SIMON FRASER UNIVERSITY
Department of Economics

Econ 305 — Prof. Kasa
Intermediate Macroeconomic Theory — Fall 2021

PROBLEM SET 2
(Solutions)

1. (25 points). Suppose Canada's production function is $Y = A \cdot K^{1/3}L^{2/3}$. Let (s, n, δ) denote the saving rate, population growth rate, and depreciation rate. For simplicity, suppose productivity, A, is constant.

   (a) Derive an expression for the steady state level of per capita income as a function of (A, s, n, δ).

   The steady state condition is

   $$sAk^{1/3} = (n+\delta)k$$

   Solving for k gives

   $$k^* = \left(\frac{aA}{n+\delta}\right)^{3/2}$$

   Substituting this back into the production function gives

   $$\begin{aligned} y^* &= A\left(\frac{sA}{n+\delta}\right)^{1/2} \\ &= A^{3/2}\left(\frac{s}{n+\delta}\right)^{1/2} \end{aligned}$$

   (b) Explain why A has a bigger effect on steady state per capita income than does s.

   An increase in A not only increases output directly (for a given k), it also encourages investment, which increases k. In contrast, an increase in s just increases k.

   (c) Discuss the implications of this finding for 'growth accounting' decompositions.

   Growth accounting measures the influence of productivity as a residual, after growth in factor inputs have been taken into account. Because part of the growth in capital is in response to productivity growth, growth accounting calculations tend to understate the importance of productivity growth.

2. (25 points). Consider a standard one-sector Solow model with a fixed savings rate s. Output is produced via the production function

   $$Y_t = K_t^{\alpha}L_t^{1-\alpha}$$

   As usual, labor is inelastically supplied and grows at the exogenous rate n (i.e., $L_{t+1} = (1+n)L_t$).

   Suppose that each unit of output produced generates Ω_t units of 'pollution', and that due to exogenous technological progress in pollution abatement, Ω_t decreases over time at rate g_a (i.e., $\Omega_{t+1}/\Omega_t = 1/(1+g_a)$). In addition, suppose that there is an 'abatement technology' that allows resources to be diverted into pollution reduction. Specifically, if θ represents the share of output used in pollution reduction, then net pollution emission, E_t, is given by

$$E_t = a(\theta)\Omega_t Y_t \tag{1}$$

where $a(\theta)$ is assumed to be a positive, decreasing function. For simplicity, assume that θ is constant and exogenous.

As usual, for notational convenience, let y_t represent net output available for consumption and capital accumulation per capita. That is,

$$y = \frac{(1-\theta)Y}{L}$$

Similarly, let k and e be capital and net pollution emission per capita. Using this notation, we have the following 'green Solow' model:

$$\begin{aligned} y_t &= (1-\theta)k_t^\alpha \\ \Delta k_{t+1} &= s(1-\theta)k_t^\alpha - (\delta+n)k_t && (2) \\ e_t &= a(\theta)\Omega_t k_t^\alpha && (3) \end{aligned}$$

where δ is the depreciation rate of capital. Evidently, from equation (2), the economy will converge to a unique steady state $k_t = k^*$. From equation (3), it is then clear that in the steady state (ie, when k_t is constant), pollution grows at a constant rate, g_E, given by

$$g_E = \frac{\Delta E_{t+1}}{E_t} = \frac{1+n}{1+g_a} - 1 \approx n - g_a$$

Of course, during the *transition* to the steady state, pollution may be growing either faster or slower than this. Now, let's <u>define</u> 'sustainable' growth to be a situation where $g_a \geq n$. That is, pollution remains bounded.

(a) It is often claimed that the time path of pollution within economies follows a so-called 'Environmental Kuznets Curve' (EKC), with pollution rising as the economy develops, and then eventually falling once the economy becomes wealthy enough (i.e., it traces out an inverted U-shape when plotted against either time or per capita income). Consider a sustainable economy, where $g_E < 0$. Under what conditions will this economy feature an EKC? (Hints: (1) Derive expressions for $\Delta k_{t+1}/k_t$ and $\Delta E_{t+1}/E_t$ as functions of k_t during the transition to the steady state, (2) The growth rate of $\Omega_t k_t^\alpha$ can be approximated by $\frac{\Delta\Omega_{t+1}}{\Omega_t} + \alpha\frac{\Delta k_{t+1}}{k_t}$). If you can't provide explicit analytical conditions, then at least try to explain intuitively how this relationship could arise.

