Topics for Today

1.) Options

- Background
- Terms + Concepts
- Payoff Profiles
- Portfolios of options
- Put-Call Parity
- A 1-period binomial option pricing formula

Background

- An option is the right, but not the obligation, to buy (or sell) an asset under specified terms.

- Options have been around for centuries. In fact, options are implicit in many, if not most, economic decisions!

- However, exchange-traded financial options began in 1973, at the Chicago Board Options Exchange (CBOE). [Not coincidentally, this was the publication date of the Black-Scholes paper].

- Options are an example of a derivative security. The value of a derivative security depends on the value of some underlying asset. [Other examples - futures, swaps, mortgage-backed securities].

- Options exist for many underlying assets, e.g., stocks, bonds, foreign exchanges and commodities.

- No one really knows how big the market is. Recent estimates of the (notional) value of the derivatives market puts it in excess of $500 trillion. [The global stock market is only around $50 trillion].

- The derivatives market is relatively unregulated (especially the Over-the-Counter (OTC) market). Exchange-traded options are guaranteed by the exchange. Most derivative positions are "off balance sheet".

Terms & Concepts

- Options are defined by 3 features:
  1.) Right to buy or sell
  2.) Strike (or exercise) price
  3.) Expiration date

- An option to buy is called a call option. An option to sell is called a put option.

- A given stock typically has many different option contracts written on it, differing by call/put, strike price, expiration date.

- A European option can only be executed at the expiration date. An American option can be exercised at any time before the expiration date.

- The price of the option is called the premium. For exchange traded options it is determined by supply & demand. It changes minute-by-minute (or millisecond-by-millisecond!).

- The premium is paid up-front, and is not recovered if the option is not exercised.

- An option is "in-the-money" if current exercise yields a profit. Otherwise, it is "out-of-the-money".
  Note - An out-of-the-money option may still have positive value!

- The seller of an option is said to "write" the option.

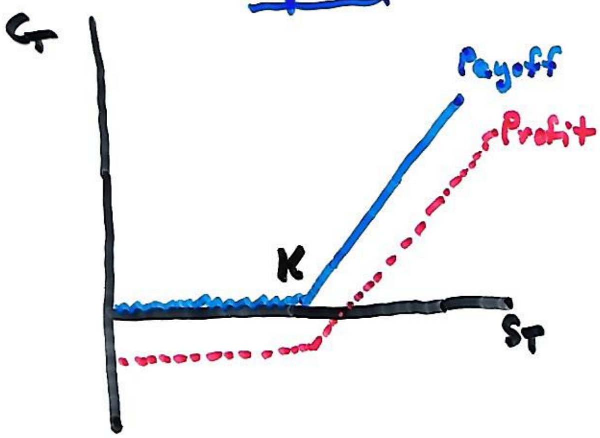
Payoff Profiles

- Let S = Stock Price, K = Strike Price, C = Value of Call, P = Value of Put

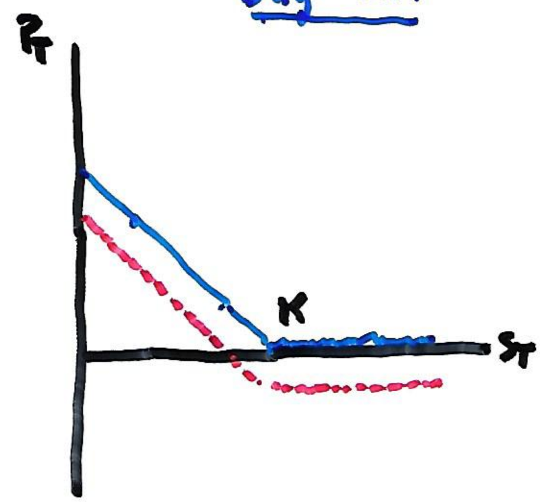
- At expiration date, T, we have:

