Topics for Today

1.) Why Can't We Just Agree to Disagree?

- Aumann (1976) - "Agreeing to Disagree".
- Motivation
- A Quote
- A Brainteaser
- An Example
- Information Partitions and Common Knowledge
- Aumann's Theorem

Motivation

- We have talked a lot about info + beliefs. So far, this info or belief has been about fundamentals (e.g., dividends).

- Some people argue that with heterogeneous beliefs + info what really matters is beliefs about other people's beliefs!

- Whether an investment is profitable doesn't depend on what you know or believe, since you have no influence on price. It depends on what everyone else thinks. So you must forecast other people's forecasts. Of course, everyone else is in the same boat, so everyone is trying to outguess what everyone else is guessing. Average opinion becomes an object of speculation. This produces an infinite regress.

- The limit of this infinite regress is called Common Knowledge. Consider 2 individuals, A + B. An event E is Common Knowledge if A + B both know it occurred. Not only that, A knows B knows, and B knows A knows. In fact, A knows that B knows that A knows, and B knows that A knows that B knows, and so on, ad infinitum.

- When is an event or info CK? Basically, when it is public. That way everyone knows that everyone knows.

- In the last couple lectures I assumed traders had common priors. We can now be more precise. Not only did they have common priors, but this common prior was common knowledge!

A Quote

Professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligence to anticipating what average opinion expects average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.

– J.M. Keynes (1936), *The General Theory of Employment, Interest and Money*

A Brainteaser

The Blue-Eyed Suicides:

There is an island of 1000 people. 900 have brown-eyes, 100 have blue eyes. There are no mirrors on the island, and the local religion forbids all discussion of eye color. Even worse, anyone who inadvertently discovers their own eye-color must commit suicide that same day.

One day, an explorer lands on the island, and is invited to speak before the whole population. Ignorant of the local customs, he commits a faux pas: "How pleasant it is", he says, "to see another pair of blue-eyes, after all these months at sea".

What happens next?

Analysis

- Suppose there was just one blue-eyed islander, call him A. A knows from the explorer's speech that there is at least one blue-eyed islander. Since he can see no other, A concludes it must be him, so he commits suicide on the *first* day.

- Now suppose there are two, A and B. A can see B, so he knows there is at least one blue-eyed islander. Likewise for B. However, on the second day, A sees that B has not committed suicide, and therefore concludes that B must also see someone with blue-eyes. Since A can see no other, A concludes he must also have blue eyes, and so he commits suicide on the *second* day. Same with B.

- By induction, all 100 blue-eyed islanders commit suicide on the 100th day. The next day, all the brown-eyed islanders do likewise.

- Interestingly, the explorer didn't tell anyone anything they didn't already know. Instead, what he did was to provide a degree of Common Knowledge. Go back to the case of 2 blue-eyed islanders. Here the info in the announcement was "*First-Order Knowledge*" even before the speech - everyone knows it's true. But it was not "Second-Order Knowledge", because A does not know that B knows (since A doesn't know his own eye-color).

- Note, if there were three blue-eyed islanders then the info in the statement *would* be 2nd-order Knowledge. Each would know that the others know there is at least one blue-eyed islander. (However, it wouldn't be 3rd-order knowledge!)

An Example: Currency Crises & Imperfect Common Knowledge

- Assume there is some value of reserves, R, below which the Central Bank devalues for sure; call it $\underline{R}$.

