

SIMON FRASER UNIVERSITY
Department of Economics

Econ 815
Financial Economics, I

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Fall 2015

PROBLEM SET 1 - Dynamic Optimization and State-Contingent Claims
(Due October 13)

1. (10 points). In class we solved a stochastic growth model with Constant Relative Risk Aversion (CRRA) preferences. This problem asks you to solve a stochastic growth model with Constant Absolute Risk Aversion (CARA) preferences.

Consider an agent who wants to solve the following problem:

$$\max_c E \int_0^\infty e^{-\rho t} u(c) dt \quad \text{where} \quad u(c) = \frac{-1}{\gamma} e^{-\gamma c}$$

The parameter γ is called the coefficient of absolute risk aversion. The capital stock, k , evolves according to the following stochastic differential equation

$$dk = (\mu k - c) \cdot dt + \sigma dW$$

where dW is the increment to a Wiener process, and μ and σ are constant parameters.

- (a) Write down the agent's (stationary) HJB equation.
- (b) Use a guess-and-verify strategy to solve the HJB equation. (Hint: Try the guess $V(k) = -\gamma^{-1} e^{Ak+B}$, where A and B are undetermined coefficients.)
- (c) Given your answer to part (b), write down the agent's optimal consumption/savings policy. Interpret your answer in terms of intertemporal substitution and precautionary saving.
2. (10 points). Consider a world with just two 'states' - Rain or Shine. There are two firms in the economy - an Umbrella manufacturer and a Sunglasses manufacturer. The share price of the Umbrella manufacturer is \$7 and the share price of the Sunglasses manufacturer is \$4. Assume their state-contingent profits are as follows:

	Rain	Shine
Umbrellas	2	1/2
Sunglasses	0	2

- (a) What are the (implicit) state-contingent claims prices (ie, the price of \$1 if and only if a given state occurs)?
- (b) Given your answer to part (a), what must be the price of a risk-free asset?