Topics for Today

1.) Assumptions of the CAPM

2.) Market Equilibrium and the CAPM
- The "Capital Market Line"

3.) Pricing Implications of the CAPM
- Betas

4.) The Security Market Line
- Systematic vs. Idiosyncratic Risk

5.) Using the CAPM

6.) Testing the CAPM

Assumptions of the CAPM

1.) One-period investment horizon (all end-of-period wealth is consumed)

2.) Homogeneous Beliefs

3.) Investors are Mean-Variance optimizers

4.) No non-traded assets (e.g., labor income)

5.) All investors can borrow + lend at the same risk-free rate.

6.) No taxes or transaction costs.

Market Equilibrium and the CAPM

- Last time we saw that when the set of assets includes a riskless asset, the MV frontier consists of a straight line from the riskless asset to a single 'tangency portfolio'. That is, we obtained a "One-Fund Theorem".

- Everybody should invest in the same portfolio of risky assets! (of course, the amount they invest will generally differ, depending on wealth + preferences).

- However, if everybody invests in the same portfolio of risky assets, how do we know that an equilibrium exists? How do we know that the demand for each asset equals its supply? This is the question Sharpe (1964) addressed.

- Sharpe argued that in order for Supply to equal Demand, the tangency portfolio must be the 'market portfolio'.

Definition: The Market Portfolio consists of the value-weighted sum of all assets. That is, the weight of each asset is its share of total market capitalization.

- Note, what matters is the market-value of each asset, not the number of shares outstanding. (i.e., price per share × Number of Shares).

- Since the sum of the wealth of all investors equals total market wealth, if each investor invests in the market Portfolio, the total demand for each asset will equal the total supply.

- <u>Example</u>: Suppose total market wealth is 1000, and there are 2 assets (a1, a2). The market value of a1 = 400 and the market value of a2 = 600. Suppose there are 50 investors, each with wealth 20. If each invests in the Market Portfolio, each invests 8 (= .4 × 20) in a1, and 12 (= .6 × 20) in a2. Total demand for a1 = $\sum_{i=1}^{50} 8 = 400$ and total demand for a2 = $\sum_{i=1}^{50} 12 = 600$. The market clears!

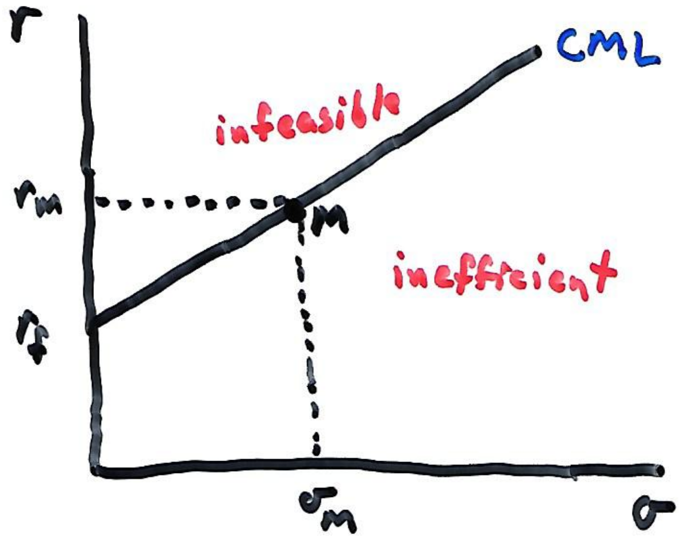
- What if investors do <u>not</u> hold the market Portfolio? Then some assets would be in excess demand ⇒ Their prices would rise ⇒ their expected returns fall ⇒ demand declines. Conversely, other assets would be in excess supply ⇒ Their prices would fall ⇒ Their expected returns rise ⇒ demand increases. (Somebody won a Nobel Prize for this!).

The Capital Market Line

- We can now refine our previous picture of the MV frontier
- Let r_m = expected return on the Market Portfolio

  σ_m = st. dev. of the Market Portfolio

- With the Market Portfolio as the tangency portfolio, we can now plot the Capital Market Line



- All efficient portfolios lie on the CML

$$\boxed{\text{slope of CML} = \frac{r_m - r_f}{\sigma_m} = \text{Price of Risk}}$$

- Let σ be the st. dev. of some asset or portfolio. Then it will be efficient iff

$$\boxed{r = r_f + \left(\frac{r_m - r_f}{\sigma_m}\right)\sigma}$$

- Note, in equilibrium, no one needs to solve the Markowitz portfolio problem. You just need to hold the Market Portfolio!

Pricing Implications of the CAPM

- Part of the reason the CAPM is so *popular* is that it makes predictions about the equilibrium returns on individual assets. That is, it is a pricing model.

Proposition: If the Market Portfolio is efficient, then the equilibrium returns on all assets is given by

$$\boxed{r_i = r_f + \beta_i (r_m - r_f)} \quad \text{The CAPM}$$

where $\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$ and $\sigma_{im} = \mathrm{cov}(r_i, r_m)$.