- Assume speculators do not observe R

- Instead, each speculator receives a noisy signal of R (uniformly distributed around R).

$$S_i \in [R - \varepsilon, R + \varepsilon] \text{ where } \varepsilon \text{ is "small"}$$

- Note, signals are correlated across speculators. Your signal tells you about R and it gives you information about other people's information.

$$\boxed{\text{Key Point: It is never Common Knowledge that the peg is sustainable.}}$$

Note: Signals can differ across individuals by at most 2ε

1.) First-Order Knowledge of Sustainability
(you know the peg is sustainable)

$$S_i \geq \underline{R} + \varepsilon$$

2.) 2nd-Order Knowledge of Sustainability
(you know that everyone else knows the peg is sustainable)

$$S_i \geq \underline{R} + 3\varepsilon$$

> smallest signal someone else could have conditional on $S_i = \underline{R} + 3\varepsilon$ is $S_i = \underline{R} + \varepsilon$

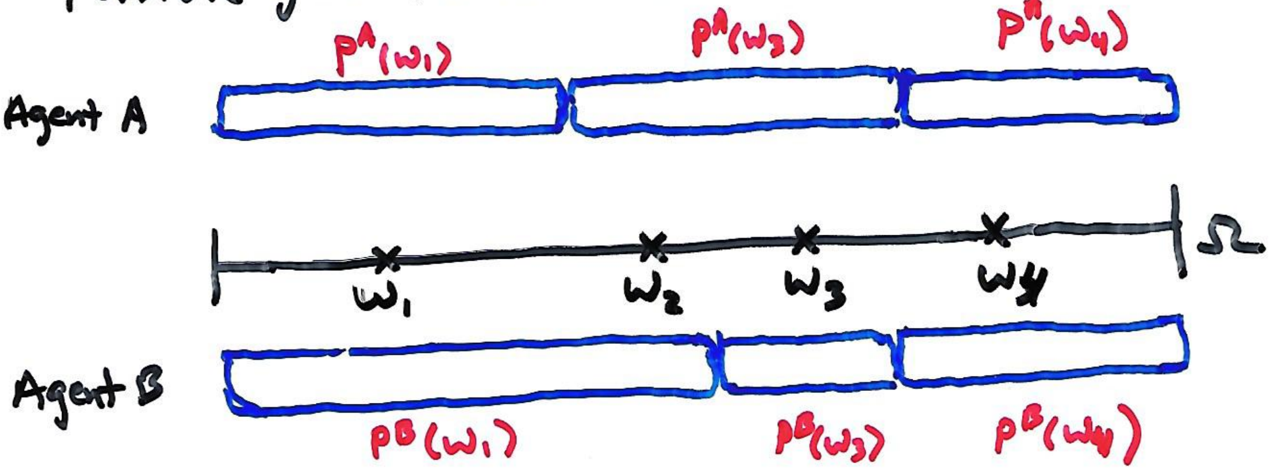
3.) 3rd-Order Knowledge of Sustainability
(you know that everyone else knows that everyone else knows the peg is sustainable)

$$S_i \geq \underline{R} + 5\varepsilon$$

$$\boxed{\text{Common Knowledge of Sustainability} \Rightarrow \text{Infinite-Order Knowledge!}}$$

Information Partitions and Common Knowledge

- Events : Set of underlying states
- Information allows agents to <u>rule out</u> certain states
- An agent's info can be represented as a partition of the state space. Information is better when the partition is "finer" (i.e., consists of smaller subsets). In general, agents have different partitions.
- Let $P^A(\omega)$ = Mr A's "possibility set" (the states A thinks are possible given the actual state ω.



- Suppose Partition's are CK and ω_1 is the state.
- A thinks any $\omega \in P^A(\omega_1)$ is possible. Hence he can rule out ω_2
- However, he knows B thinks ω_2 is possible
- Therefore event $P^A(\omega_1)$ is <u>not</u> CK. What about $P^B(\omega_1)$?
- Consider ω_3. A & B both know that ω_3 is not the true state. However, A knows B might think A thinks ω_3 is possible. Therefore, $P^B(\omega_1)$ is <u>not</u> CK either
- What <u>is</u> CK is $P^B(\omega_1) \cup P^B(\omega_3)$ [and $P^A(\omega_1) \cup P^A(\omega_3)$]. Events outside this set are CK. Note, CK is in general a "coarsening" of each agent's info partition.

