SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 | Prof. Kasa
Financial Economics, I | Fall 2015

FINAL EXAM
(Solutions)

Answer the following True, False, or Uncertain. Explain Why. 8 points each.

1. Asset price bubbles indicate that investors are irrational.

   FALSE. *In class we discussed two theories of bubbles, both of which presume agents are 'rational'. The first was the so-called rational bubble model, where the bubble simply represents the solution to the homogeneous part of the asset price difference equation. Prices rise simply because people expect them to rise. This is not a satisfactory model of bubbles (in my opinion), for two reasons: (1) Transversality conditions often rule them out (so in fact they are* <u>*not*</u> *rational), and (2) They do not generate any asset trading, which is not consistent with the data. The second theory we discussed was based on heterogeneous expectations, and the presence of short-sale constraints, which gives rise to a resale option value, which can be interpreted as a 'bubble'.*

2. Options are riskless investments.

   FALSE. *This is actually kind of tricky. There is an important difference between no arbitrage/risk-neutral pricing and riskless assets. We were able to use risk-neutral pricing when deriving the BS option formula because the option was a 'redundant' security. The riskless asset and the stock already completely span the state space (given continuous trading), meaning that we can form a dynamically adjusted portfolio of them that perfectly replicates the payoff on the option. (Or equivalently, we can form a portfolio of the stock and the option that is riskless). That enables us to calculate the no arbitrage price of the option. However, this does* <u>*not*</u> *imply the option itself is riskless! (As many option traders have painfully discovered for themselves). Another way to think about it - it is the* <u>*portfolio*</u> *consisting of the option and stock that is riskless, not the individual constituents of the portfolio. Each of them separately can be quite risky.*

3. An increase in a stock's variance reduces the value of options written on that stock.

   FALSE. *It's precisely the opposite. The value of an option depends on the* <u>*tail*</u> *of the underlying asset's price distribution (right tail for a call, left tail for a put). If you hold an option, you want a* <u>*greater*</u> *variance (holding price constant!), since that makes it more likely the option will finish in-the-money. (If the price goes the wrong direction, you simply do not exercise).*

4. If investors have common prior beliefs, then they won't trade with each other.

   FALSE/UNCERTAIN. *Under certain assumptions, Tirole showed that common priors rule out trading. However, one of his key conditions was the existence of an initially Pareto optimal allocation, so that the* <u>*only*</u> *reason people would trade was for speculative gain. His No Trade theorem is perfectly consistent with trade motivated by (mutually beneficial) risk-sharing.*

5. If markets are efficient, then stock prices reveal all publicly available information.

   FALSE/UNCERTAIN. *Grossman and Stiglitz argued that if information is costly to produce and acquire, then prices cannot perfectly reveal information. If they did, that would destroy the incentives to acquire the information. On the other hand, if information is 'free', then you could argue that efficient markets should 'reveal' (or more properly, 'aggregate') the information.*

   Short answer questions:

6. (30 points). Use an arbitrage argument to derive the Black-Scholes PDE. Outline a simple Monte Carlo simulation strategy for solving this PDE.

   See class notes.

7. (30 points). Consider an economy with two possible states, good and bad. Each state occurs with probability 1/2. In the good state, each individual's consumption is 1. In the bad state, a fraction λ of the population consumes $1 - (\phi/\lambda)$ and the rest consumes 1, where $0 < \phi < 1$ and $\phi \leq \lambda \leq 1$. Note that ϕ represents the fall in *average* consumption in the bad state, while λ measures how broadly that reduction is shared. For example, if $\phi = \lambda = 1/2$, then in the bad state aggregate consumption falls by 50%, but half the population loses nothing, while the other half loses everything.

   Consider two assets (Arrow securities), one that pays 1 unit in the good state, and one that pays 1 unit in the bad state. Let P denote the price of the bad state asset relative to the good state asset.

   (a) As in the Lucas (1978) model, the equilibrium price must adjust so that everyone is willing to consume their endowment. Before uncertainty is realized, individuals can buy and sell the two assets so as to equate expected marginal utilities across states. Write down the Euler equation that makes individuals indifferent between holding the good state or bad state asset. (Hint: In equilibrium, an agent can't make himself better off by selling the bad state asset, and using the proceeds to buy the good state asset).

   Let P_b be the price of the bad state Arrow security, and P_g be the price of the good state Arrow security. Equating expected marginal utility per dollar gives the Euler equation

   $$\frac{\lambda U'(1-\phi/\lambda) + (1-\lambda)U'(1)}{P_b} = \frac{U'(1)}{P_g}$$

   (b) Use the result in part (a) to derive an expression for the equilibrium value of P. (Hint: Your answer should be in terms of $U'(1)$, $U'(1-(\phi/\lambda))$, and λ.)

   Let $P = P_b/P_g$ be the relative price of the bad state Arrow security. The above Euler equation then implies

   $$P = \frac{\lambda U'(1-\phi/\lambda) + (1-\lambda)U'(1)}{U'(1)}$$

   (c) Prove that if $U''' > 0$, then $\partial P/\partial\lambda < 0$. Explain what this means in words, and interpret this result economically. Do you think this might help explain the equity premium puzzle?

   Differentiating the previous expression w.r.t. λ gives

   $$\frac{\partial P}{\partial \lambda} = \frac{U'(1-\phi/\lambda) - U'(1) + \lambda U''(1-\phi/\lambda)\frac{\phi}{\lambda^2}}{U'(1)}$$

   Therefore

   $$\text{sgn}\left(\frac{\partial P}{\partial \lambda}\right) = \text{sgn}\left(U'(1-\phi/\lambda) - U'(1) + U''(1-\phi/\lambda)\frac{\phi}{\lambda}\right)$$

A 2nd order Taylor series of $U'(1)$ around $U'(1-\phi/\lambda)$ gives

$$U'(1) \approx U'(1-\phi/\lambda) + U''(1-\phi/\lambda)(\frac{\phi}{\lambda}) + \frac{1}{2}U'''(1-\phi/\lambda)(\frac{\phi}{\lambda})^2$$

Using this to substitute for $U'(1)$ in the previous expression gives

$$\operatorname{sgn}\left(\frac{\partial P}{\partial \lambda}\right) = \operatorname{sgn}\left(-\frac{1}{2}U'''(1-\phi/\lambda)(\frac{\phi}{\lambda})^2\right)$$

and the result is proved. Intuitively, as λ declines down to ϕ, a given decline in average consumption is being concentrated on a smaller and smaller fraction of the population. That is, you are facing a smaller probability of experiencing a larger loss. If $U''' > 0$ then people are especially averse to this, and so the price of the bad state asset gets bid up. The main point of this question is to highlight one potential limitation of representative agent asset pricing. The cross-sectional <u>*distribution*</u> *of risk can matter a lot. (Note: This question is based on an old paper by Mankiw (1986, JFE)).*