Soln. to PS1/Q1

a.) **HJB Eq.**: $\rho V(B) = \max_c \left\{ -\frac{1}{\gamma} e^{-\gamma c} + V_B \cdot (rB + \bar{y} - c) + \frac{1}{2}\sigma^2 V_{BB} \right\}$

b.) **FOC**: $e^{-\gamma c} = V_B$ Guess: $V = -\frac{1}{\gamma} e^{-\gamma A_1 B + A_2} \Rightarrow V_B = A_1 e^{-\gamma A_1 B + A_2}$

$$\Leftrightarrow c = -\frac{1}{\gamma}\left[\log(A_1) + A_2\right] + A_1 B$$

sub back into HJB

$$-\frac{\rho}{\gamma} e^{-\gamma A_1 B + A_2} = -\frac{1}{\gamma} e^{\log(A_1) - \gamma A_1 B + A_2} + A_1 e^{-\gamma A_1 B + A_2}\left[rB + \bar{y} + \frac{1}{\gamma}(\log(A_1) + A_2) - A_1 B\right] - \frac{1}{2}\sigma^2 A_1^2 \gamma e^{-\gamma A_1 B + A_2}$$

$$\Rightarrow \rho = A_1 - \gamma A_1\left[rB + \bar{y} + \frac{1}{\gamma}(\log(A_1) + A_2) - A_1 B\right] + \frac{1}{2}\sigma^2 A_1^2 \gamma^2$$

Match Coeffs. $\Rightarrow$ $A_1 = r$

$$A_2 = 1 - \rho/r - \gamma\bar{y} - \log(r) + \frac{1}{2}\sigma^2\gamma^2 r$$

c.)

Therefore, $\boxed{c = rB + \bar{y} + \underbrace{\frac{1}{\gamma}(\rho/r - 1)}_{\text{intertemp. subst.}} - \underbrace{\frac{1}{2}\sigma^2\gamma r}_{\text{precautionary saving}}}$

d.) Using the mkt. clearing conds. $\int dB_i = 0$ $\int B_i = 0$ $\int dZ_i = 0$ in the budget constraint gives the following eq. for the mkt.-clearing r.

$$\bar{y} - \int c_i = 0 \Rightarrow \frac{1}{\gamma}(\rho/r - 1) = \frac{1}{2}\sigma^2\gamma r$$

$$\Rightarrow \boxed{\frac{1}{2}\sigma^2\gamma^2 r^2 = \rho - r}$$

e.) $(1 + \sigma^2\gamma^2 r)\frac{dr}{d\sigma^2} = -\frac{1}{2}\gamma^2 r^2$ ✓

$$(1 + \sigma^2\gamma^2 r)\frac{dr}{d\gamma} = -\gamma\sigma^2 r^2$$