Topics for Today

1.) Speculative Trading in Financial Markets

- Motivation
- An Example from Milgrom + Stokey (1982)
- Tirole's No Trade Theorem
- Ruling Out Speculative Bubbles

Motivation

- At the beginning of the course we saw from Arrow (1964) how dynamic trading of securities could be used to achieve and sustain a Pareto Optimal allocation of risk.

- Now at the end of the course we'll see that there is a sense in which this is the *only* reason we should observe trade in financial mkts.

- From Harrison & Kreps (1978), we know agents will trade if they have different priors. However, unless their prior beliefs are "dogmatic", this trade should dissipate over time as agents learn and their beliefs "merge" together.

- Today we'll see that asymmetric info, *by itself*, cannot generate trade (among rational individuals). Tirole (1982) shows that if agents have already achieved a Pareto optimal allocation of risk, then any future introduction of asymmetric info will *not* generate trade. Instead, prices effectively reveal all relevent info *without* trade.

- The intuition is simple - If a pre-existing allocation is Pareto Optimal, then additional trade becomes a Zero-sum game. If new info enters the market, the *only* reason someone would want to trade is the anticipation of a speculative gain (necessarily, at someone else's expense). Just as not everyone can be above average, not everyone can expect to profit from trade (given common priors). Since everyone knows this, no one trades. [Yet another example of Akerlof's Lemons Problem!].

An Example from Milgrom + Stokey (1982) - "Information, Trade, + Common Knowle

- All No Trade Theorems rely on CK assumptions. Common Knowledge is based on quite sophisticated reasoning. We saw that last time.

- Milgrom + Stokey base their No Trade Theorem more directly on Aumann's Agreement Theorem, which we proved last time, so an example from their paper is a good place to start.

- Consider 2 agents: Mr. 1 and Mr. 2.

  Let θ = payoff relevant state (e.g., asset payoff)

  x = signal correlated with θ

- Suppose agents have the following common prior for the joint distribution of (θ, x):

	$\theta = 1$	$\theta = 2$
$x = 1$	.20	.05
$x = 2$	.05	.15
$x = 3$	.05	.05
$x = 4$	.15	.05
$x = 5$	.05	.20

- Note, $x = \{1,4\}$ are favorable for $\theta = 1$ and $\{2,5\}$ are favorable for $\theta = 2$

- Suppose agents have the following information partitions:

P_1 : $\{x = 1 \text{ or } 2\}$, $\{x = 3 \text{ or } 4\}$, $\{x = 5\}$

P_2 : $\{x = 1\}$, $\{x = 2 \text{ or } 3\}$, $\{x = 4 \text{ or } 5\}$

- These partitions are CK - You don't know what the other guy knows, but you know what he could know.

- Consider the following bet:

  If $\theta = 1$, then Mr 2 pays \$1 to Mr. 1

  If $\theta = 2$, then Mr 1 pays \$1 to Mr. 2

- Suppose $x = 3$ occurs, and consider the following increasingly sophisticated levels of reasoning:

Case A: Optimize given your own info.

Case B: Optimize given your own info + what you know about what the other guy knows given your info.

Case C: Optimize given CK of rationality + priors

Analysis

Case A:

At $x = 3$, $P(\theta = 1 \mid P_1) = 2/3$

(Mr 1 Knows $x = \{3, 4\} \Rightarrow \frac{P(\theta=1)}{P(\theta=2)} = \frac{.20}{.10} \Rightarrow P(\theta=1) = 2/3$)

Similarly, $P(\theta = 2 \mid P_2) = 2/3$

(Mr 2 Knows $x = \{2, 3\} \Rightarrow \frac{P(\theta=2)}{P(\theta=1)} = \frac{.20}{.10} \Rightarrow P(\theta=2) = 2/3$)

Therefore, both believe they will win the bet, and trade occurs.

Case B:

Mr 1 reasons as follows - "I Know $x = \{3, 4\}$. If $x = 3$, then Mr 2 thinks $x = \{2, 3\}$, which implies $P(\theta=2 \mid P_2) = 2/3$, so I know Mr 2 will accept the bet. If $x = 4$, then Mr 2 thinks $x = \{4, 5\}$, which implies $P(\theta=2 \mid P_2) = 5/9$ (since $\frac{P(\theta=2)}{P(\theta=1)} = \frac{.25}{.20} = 5/4$)

So again I expect him to accept. Therefore, Mr 2's acceptance tells me nothing new, so I will accept."

This same line of reasoning applies from Mr 2's perspective. He Knows $x = \{2, 3\}$. If $x = 2$, then Mr. 1 thinks $x = \{1, 2\} \Rightarrow P(\theta = 1 \mid P_1) = 5/9$, while if $x = 3$, then Mr 1 thinks $x = \{3, 4\} \Rightarrow P(\theta = 1 \mid P_1) = 2/3$. So Mr 2 expects Mr 1 to accept as well.

