Topics for Today

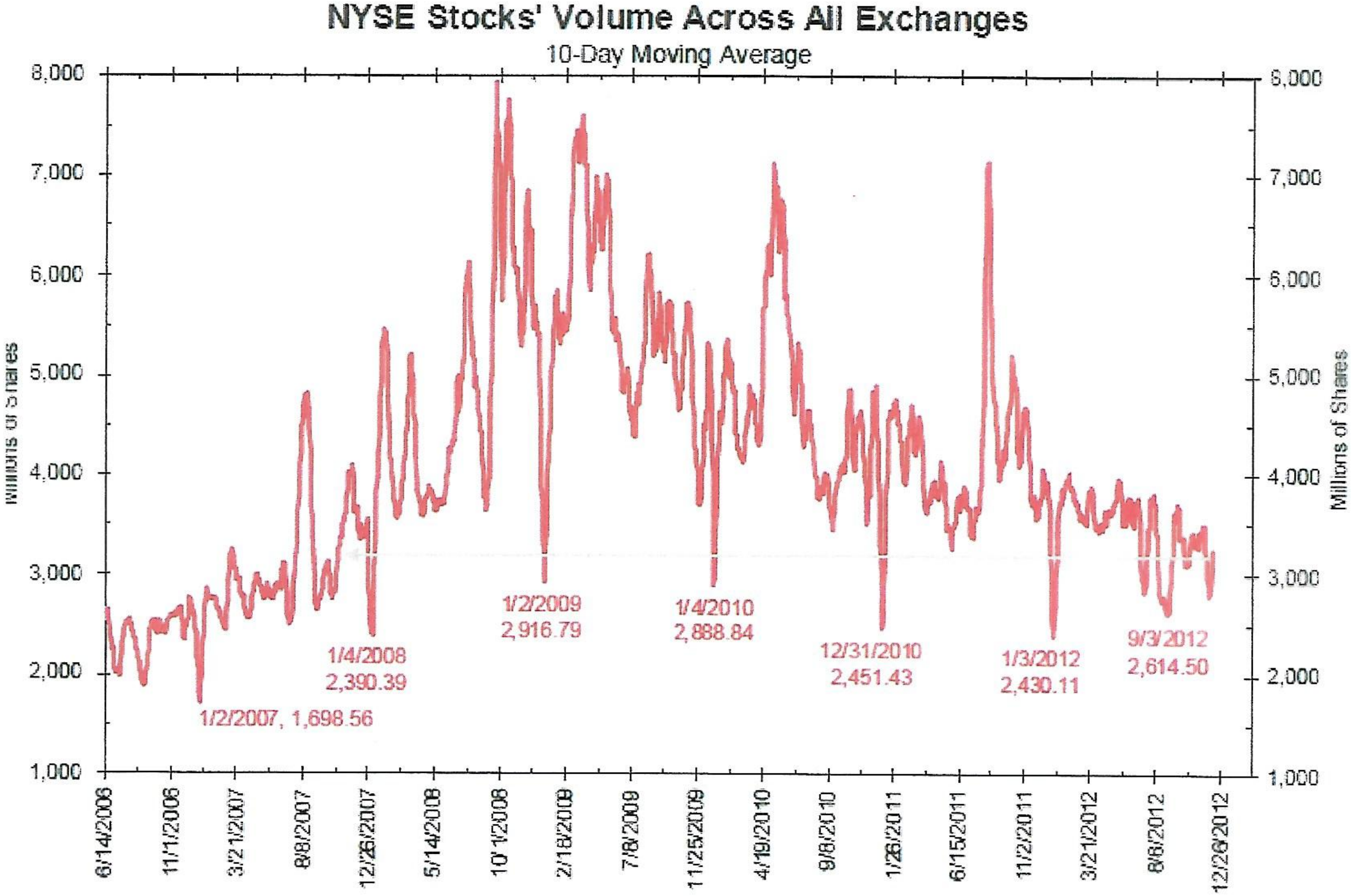
1.) Trading Volume

2.) A Simple Version of Harrison + Kreps (1978)

3.) Harrison + Kreps (1978)

4.) The Scheinkman-Xiong (2003) Model (Bubbles?)

5.) Caveats + Qualifications


NYSE Stocks' Volume Across All Exchanges
10-Day Moving Average
Millions of Shares
Millions of Shares
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
1/2/2007, 1,698.56
1/4/2008
2,390.39
1/2/2009
2,916.79
1/4/2010
2,888.84
12/31/2010
2,451.43
1/3/2012
2,430.11
9/3/2012
2,614.50
6/14/2006
11/1/2006
3/21/2007
8/8/2007
12/26/2007
5/14/2008
10/1/2008
2/18/2009
7/8/2009
11/25/2009
4/19/2010
9/8/2010
1/26/2011
6/15/2011
11/2/2011
3/21/2012
8/8/2012
12/26/2012

Trading Volume

- Trading volume is enormous. Billions of shares change hands every day on the NYSE. Volume in the forex market is even larger.

