SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 | Prof. Kasa
Financial Economics, I | Fall 2020

FINAL EXAM
Due December 7, 6pm

Answer the following True, False, or Uncertain. Explain Why. 10 points each.

1. Buying a call option on a stock is riskier than buying the stock itself.
2. Rational individuals cannot agree to disagree.
3. If markets are efficient, stock prices reveal all available information.
4. The Lucas model predicts that stock and bond prices are positively correlated.

Short answer questions:

5. (20 points). **Asian Options**. In class we focused on 'plain vanilla' options. In practice, many so-called 'exotic' options are traded. One example is an 'Asian option'. These options are *path dependent*, i.e., their value depends on the entire time path of the stock, not just its terminal value. As a result, one cannot compute their no arbitrage value using a simple analytical formula. Instead, one must use a numerical, monte carlo simulation approach. This question asks you to do this.

   In particular, consider a stock which has a price that follows the geometric Brownian motion process:

$$\frac{dS}{S} = \mu dt + \sigma dW$$

   where $\mu = .12$ and $\sigma = .20$. Suppose the current stock price is \$42, and suppose we are interested in the value of a 1-year (European) Asian call option on this stock, where the strike price is given by the average value of the stock during the life of the option. That is, $K = S_{\text{avg}}$. Hence, the value of the Asian call at expiration is given by $C = \max[S_T - S_{\text{avg}}, 0]$. Assume the risk-free rate is constant, and equal to 10%.

   Write (and execute) a computer program to calculate the value of this option. (Use whatever software you want).

   Hint 1: Your code should consist of two nested loops. The outer loop performs the monte carlo replications, the inner loop performs the stock price simulations. Unless your computer is really slow, set the number of monte carlo replications to $N = 1000$.

   Hint 2: Based on Black-Scholes logic, what should be the drift rate of the stock in your program?

6. (20 points). **News and the Stock Market**. Consider a Lucas (1978) asset pricing model where the representative agent has preferences given by

$$U = E_t \sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\gamma}}{1-\gamma}$$

The agent can buy shares in a 'tree', which yields a stream nondurable fruit/dividends d_{t+j}. In equilibrium, consumption equals dividends, which equals output, y_{t+j}. Let P_t be the time-t (ex-dividend) share price (ie., the price after current period dividends have been paid, so that a current share purchase only entitles you to the stream of dividends starting next period).

(a) Write down the agent's Euler equation characterizing his optimal portfolio decision.

(b) Impose the equilibrium condition, $d_t = y_t$, and derive a first order stochastic difference equation for P_t.

(c) Iterate this equation forward and derive an expression for P_t in terms of y_t and the expected present discounted value of future y_t. (Impose the 'no bubbles' condition to rule out explosive prices).

(d) Does 'good news' about future output always produce an increase in stock prices? Under what conditions on γ will this be the case? Interpret your results.

7. (20 points). **The Term Structure of Interest Rates**. An important segment of the financial market is the 'fixed-income market'. The two key factors in this market are credit/default risk and the term structure of interest rates. The term structure is characterized by the *yield curve*, which plots the yield to maturity of different bonds against their maturities. This question asks you to think about the determinants of the yield curve.

Consider an economy populated by a large number of identical households with utility function

$$E_0 \sum_{t=0}^{\infty} \frac{C_t^{1-\gamma}}{1-\gamma}$$

Each household owns a Lucas tree, which yields a stream of consumption following the process

$$C_{t+1} = C^* C_t^{\alpha} \varepsilon_{t+1} \qquad 0 < \alpha < 1$$

where $\log(\varepsilon_t)$ is an i.i.d. sequence of $N(0, \sigma^2)$ random variables. Assume that, in addition to shares in trees, agents can trade bonds of all maturities. (Note, a j-period bond is a claim to a unit of consumption j periods in the future).

(a) Use the household's Euler equation to calculate the term structure of interest rates. (Hint 1: Remember, the yield on a bond is the reciprocal of its price, $R_{t,j} = (1/P_{t,j})^{1/j}$, where $P_{t,j}$ is the time-t price of a j-period bond, and $R_{t,j}$ is the annualized yield. Hint 2: Take logs of the consumption process, and rewrite the investor's Euler equation in terms of the log and exp functions. Then use the formula for the mean of a lognormal random variable).

(b) When does the yield curve slope up? When does it slope down? Explain.

(c) Economist A argues that economic theory predicts that the variance of the log of short-term interest rates is always lower than the variance of the log of long-term interest rates, because short rates are 'riskier'. Do you agree? Use the results from part (a) to justify your answer.

(d) Economist B claims that short-term interest rates, i.e., $j = 1$, are 'more responsive' to the state of the economy, i.e., C_t, than are long-term interest rate, i.e., j large. Do you agree? Explain.

(e) Economist C claims the Fed should lower interest rates because whenever interest rates are low, consumption is high. Do you agree? Explain.

(f) Economist D claims that in economies where output and consumption are persistent (i.e., $\alpha \approx 1$), changes in output and consumption do not affect interest rates. Do you agree? Explain intuitively.