From eq. (2), we have the following expression for the (percentage) change in k, during the transition to the steady state

$$\frac{\Delta k_{t+1}}{k_t} = s(1-\theta)k_t^{\alpha-1} - (\delta+n)$$

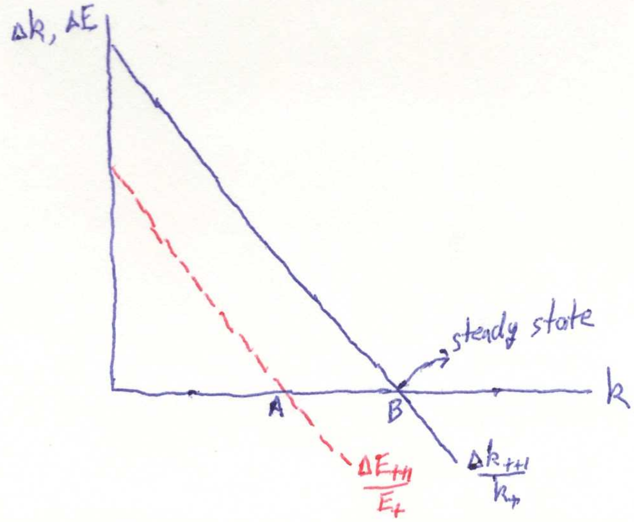
Note this is a declining function of k, which is positive for small values of k, and negative for large values of k. The steady state occurs when it crosses the horizontal axis. Next, by definition, we have the following expression for the (percentage) change in pollution emissions during the transition to the steady state

$$\frac{\Delta E_{t+1}}{E_t} = \frac{\Omega_{t+1}Y_{t+1} - \Omega_t Y_t}{\Omega_t Y_t}$$

Using the given hints and previous results, we can write this also as a function of k

$$\frac{\Delta E_{t+1}}{E_t} = \alpha(1+n)[s(1-\theta)k_t^{\alpha-1} - (\delta+n)] + n - g_a$$

Note that this has this has the same basic shape as the the capital accumulation function, but it crosses the horizontal axis before it. This is because $n < g_a$ when the economy is 'sustainable'. In sum, we can depict the paths of capital and pollution in the following diagram



Notice that if the economy is initially poor, with a relatively low value of k, (ie, to the left of point A), then during the transition both output and pollution increase as the economy develops. Then, since the $\Delta E_{t+1}/E_t$ line hits zero first, pollution begins to decline, while per capita income continues to grow. This would produce an Environmental Kuznets Curve. However, if the economy starts out relatively rich (ie, to the right of point A but to the left of point B), the economy continues to grow, but pollution declines monotonically. In this case, there would be no EKC. The intuition here is that due to diminishing returns, output growth is faster in poor countries. At the same time, since pollution growth is just proportional to output growth, pollution emissions grow rapidly in poor countries. The good news is that 'productivity' in pollution abatement is constant, and independent of the level of income. Eventually, productivity growth in pollution abatement comes to dominate the output effect, since output growth declines over time.

(b) How does an increase in abatement effort (i.e., an increase in θ) affect the time path of pollution? Explain intuitively, and relate your conclusions to how a standard Solow model reacts to an increase in the savings rate. (Hint: You do not need to solve for anything. Just sketch out a time path).

In this model, changes in 'abatement effort', $a(\theta)$, are just like changes in the saving rate in the regular Solow model. They produce temporary changes in the growth rate of pollution, but not long lasting changes. Those are determined entirely by the underlying growth rates of output and abatement productivity. The picture would look just like the economy's response to a (permanent) change in the saving rate. There would be a sudden drop in the growth rate of pollution, but it would eventually climb back to its long-run balanced growth path.