SIMON FRASER UNIVERSITY
Department of Economics

Econ 305 Prof. Kasa
Intermediate Macroeconomic Theory Fall 2021

PROBLEM SET 2
(Due October 28)

1. (25 points). Suppose Canada's production function is $Y = A \cdot K^{1/3}L^{2/3}$. Let (s, n, δ) denote the saving rate, population growth rate, and depreciation rate. For simplicity, suppose productivity, A, is constant.

   (a) Derive an expression for the steady state level of per capita income as a function of (A, s, n, δ).

   (b) Explain why A has a bigger effect on steady state per capita income than does s.

   (c) Discuss the implications of this finding for 'growth accounting' decompositions.

2. (25 points). Consider a standard one-sector Solow model with a fixed savings rate s. Output is produced via the production function

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

As usual, labor is inelastically supplied and grows at the exogenous rate n (i.e., $L_{t+1} = (1+n)L_t$).

Suppose that each unit of output produced generates Ω_t units of 'pollution', and that due to exogenous technological progress in pollution abatement, Ω_t decreases over time at rate g_a (i.e., $\Omega_{t+1}/\Omega_t = 1/(1+g_a)$). In addition, suppose that there is an 'abatement technology' that allows resources to be diverted into pollution reduction. Specifically, if θ represents the share of output used in pollution reduction, then net pollution emission, E_t, is given by

$$E_t = a(\theta)\Omega_t Y_t \tag{1}$$

where $a(\theta)$ is assumed to be a positive, decreasing function. For simplicity, assume that θ is constant and exogenous.

As usual, for notational convenience, let y_t represent net output available for consumption and capital accumulation per capita. That is,

$$y = \frac{(1-\theta)Y}{L}$$

Similarly, let k and e be capital and net pollution emission per capita. Using this notation, we have the following 'green Solow' model:

$$\begin{aligned} y_t &= (1-\theta)k_t^{\alpha} \\ \Delta k_{t+1} &= s(1-\theta)k_t^{\alpha} - (\delta + n)k_t \qquad (2) \\ e_t &= a(\theta)\Omega_t k_t^{\alpha} \qquad (3) \end{aligned}$$

where δ is the depreciation rate of capital. Evidently, from equation (2), the economy will converge to a unique steady state $k_t = k^*$. From equation (3), it is then clear that in the steady state (ie, when k_t is constant), pollution grows at a constant rate, g_E, given by

$$g_E = \frac{\Delta E_{t+1}}{E_t} = \frac{1+n}{1+g_a} - 1 \approx n - g_a$$

Of course, during the *transition* to the steady state, pollution may be growing either faster or slower than this. Now, let's define 'sustainable' growth to be a situation where $g_a \geq n$. That is, pollution remains bounded.

(a) It is often claimed that the time path of pollution within economies follows a so-called 'Environmental Kuznets Curve' (EKC), with pollution rising as the economy develops, and then eventually falling once the economy becomes wealthy enough (i.e., it traces out an inverted U-shape when plotted against either time or per capita income). Consider a sustainable economy, where $g_E < 0$. Under what conditions will this economy feature an EKC? (Hints: (1) Derive expressions for $\Delta k_{t+1}/k_t$ and $\Delta E_{t+1}/E_t$ as functions of k_t during the transition to the steady state, (2) The growth rate of $\Omega_t k_t^\alpha$ can be approximated by $\frac{\Delta \Omega_{t+1}}{\Omega_t} + \alpha \frac{\Delta k_{t+1}}{k_t}$). If you can't provide explicit analytical conditions, then at least try to explain intuitively how this relationship could arise.

(b) How does an increase in abatement effort (i.e., an increase in θ) affect the time path of pollution? Explain intuitively, and relate your conclusions to how a standard Solow model reacts to an increase in the savings rate. (Hint: You do not need to solve for anything. Just sketch out a time path).