- Note, # of shares traded is a bit misleading, since it is not scale or unit invariant. A better measure is turnover rate, i.e., what percentage of the value or # of shares is traded over a given time period. Weekly turnover rates are in the range 1-2% on the NYSE.

- There are intriguing correlations between prices and trading volume. For example, volume is positively correlated with the magnitude of price changes (both positive + negative).

- There are 4 basic reasons why people might trade:

  1.) Dynamic Spanning/Hedging (Portfolio Rebalancing)

  2.) Asymmetric Info. (Requires background noise).

  3.) Heterogeneous Beliefs (Differences in opinion).

  4.) They're Crazy (or they just like to trade).

- They only one we've discussed so far is dynamic spanning (Remember Arrow (1964)). However, this seems completely inadequate to explain the magnitude of volume + its high frequency volatility.

- In principle, asym info. could be important. But the fact that trading responds to public info. suggests differences in opinion are important.

A Simple Version of Harrison & Kreps (1978)

Assumptions

1.) Single Asset, in fixed supply, normalized to 1

2.) Asset yields a sequence of dividends at discrete dates $t = 1, 2, 3, \ldots$

3.) Two ∞-lived, risk-neutral, agent types: Mr. E and Mr. O

4.) Agents have common discount factor β, & behave competitively.

5.) **Beliefs**

E believes $d_t = \begin{cases} 1 & \text{if } t \text{ is even} \\ 0 & \text{if } t \text{ is odd} \end{cases}$

O believes $d_t = \begin{cases} 0 & \text{if } t \text{ is even} \\ 1 & \text{if } t \text{ is odd} \end{cases}$

6.) No Learning. Beliefs are never revised. (Hence, we don't even need to specify what the d_t process really is!)

7.) No Capital limits (infinite wealth). Agents can always buy back all shares.

8.) Dividends are nonstorable

9.) Trading occurs at the beginning of each period, before dividends are announced.

- People often define the "fundamental value" of an asset as the expected PDV of its stream of dividends.

- This example will show that this is not correct if agents have heterogeneous beliefs.

- With (fluctuating) heterogeneous beliefs, prices will be higher than the expected PDV of their cash flows. This is because they will embody a resale option value.

- Consider period 1. Let V_0 = Mr. O's valuation and V_E : Mr. E's valuation. If agents must hold onto the asset:

$$V_0 = 1 + \beta^2 + \beta^4 + \cdots = \frac{1}{1-\beta^2}$$

$$V_E = \beta + \beta^3 + \beta^5 + \cdots = \frac{\beta}{1-\beta^2}$$

- Since Mr. O places a higher value on the asset, he will acquire all of it. (Remember, Mr. E cannot short sell).

- What about period 2? Now the roles are just reversed:

$V_0 = \frac{\beta}{1-\beta^2}$ $V_E = \frac{1}{1-\beta^2}$ $\Rightarrow$ Mr. E buys the asset from Mr. O

- So it would appear the asset price is constant at

$$P = \frac{1}{1-\beta^2}.$$

- Is $P = \frac{1}{1-\beta^2}$ really the competitive equilibrium price?

- Suppose $t=1$, and Mr. O considers the following strategy:

- Speculative Trading Strategy: Buy the asset now, collect the dividends I expect this period, then sell at $t=2$ for $P = \frac{1}{1-\beta^2}$ when I expect no dividends.

$$\text{Round 1 Expected Discounted Payoff} = 1 + \beta \frac{1}{1-\beta^2} = \frac{1+\beta-\beta^2}{1-\beta^2} > \frac{1}{1-\beta^2}$$

- Competition among O-types will therefore force the price up to $P = \frac{1+\beta-\beta^2}{1-\beta^2}$ at $t=1$. Next period, the same reasoning applies to Mr. E.

- Hence, we now predict a constant price of $P = \frac{1+\beta-\beta^2}{1-\beta^2}$. But if this is the price, what is the expected payoff from Mr. O's original strategy?

$$\text{Round 2 Expected Discounted Payoff} = 1 + \beta \frac{1+\beta-\beta^2}{1-\beta^2} = \frac{1+\beta-\beta^3}{1-\beta^2} > \frac{1+\beta-\beta^2}{1-\beta^2}$$

- Again, competition forces the price up to this higher payoff.