Case C:

Mr 1 now reasons as follows - "If $x=1$, Mr 2 Knows $x=1$, and will therefore refuse the bet (since $P(\theta=2|x=1)=.05$). Hence, if Mr 2 accepts, I know $x \neq 1$. Therefore, if I observe $\{1,2\}$ and Mr 2 accepts, then $x=2$. But $x=2$ is bad for me (since $P(\theta=1|x=2)=.05$). Therefore, if I observe $\{1,2\}$ I should refuse the bet. I will obviously reject as well if I observe $\{x=5\}$ (since $P(\theta=1|x=5)=.05$)."

Mr 2 applies the same reasoning to refuse the bet if he observes $\{4,5\}$. For if $\{x=5\}$, Mr 1 Knows it, and will therefore reject, so Mr 1's acceptance signals $x=4$, which is bad for Mr 2. Therefore, Mr 2 rejects if $x=\{4,5\}$. He will also clearly reject if $\{x=1\}$

We can therefore conclude the following:

Mr 1 rejects if he observes $\{1,2\}$ or $\{5\}$

Mr 2 rejects if he observes $\{4,5\}$ or $\{1\}$

Hence, the bet is only accepted by both if $x=3$. But if $x=3$, the bet is a "fair bet" (50/50), and will be rejected by both if they are only slightly risk averse

$\Longrightarrow$ No Trade!

Tirole's (1982) No Trade Theorem

- The Milgrom-Stokey argument is clearly quite subtle + complex. But it's the sort of process small groups of rational individuals must go through given CK of priors.

- Financial markets are (arguably) anonymous. Individuals don't reason about what other individual traders know or do. Instead, agents draw inferences from market-clearing prices.

- Tirole provides a No Trade Theorem in a dynamic market setting. He proves that if the following conditions apply:

  1.) Finite Number of Potential Traders

  2.) Common Knowledge of Rationality

  3.) Common Knowledge of a Common Prior

  4.) Pareto Optimal Allocation prior to info arrival

  Then no trade will occur, even if agents have private info.

- The key condition is (4). It makes trade a Zero-Sum Game. Moreover, this fact is Common Knowledge. Common Knowledge that trade is a Zero-Sum game is the key part of his proof.

- Let s_t^i = Mr i's signal at time-t

  $S(P_t)$ = What can be inferred about the signal profile given mkt. clearing prices.

Proposition 2 (Tirole) : Consider an asset with fixed supply, $\bar{x}$.

Given the above assumptions,

$$P_t = E[\gamma d_{t+1} + \gamma P_{t+1} \mid s_t^i, S(P_t)] \quad \forall \text{ traders } i$$

Therefore, no trader can expect to profit by trading on his private info, s_t^i.

Proof :

Let x_t^i = Mr i's asset holdings

Let $g_t^i = -P_t \Delta x_t^i$ and ${}_t g_{t+1}^i = [P_{t+1} + d_{t+1}] \Delta x_t^i$ denote the change in Mr i's cash flows at t and $t+1$ resulting from his time-t trade $\Delta x_t^i = x_t^i - x_{t-1}^i$

From Market-Clearing,

$$\sum_i g_t^i = 0 \quad \text{and} \quad \sum_i {}_t g_{t+1}^i = 0$$

> since $\sum \Delta x_t^i = 0$

Therefore,

$$\sum_i [g_t^i + \gamma \, {}_t g_{t+1}^i] = 0$$

- By "Conditioning down"

$$\sum_i E[g^i_t + \gamma_t g^i_{t+1} \mid S_t] = 0$$

> if something equals 0 for all realizations, it equals 0 in expectation

- From CK of rationality and Pareto Optimality,

$$E[g^i_t + \gamma_t g^i_{t+1} \mid s^i_t, S_t] \geq 0$$

- By the "law-of-iterated expectations" (integrating over s^i_t),

$$E[g^i_t + \gamma_t g^i_{t+1} \mid S_t] = E[E(g^i_t + \gamma_t g^i_{t+1} \mid s^i_t, S_t) \mid S_t] \geq 0$$

- Therefore, we know
  - a.) Mkt. Clearing requires $\sum_i E[g^i_t + \gamma_t g^i_{t+1} \mid S_t] = 0$
  - b.) But each element of the sum must be non-negative
  - c.) Therefore, they all must be 0.
  - d.) Therefore $E[g^i_t + \gamma_t g^i_{t+1} \mid s^i_t, S_t] = 0 \quad \forall i$

Comments

1.) Tirole doesn't say much about γ. Implicitly, it is a dynamic stochastic discount factor. With Pareto Optimality, it is common across traders. That is sufficient for Tirole's argument to apply.

2.) Later, Tirole uses the same argument to rule out speculative bubbles when there are a finite number of traders.