’s preferences are given by

\[ U(C_1, C_2, \ell_1, \ell_2) = C_1 + \gamma \ell_1^{1/3} + \beta \{C_2 + \gamma \ell_2^{1/3}\} \]  

where \(C_1\) and \(C_1\) denote consumption in the first and second time period, \(\ell_1\) and \(\ell_2\) denote leisure in
the first and second time period, \(\gamma\) is a fixed parameter summarizing the relative preference for leisure,
and \(\beta < 1\) is a fixed parameter summarizing the household’s time preference. Output in each period
is produced with the following Cobb-Douglas production function:

\[ Y_i = z_i K_i^{2/3} N_i^{1/3} \quad i = 1, 2 \]  

where \(z_i\) denotes total factor productivity in period-\(i\). The economy begins with a fixed amount
of capital, \(K_1\), in period 1. This capital can be increased by investing in the first period, so that
\(K_2 = K_1 + I_1\). Notice for simplicity we’ve assumed that capital does not depreciate (i.e., \(\delta = 0\)).
As usual, the household confronts the following time constraint each period, \(\ell_i + N_i = h\), where \(h\) is
the total time available in each period. Finally, for simplicity, suppose there is no government in this
economy, and that all markets are perfectly competitive.

Calculate the competitive equilibrium values of consumption, employment and investment in each pe-
riod. Also, derive expressions for the market clearing wage rates and interest rates. How do these
variables depend on current and future productivity? [Hints: (i) Rather than look for market-clearing
wage rates and interest rates, use the ‘second welfare theorem’, and compute the competitive equi-
librium quantities by solving a ‘social planner’s problem’. That is, maximize the household’s utility
subject to the economy’s technology and resource constraints. There are 5 constraints: \(C_i + I_i = Y_i\),
\(\ell_i + N_i = h\) and \(K_2 = K_1 + I_1\), where \(Y_i\) is given by equation (2). That is, there are 2 aggregate
resource constraints (i.e., the National Income Accounting identity), 2 time constraints, and a capital
accumulation equation. (ii) Use the constraints to sub out \((C_1, C_2, \ell_1, \ell_2)\) and then solve an uncon-
strained maximization problem over \((N_1, N_2, I_1)\). (iii) Notice that since the economy ends in period 2,
It makes no sense to invest in period 2. That is, we know \(I_2 = 0\), so that \(C_2 = Y_2\).]