- Proceeding iteratively, we conclude

$$\bar{P} = \lim_{n\to\infty} \frac{1+\beta-\beta^n}{1-\beta^2} = \frac{1+\beta}{1-\beta^2} = \frac{1}{1-\beta} > \frac{1}{1-\beta^2}$$

• Note, with $\bar{P} = \frac{1}{1-\beta}$, there is no longer any advantage to the buy + sell strategy:

$$1 + \beta\left(\frac{1}{1-\beta}\right) = \frac{1}{1-\beta}$$

We have found an equilibrium.

Comments

1.) It may seem restrictive to limit the trading strategy to selling after just 1 period, but it's not.

Doob's Optimal Stopping Theorem: Expected value of a martingale at any stopping time is equal to its initial expected value. That is, if a game is fair, there are no profitable (in expectation) trading strategies.

2.) There is still a sense in which price is the expected PDV of dividends. At each date we just use the expectation of the currently most optimistic agent:

$$P = 1 + \beta + \beta^2 + \cdots = \frac{1}{1-\beta}$$

3.) The difference between the "buy + hold" price and the speculative trading price

$$\frac{1+\beta}{1-\beta^2} - \frac{1}{1-\beta^2} = \frac{\beta}{1-\beta^2} \rightarrow \text{resale option value}$$

can be interpreted as a resale option value

Harrison + Kreps (1978)

- The previous example was a little silly, because such dogmatic beliefs about dividends would be easily rejected by experience.
- HK show that the same logic applies when agents hold "more reasonable" beliefs, which would be harder (i.e., take longer) to reject.
- Let's now adopt the HK notation. The 2 traders are Mr. 1 and Mr. 2. Let $\gamma = .75$ be the common discount factor. Dividends follow a 2-state Markov Chain. Let state 0 be the 'bad' state, when $d_t(0) = 0$, and let state 1 be the 'good' state, $d_t(1) = 1$. Dividends are not received until next period.
- Beliefs are described by the following state transition matrices:

$$Q^1 = \begin{bmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{bmatrix} \quad \text{\} Mr. 1's beliefs}$$

$$Q^2 = \begin{bmatrix} 2/3 & 1/3 \\ 1/4 & 3/4 \end{bmatrix} \quad \text{\} Mr. 2's beliefs}$$

- The important point here is that Mr. 1 is more optimistic than Mr. 2 in state 0 (since he believes there is a higher prob. of being in state 1 tomorrow [$1/2 > 1/3$]). Conversely, Mr. 2 is relatively optimistic in state 1 (since $3/4 > 1/3$).

- As in the previous example, let's first compute the "buy & hold" prices:

$$P^i = (\gamma Q^i + \gamma^2 Q^{i^2} + \cdots)\begin{pmatrix}0\\1\end{pmatrix} = \gamma Q^i (I - \gamma Q^i)^{-1}\begin{pmatrix}0\\1\end{pmatrix}$$

This gives

$$P^1 = \begin{pmatrix}4/3\\11/9\end{pmatrix} = \begin{pmatrix}1.33\\1.22\end{pmatrix}$$

$$P^2 = \begin{pmatrix}16/11\\21/11\end{pmatrix} = \begin{pmatrix}1.45\\1.91\end{pmatrix}$$

- Note, Mr. 2 has a higher buy & hold price in <u>both</u> states. (He thinks the dividend process spends more time in the good state). A naive interpretation would suggest Mr. 2 buys the asset and holds it forever (i.e., there is no trading).

- Since $P^1(0) < P^2(0)$, it would seem that Mr. 1 would have no desire to buy in state 0. However, suppose he embarks on the following strategy:

<u>Speculative Trading Strategy</u>: Buy in state 0, and then sell (for $P = 1.91$) the first time you enter state 1 (where Mr 1 becomes relatively pessimistic).

$$\text{Round 1 Expected Discounted Payoff} = \left[\gamma \cdot \tfrac{1}{2} + \gamma^2 \left(\tfrac{1}{2}\right)^2 + \cdots\right](1 + 1.91) = 1.75$$

- Note, this is <u>higher</u> than $P^2(0)$!

- Proceeding iteratively as before

$$P_t^n(x_t) = \max_{a \in A} E^a \left\{ \gamma d_{t+1}(X_{t+1}) + \gamma P_{t+1}^{n-1}(X_{t+1}) \mid x_t \right\}$$

where we can start at $P_t^0 = 0$.

