

SIMON FRASER UNIVERSITY
Department of Economics

Econ 815
Financial Economics, I

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Fall 2016

PROBLEM SET 1 - Dynamic Optimization and State-Contingent Claims
(Due October 13)

1. (10 points). **Uncertainty and Exhaustible Resources.** Consider an exhaustible resource that evolves according to the following stochastic differential equation:

$$dR = -xR \cdot dt + \sigma R \cdot dW$$

where R is the current stock of the resource, and x is the current *rate* of extraction. Hence, xR is the current rate of consumption. As usual, dW represents the increment to a Wiener process. This question asks you to study the effects of uncertainty, as parameterized by σ , on the optimal rate of extraction.

Suppose the objective is to maximize the expected discounted utility from the use of the resource,

$$\max_x E \left\{ \int_0^\infty e^{-rt} u(xR) dt \right\} \quad R(0) = R \text{ given}$$

and that preferences take the Constant Relative Risk Aversion form $u(xR) = \frac{(xR)^{1-\gamma}}{1-\gamma}$, where γ is the coefficient of relative risk aversion. For simplicity, suppose the interest rate r is fixed over time.

- (a) Write down the (stationary) HJB equation.
 - (b) Use a guess-and-verify strategy to solve the HJB equation. (Hint: Try the guess $V(R) = A \frac{R^{1-\gamma}}{1-\gamma}$, where A is an undetermined coefficient).
 - (c) Given your answer to part (b), derive the optimal extraction policy x . Does it depend on R ? Why or why not? Will the resource ever be depleted?
 - (d) How does x depend on r ? Explain. (Hint: Google ‘Hotelling’s Rule’).
 - (e) How does x depend on σ^2 ? Does greater uncertainty increase or decrease the optimal rate of extraction? How does your answer depend on the value of γ ?
2. (10 points). Consider a world with just two ‘states’ - Clinton or Trump. There are two firms in the economy - a manufacturer of email servers and a casino. The share price of the server manufacturer is \$34 and the share price of the casino is \$28. Assume their state-contingent profits are as follows:

	Clinton	Trump
Servers	4	1/2
Casino	1	5

- (a) What are the (implicit) state-contingent claims prices (ie, the price of \$1 if and only if a given state occurs)?
- (b) Given your answer to part (a), what must be the price of a risk-free asset?