Aumann's Theorem

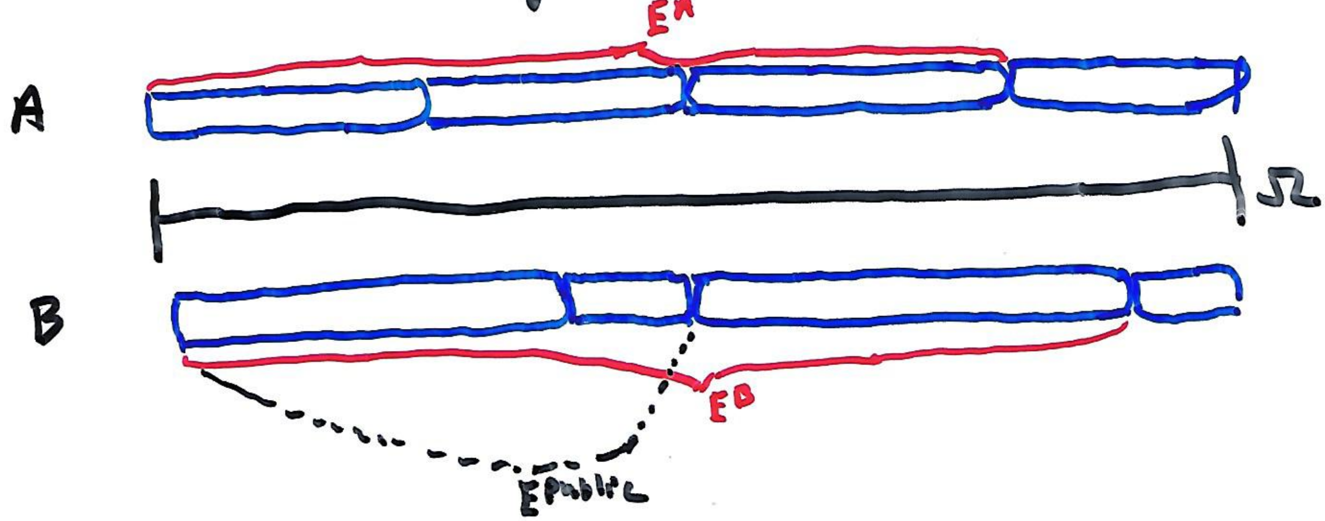
- Suppose the following is true:

1.) Agent's are rational
2.) This rationality is CK
3.) Agents have common priors
4.) This common prior is CK

Then Aumann's Theorem says the following:

If agents' posteriors are CK, then they must be the <u>same</u>, even if they are based on different information. That is, agents <u>cannot</u> agree to disagree.

- Consider the following picture



- Let q_A = A's posterior for the event E
  q_B = B's posterior for the event E

- These posteriors partition each agent's info set into those sets that generate q_i and those that don't. Call them E^A + E^B.

- From the previous slide, if posteriors are CK, then ω must be in the "meet" of E^A + E^B. Call this E^{public}. Since $q^A = \frac{P(E \cap P_i^A)}{P(P_i^A)}$ for all $P_i \in E^{public}$ [oops, better get another slide!]

- Since $q^A = \dfrac{P(E \cap P_i^A)}{P(P_i^A)}$ for all $P_i^A \in E^{public}$

we know $q^A P(E^{public}) = \sum P(E \cap P_i^A) = P(E \cap E^{public})$

(since $\sum P(P_i^A) = P(E^{public})$)

- Now, the same applies to B.

$$q^B = \frac{P(E \cap P_i^B)}{P(P_i^B)} \quad \text{for all } P_i^B \in E^{public}$$

$$\Rightarrow \quad q^B P(E^{public}) = \sum P(E \cap P_i^B) = P(E \cap E^{public})$$

$$\Rightarrow \quad q^A = q^B \quad !$$

Comments

1.) Saying their posteriors are the same is not the same as saying they have the same info. Sharing posteriors is not the same thing as sharing info. If they pooled info, the common posterior would in general be different.

2.) Aumann doesn't say much about how posteriors are supposed to become CK. A dynamic process of reporting and revising can sometimes work.