$$\boxed{C_T = \max\{S_T - K, 0\}} \qquad \boxed{P_T = \max\{K - S_T, 0\}}$$

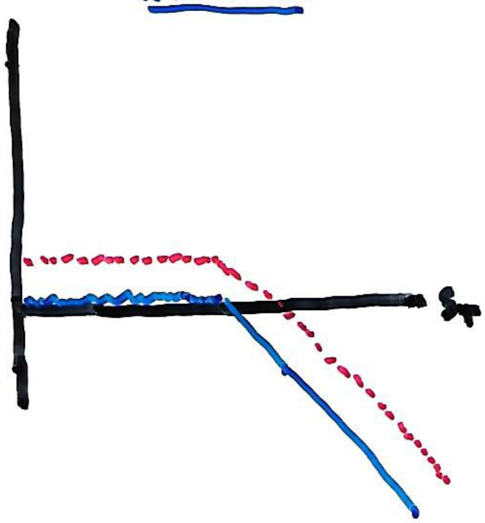
Buy Call



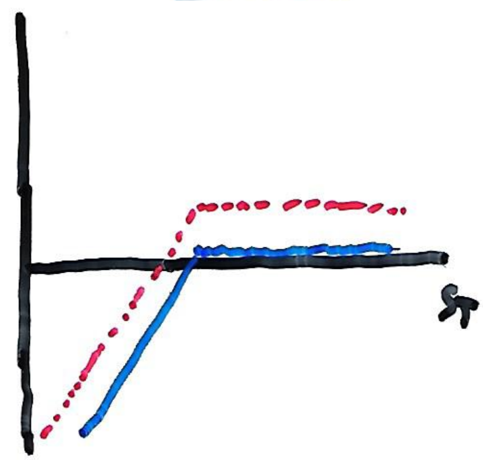
Buy Put



Write Call

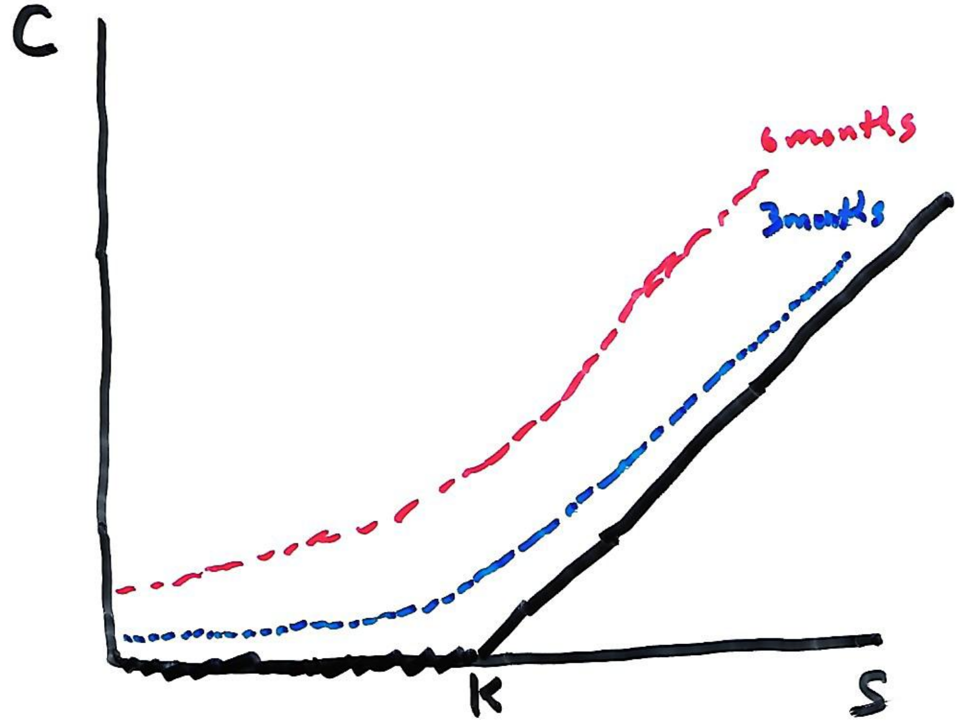


Write Put



- Note, the writer of a call option faces unbounded losses.

• Before expiration, the value of the option will exceed its expiration value,

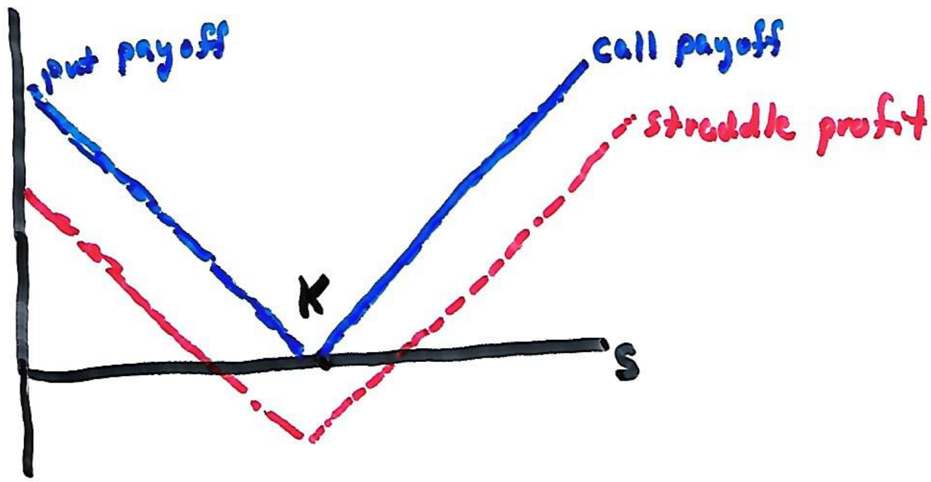


• The Black-Scholes formula provides an equation describing these curves, how they evolve over time, and how they shift in response to changes in model parameters (e.g., strike price or price volatility).