Proof: Consider a portfolio consisting of asset i and the Market Portfolio. Let α = share invested in asset i. Then,

$$r_\alpha = \alpha r_i + (1-\alpha) r_m$$

$$\sigma_\alpha = \sqrt{\alpha^2 \sigma_i^2 + 2\alpha(1-\alpha)\sigma_{im} + (1-\alpha)^2 \sigma_m^2}$$

As α varies, we trace out a curve from r_i to r_m. This curve must become tangent to M as $\alpha \to 0$. If it crossed the CML, then this would imply that M was not efficient.

$$\frac{dr_\alpha}{d\alpha} = r_i - r_m \qquad \frac{d\sigma_\alpha^2}{d\alpha} = 2\sigma_\alpha \frac{d\sigma_\alpha}{d\alpha} = 2\alpha\sigma_i^2 + (2-4\alpha)\sigma_{im} - 2(1-\alpha)\sigma_m^2$$

Therefore, $$\left.\frac{dr_\alpha}{d\sigma_\alpha}\right|_{\alpha=0} = \frac{dr_\alpha/d\alpha|_{\alpha=0}}{d\sigma_\alpha/d\alpha|_{\alpha=0}} = \frac{(r_i - r_m)\sigma_m}{\sigma_{im} - \sigma_m^2}$$

Tangency requires this to be the same as the slope of the CML

$$\frac{r_m - r_f}{\sigma_m} = \frac{(r_i - r_m)\sigma_m}{\sigma_{im} - \sigma_m^2} \Longrightarrow r_i = r_f + \frac{\sigma_{im}}{\sigma_m^2}(r_m - r_f)!$$

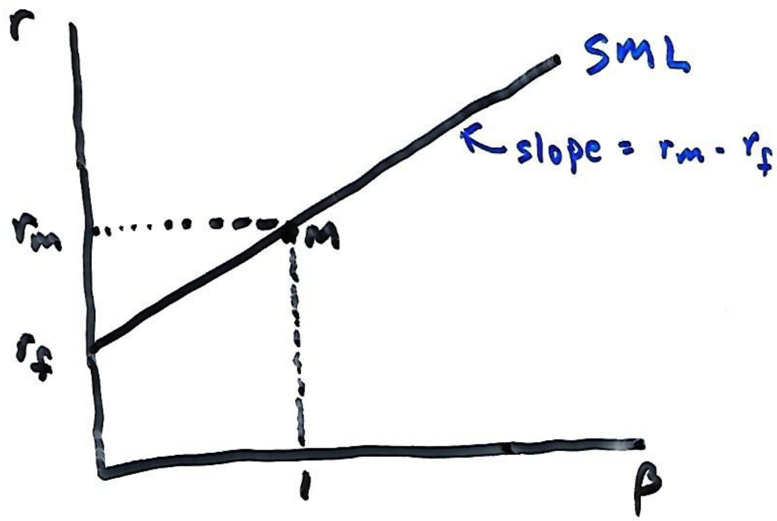
We just proved the CAPM

The Security Market Line

- According to the CAPM, an asset's β provides the appropriate measure of an asset's risk.

- By definition, $\beta_m = 1$. Assets with $\beta > 1$ move more than the market, and so they require a higher expected return than the market. Conversely, assets with $\beta < 1$ move less than the market, and so receive lower expected returns.

- According to the CAPM, if we plot asset mean returns against their β's, we should get a straight line. This predicted relationship is called the Security Market Line.



- Key Point: Risk is measured by a covariance, not a variance. The part of ~~an~~ asset's variance that is uncorrelated with the market is diversifiable, and will not be compensated by the market. From the CAPM,

$$r_i = r_f + \beta_i(r_m - r_f) + \varepsilon_i \implies \sigma_i^2 = \underbrace{\beta^2\sigma_m^2}_{\text{systematic risk}} + \underbrace{var(\varepsilon_i)}_{\text{idiosyncratic risk}}$$

- Implication: An asset can be very volatile and still have a low expected return, if it is weakly correlated with the market.

Using the CAPM

- The CAPM has many other uses, besides its use as an equilibrium pricing model.
- Here are a couple:

1.) **Performance Evaluation**: In practice, not everyone holds the Market Portfolio. However, given the presence of index funds, which proxy the Market Portfolio, we can use the CAPM to evaluate whether portfolio managers are "beating the market". We know we can't just look at average returns, since high returns might be just compensating for a lot of risk. Is a portfolio manager getting positive risk-adjusted excess returns? Or, is he underperforming? We just need to plot his average (r, σ) on the CML!

2.) **Project Evaluation**: Often assets are non-traded, or one must decide whether to undertake a risky project. How much should you pay for a non-traded asset? How should you decide whether to undertake a risky project? If we can form an estimate of the β of the cash flows, we can use the CAPM to compute an appropriate risk-adjusted discount rate, which can then be used to compute net Present Values.

Testing the CAPM

- Many thousands of finance professors have spent their careers testing the CAPM. It has probably been tested more than Quantum Mechanics + General Relativity!

- Initial tests in the early 1970s were favorable. Since then, numerous "anomalies" have been detected. Among the best known:
  1.) The January effect
  2.) The Small Firm effect
  3.) Book-to-Market Value effect
  4.) Mean Reversion/Momentum effects (violates i.i.d. assumption).

- The most common testing strategy consists of the following 3 steps:
  1.) Use Time-Series regressions to estimate each asset's β
  2.) Run (a sequence of) cross-sectional regressions of returns on β's.
  3.) Check whether the intercepts in the cross-sectional regressions are 0. Compare the slope to the market excess return.

- Because β's are estimated, they contain measurement error. Using them as regressors in the 2nd-stage cross-sectional regressions produces "attenuation bias". In response, most tests use grouped data (i.e., portfolios) in an effort to mitigate measurement error.