

SIMON FRASER UNIVERSITY
Department of Economics

Econ 815
Financial Economics, I

Prof. Kasa
Fall 2017

PROBLEM SET 1 - Dynamic Optimization and State-Contingent Claims
(Due October 19)

1. (10 points). **Uncertainty and Saving.** Consider a household with the following budget constraint:

$$dw = (\mu w - c) \cdot dt + \sigma w \cdot dB$$

where w is the household's current wealth, and c is the current rate of consumption. The constant parameter μ is the rate of return on investment, and σ is the standard deviation of the return. As usual, dB represents the increment to a Brownian motion (or Weiner) process. This question asks you to study the effects of uncertainty, as parameterized by σ , on the optimal savings rate.

Suppose the household has an infinite horizon, and its objective is to maximize expected discounted utility, with preferences that exhibit constant relative risk aversion,

$$\max_c E \left\{ \int_0^\infty e^{-rt} \frac{c^{1-\gamma}}{1-\gamma} dt \right\} \quad w(0) = w_0 \text{ given}$$

where γ is the coefficient of relative risk aversion. For simplicity, suppose the rate of time preference r is fixed over time.

- (a) Write down the (stationary) HJB equation.
- (b) Use a guess-and-verify strategy to solve the HJB equation. (Hint: Try the guess $V(w) = A \frac{w^{1-\gamma}}{1-\gamma}$, where A is an undetermined coefficient). (Hint: If you're having trouble, see Lecture Slides 4 (pages 5-6)).
- (c) Given your answer to part (b), derive the optimal savings rate $s = 1 - c/w$. Does it depend on w ? Why or why not?
- (d) How does s depend on σ^2 ? Does greater uncertainty increase or decrease the optimal savings rate? How does your answer depend on the value of γ ? Explain intuitively.
2. (10 points). Consider a world with just two 'states' - Clinton or Trump. There are two firms in the economy - a manufacturer of email servers and a casino. The share price of Apple is \$21 and the share price of Samsung is \$20. Assume their state-contingent profits are as follows:

	War	Peace
Apple	3	5
Samsung	1	6

- (a) What are the (implicit) state-contingent claims prices (ie, the price of \$1 if and only if a given state occurs)?
- (b) Given your answer to part (a), what must be the price of a risk-free asset?