Portfolios of Options

- Part of the reason options are so popular is that by combining them into portfolios, you can construct very flexible payoffs. Here are 2 examples:

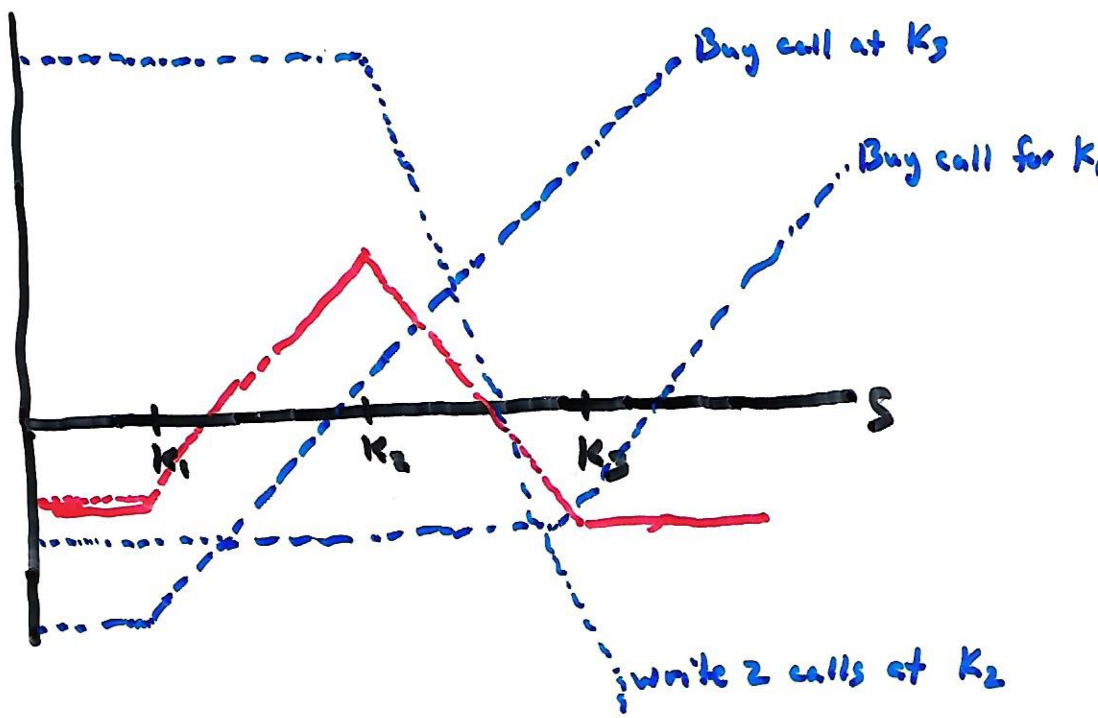
① **Straddle**: Buy both a call + put at the same strike price



- A straddle is a bet on volatility. It pays off if prices move more than expected (in either direction).

② **Butterfly Spread**: Buy 2 calls at different strike prices + write 2 calls at an intermediate strike price.

Now you're betting against volatility, while capping your losses.