- In the limit, we get the following fixed point condition

$$P(0) = \gamma \max \{ \underset{\text{Mr. 1}}{\tfrac{1}{2} \cdot P(0) + \tfrac{1}{2}(1 + P(1))}, \underset{\text{Mr. 2}}{\tfrac{2}{3} P(0) + \tfrac{1}{3}(1 + P(1))} \}$$

$$P(1) = \gamma \max \{ \tfrac{2}{3} P(0) + \tfrac{1}{3}(1 + P(1)), \tfrac{1}{4} P(0) + \tfrac{3}{4}(1 + P(1)) \}$$

- Conjecture that Mr. 1 buys in 0 and Mr. 2 buys in 1. This gives 2 equations in 2 unknowns

$$P(0) = \gamma [\tfrac{1}{2} P(0) + \tfrac{1}{2}(1 + P(1))]$$
$$P(1) = \gamma [\tfrac{1}{4} P(0) + \tfrac{3}{4}(1 + P(1))]$$
$$\Rightarrow \begin{pmatrix} P(0) \\ P(1) \end{pmatrix} = \begin{pmatrix} \frac{24}{13} \\ 27/13 \end{pmatrix} = \begin{pmatrix} 1.85 \\ 2.04 \end{pmatrix}$$

- Note, price fluctuates over time <u>and</u> there is persistent trading volume. However, there is no <u>correlation</u> between price + volume here, since trading volume is constant. Also, note that speculative price fluctuations could be either greater or smaller than without trading

Scheinkman & Xiong (2003)

- SX argue that the KH model can explain bubbles. They define a bubble as the value of the resale option, and use Black-Scholes option pricing techniques to show that it exhibits cyclical fluctuations. Their paper is one of the most influential papers in finance during the past 10-20 years.

- It's an ∞-horizon model, in continuous-time. However, the basic idea can be conveyed with just 3 discrete periods.

- $d_t \in \{d_L, d_H\}$ is i.i.d.

  $\bar{d} = E(d) = .5\, d_L + .5\, d_H$

- 2 Groups of traders: A is 'rational'
  B is 'overconfident'

- In particular, B observers a "signal", which they believe has predictive content, when in fact it doesn't.

- $S_t \in \{0, 1, 2\}$.

- $P_r[d_{t+1} = d_H \mid S_t] = .5 + .25[S_t - 1]$ } B's beliefs

- B's beliefs are unbiased (in contrast to KH). Specifically, $S_t = 0$ with prob q, $S_t = 2$ with prob q, $S_t = 1$ with prob $1-2q$.

- When $S_t = 0$ or 2 B is overconfident. (Precision of beliefs too high).

- Trade occurs after dividend is paid (ex-dividend)
- Hence, $P_3 = 0$
- Period 1 buy + hold price

$$\bar{P}_1 = (\beta + \beta^2)\bar{d}$$

- When $S_2 = 2$, B are willing to pay

$$\beta[.75 d_H + .25 d_L] = \beta[\bar{d} + .25(d_H - d_L)] > \beta\bar{d}$$

- Hence,

$$EP_2 = q\beta[\bar{d} + .25(d_H - d_L)] + (1-q)\beta\bar{d}$$
$$= \beta[\bar{d} + .25q(d_H - d_L)] > \beta\bar{d}$$

- Although P_2 occasionally exceeds the 'fundamental value', this is not a bubble. It just reflects the fact that prices are determined by the most optimistic trader.
- However, a bubble can emerge in period 1, since the buyer in period 1 can re-sell to an optimistic buyer in period 2.

- Conditional Period 1 prices,

$$P_1(0) = P_1(1) = \beta(\bar{d} + E P_2)$$

$$P_1(2) = \beta[\bar{d} + .25(d_H - d_L) + E P_2]$$

- Hence, before s_1 is observed

$$E P_1 = q\beta[\bar{d} + .25(d_H - d_L) + E P_2] + (1-q)\beta[\bar{d} + E P_2]$$

$$= \beta[\bar{d} + .25q(d_H - d_L) + E P_2]$$

$$= (\beta + \beta^2)[\bar{d} + .25q(d_H - d_L)]$$

- Note, average price in Period 1 exceeds the "fundamental value" by the amount

$$(\beta + \beta^2) \times .25q(d_H - d_L)$$

The term $\beta \times .25q(d_H - d_L)$ simply reflects the fact that price will be determined by relative optimists. However, the term $\beta^2 \times .25q(d_H - d_L)$ is a bubble component. It reflects the fact that a buyer in Period 1 can resell to an optimist in Period 2.

- Note that the size of the bubble increases when the probability, q, of disagreement increases. Also, the size of the bubble decreases when the riskless interest rate increases (i.e., when β falls). Trading volume also increases with q.