SIMON FRASER UNIVERSITY
Department of Economics

Econ 815
Financial Economics, I

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FINAL EXAM

Answer the following True, False, or Uncertain. Explain Why. 8 points each.

1. Buying both a call and a put at the same strike price produces a riskless return.
2. Bubbles cannot exist in finite-maturity asset prices.
3. A higher coefficient of relative risk aversion will cause the risk-free rate of return to increase.
4. If markets are efficient, then stock prices follow random walks.
5. If information is costly, then markets cannot be complete (in the Arrow-Debreu sense).

Short answer questions:

6. (30 points). **Knockout Options**. In class we focused on 'plain vanilla' options. In practice, many so-called 'exotic' options are traded. One example is a 'knockout option'. These options terminate worthless if the price of the underlying asset hits a threshold. For calls they are called 'down and out' options, because the option is terminated if the stock price falls to a certain level. In general, their values must be computed numerically, but let's consider a simple example that can be computed analytically. In particular, let's consider a *perpetual* down and out option on a non-dividend paying stock (i.e., as long as the stock price remains above the threshold, it never expires). In this case, the Black-Scholes PDE becomes the following ODE (note we just drop the $\partial C/\partial t$ term):

$$\frac{1}{2}\sigma^2 S^2 C''(S) + rSC'(S) - rC(S) = 0$$

Suppose the threshold is L. The knockout feature gives us the boundary condition $C(L) = 0$. We also know that $C(S) \to S$ as $S \to \infty$.

(a) Show that the general form of the solution is

$$C(S) = a_1 S + a_2 S^{-\gamma}$$

where (a_1, a_2) are constants and $\gamma = 2r/\sigma^2$. (Hint: Guess a solution of the form $C(S) = S^\alpha$, and show that α solves a quadratic equation).

(b) Use the boundary conditions to compute (a_1, a_2). Show that the resulting down and out call option has the value

$$C(S) = S - L\left(\frac{S}{L}\right)^{-\gamma}$$

(c) Note the value of a perpetual call is just S (if you can buy the stock whenever you want, with no knockout, then the price of that option is simply the current stock price!). Given this, provide an economic interpretation of your formula.

7. (30 points). **News and the Stock Market**. Consider a Lucas (1978) asset pricing model where the representative agent has preferences given by

$$U = E_t \sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\gamma}}{1-\gamma}$$

The agent can buy shares in a 'tree', which yields a stream nondurable fruit/dividends d_{t+j}. In equilibrium, consumption equals dividends, which equals output, y_{t+j}. Let P_t be the time-t (ex-dividend) share price (ie., the price after current period dividends have been paid, so that a current share purchase only entitles you to the stream of dividends starting next period).

(a) Write down the agent's Euler equation characterizing his optimal portfolio decision.

(b) Impose the equilibrium condition, $d_t = y_t$, and derive a first order stochastic difference equation for P_t.

(c) Iterate this equation forward and derive an expression for P_t in terms of y_t and the expected present discounted value of future y_t. (Impose the 'no bubbles' condition to rule out explosive prices).

(d) Does 'good news' about future output always produce an increase in stock prices? Under what conditions on γ will this be the case? Interpret your results.