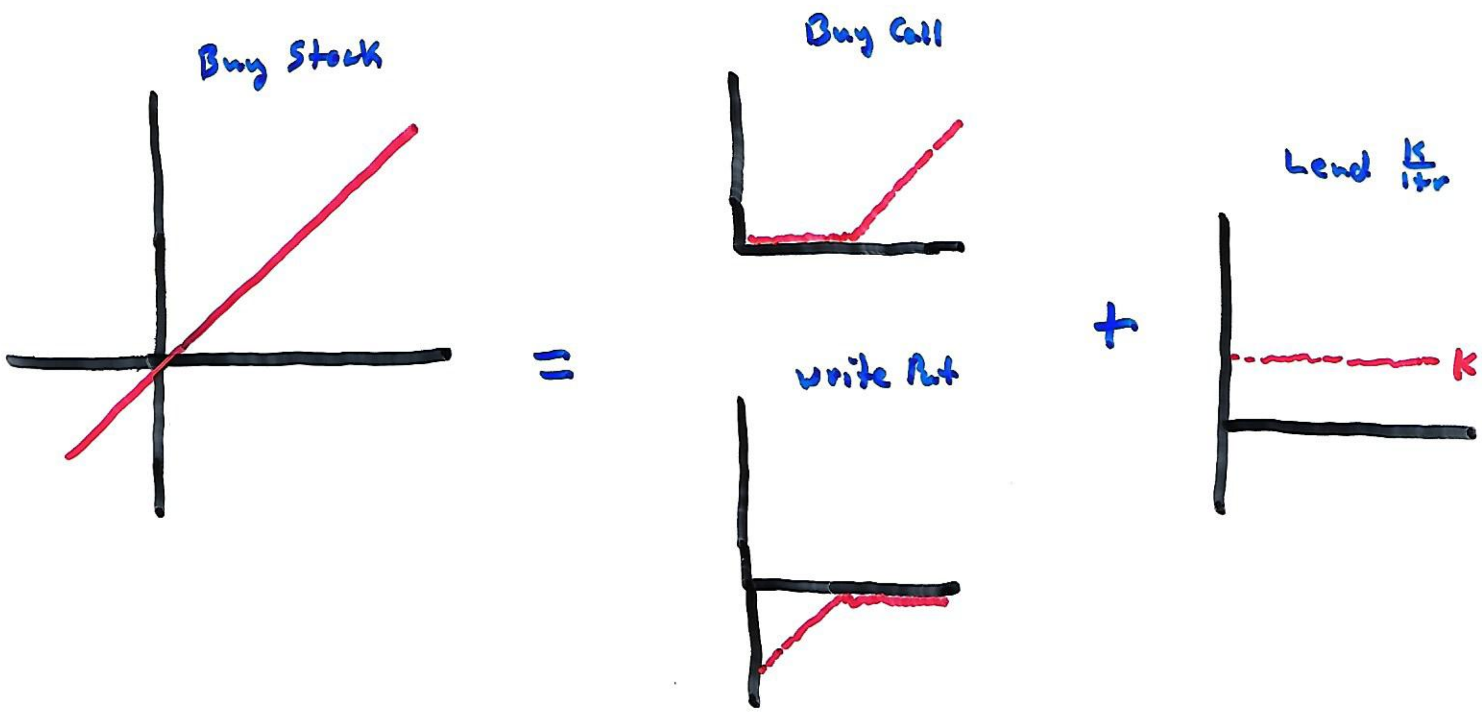


Put-Call Parity

- We will focus on European call options. This is not very restrictive, for 2 reasons:

1.) American call options (on non-dividend paying stocks) are never exercised early. [It is better to sell the option than to exercise early].

2.) Put-Call Parity: $S = C - P + \frac{K}{1+r}$



- Buying a call at K, writing a Put at K, and lending $\frac{K}{1+r}$ produces same payoff at T as just buying the stock! Hence, if we know the call price, we can infer (via no arbitrage) the put price.

A 1-period Binomial Option Pricing Formula

- Black + Scholes value options by constructing a payoff replicating portfolio. This then gives a no arbitrage valuation for the call option.

- Doing this over arbitrary time intervals takes some mathematics, but for 1-period options, all we need is a little algebra.

- Let S = initial stock price
  K = strike price
  $R = 1+r$ = (Gross) interest rate

- Suppose next period S goes up to $u \cdot S$ with prob p, and goes down to $d \cdot S$ with prob $(1-p)$.

- We Know

$$C_u = \max\{u \cdot S - K, 0\}$$ > value of option if S goes up

$$C_d = \max\{d \cdot S - K, 0\}$$ > value of option if S goes down

- Consider forming a portfolio consisting of x dollars worth of stock and b dollars worth of bonds. The value of this portfolio depends on the value of next period's stock price

$u \cdot x + Rb$ if S goes up

$d \cdot x + Rb$ if S goes down

> value of portfolio next period

- We want to pick x and b to replicate the call option payoff

$$u \cdot x + Rb = C_u$$
$$d \cdot x + Rb = C_d$$

Solving for (x, b):

$$x = \frac{C_u - C_d}{u - d} \qquad b = \frac{u \cdot C_d - d \cdot C_u}{R(u - d)}$$

- The current value of the portfolio is then,

$$\boxed{x + b = \frac{1}{R}\left(\frac{R - d}{u - d} C_u + \frac{u - R}{u - d} C_d\right)}$$

- By no arbitrage, this must also be the value of the call option!

<u>Comments</u>

1.) Define $q = \frac{R-d}{u-d}$. Since it must be the case that $u > R > d$, we know $0 < q < 1$. Hence, we can think of q as a probability, and write

$$\boxed{C = \frac{1}{R}\left[q \cdot C_u + (1-q) C_d\right] = \frac{1}{R}\hat{E}\left[C(T)\right]}$$

These are called "<u>risk-neutral probabilities</u>".

2.) Notice that p does <u>not</u> appear in the formula, which seems surprising. Shouldn't the option to buy depend on the likelihood of the price going up!