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Econ 305 Prof. Kasa
Intermediate Macroeconomic Theory Spring 2021

PROBLEM SET 2
(Solutions)

1. (25 points). Suppose Canada's production function is $Y = A \cdot K^{1/3}L^{2/3}$, and that Canada is currently in a steady with a savings rate of 20% (i.e., $s = .20$), a population growth rate of 1.7% (i.e., $n = .017$), and a depreciation rate of 5% (i.e., $\delta = .05$). For simplicity, suppose productivity, A, is constant.

   (a) If per capita income in Canada is \$30,000, what must be the value of A? (Hint: Derive a formula relating steady state per capita output to A, s, n and δ and then solve for A).
   We can write the per capita production function as $y = Ak^{1/3}$. Use this in the steady state condition, $sy = (n + \delta)k$ to solve for the steady state capital/labor ratio

$$k^* = \left(\frac{sA}{n+\delta}\right)^{3/2}$$

   Now substitute this back into the per capita production function and solve for y^*

$$y^* = A^{3/2}\left(\frac{s}{n+\delta}\right)^{1/2}$$

   Finally, plug in the given data and solve for A

$$A = \left(\frac{30,000}{\sqrt{.2/.067}}\right)^{2/3} \approx 670.5$$

   (b) Demographers predict that Canada's population growth rate could drop to zero soon. Use the Solow model to forecast the consequences of this demographic shift on total output, per capita output, the growth rate of total and per capita output, and the wage rate. Consider the effects in both the new steady state and in the transition to the new steady state. Illustrate your answer with a graph.

When n drops to 0, the break-even investment line rotates down and you can see that the steady state capital/labor ratio will rise, and therefore, so will the steady state level of output per capita. In particular, the new steady state per capita output will be

$$y^* = (670.5)^{1.5}\sqrt{.2/.05} \approx \$34,724$$

The growth rate of total output will fall from 1.7% to 0. Finally, because the steady state capital/labor ratio will be higher, the steady state wage rate will rise.

2. (25 points). Consider a Cobb-Douglas production function with three inputs: K is capital (i.e., number of machines), L is labor (e.g., number of workers), and H is 'human capital' (e.g., number of college degrees among the workers). Normalizing the scale factor A to one gives us:

$$Y = K^{1/3}L^{1/3}H^{1/3}$$

(a) Derive an expression for the marginal product of labor. How does an increase in the amount of human capital affect the marginal product of labor?

The marginal product of unskilled labor is given by

$$MPL = (1/3)K^{1/3}L^{-2/3}H^{1/3}$$

Note that an increase in H increases the marginal product of labor. Having skilled workers around enhances the productivity of unskilled workers.

(b) Derive an expression for the marginal product of human capital. How does an increase in the amount of human capital affect the marginal product of human capital?

The marginal product of skilled labor is given by

$$MPH = (1/3)K^{1/3}L^{1/3}H^{-2/3}$$

Note that an increase in H decreases the marginal product of human capital. This just reflects diminishing returns.

(c) What is the income share paid to labor? What is the income share paid to human capital? In the National Income Accounts of this economy, what share of total income do you think workers would appear to receive? (*Hint*: Consider where the return to human capital shows up).

A skilled worker not only earns the return to human capital, but she also receives the return to raw labor as well. (Presumably, after you get a college degree, you don't forget how to use a stapler!) Thus, if we let W_s denote the skilled wage, we have

$$W_s = MPL + MPH$$

(d) An unskilled worker earns the marginal product of labor, whereas a skilled worker earns the marginal product of labor plus the marginal product of human capital. Using the answers to (a) and (b), find the ratio of the skilled wage to the unskilled wage. How does an increase in the amount of human capital affect this ratio? Explain.

Now letting W_u denote the unskilled wage, we get the following expression for the wage premium for skilled labor:

$$\frac{W_s}{W_u} = \frac{MPL + MPH}{MPL} = 1 + \frac{MPH}{MPL} = 1 + \frac{L}{H}$$

Thus, increases in human capital reduce the skill premium. Having more college graduates around not only bids down wages for jobs needing a college education, it also tends to raise the wages of unskilled workers as well, since their productivity rises. For both reasons the skill premium falls with increases in H. (Of course, this assumes an exogenous increase in H!).

(e) Some people advocate government funding of college scholarships as a way of creating a more egalitarian society. Others argue that scholarships help only those who are able to go to college. Do your answers to the above questions shed light on this debate?

These answers suggest that college scholarships equalize the income distribution.