SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 — Prof. Kasa
Financial Economics, I — Summer 2019

PROBLEM SET 1 - Dynamic Optimization and State-Contingent Claims
(Due June 13)

1. (10 points). The Merton model has two limitations: (1) it views asset returns as exogenous, and (2) it ignores nonmarket income. This question relaxes both of these assumptions. It is an example of what Ljungqvist & Sargent call a 'Bewley model'.

   Suppose there are 'large number' of agents. Agents can borrow and lend at a market-determined risk free rate. Market-clearing requires that the aggregate saving in the economy be zero. Each agent has risky labor income that follows the process

$$dY_i = \bar{y}dt + \sigma dZ_i$$

   where dZ_i is an increment to a Brownian motion. The dZ_i's are uncorrelated across agents. Each agent wants to solve the following problem:

$$\max_c E\int_0^\infty e^{-\rho t}u(c)dt \qquad \text{where} \qquad u(c) = \frac{-1}{\gamma}e^{-\gamma c}$$

   The parameter γ is called the coefficient of absolute risk aversion. An agent's bond holdings, $B_{i,t}$, evolves according to the following stochastic differential equation

$$dB_i = (rB_i + \bar{y} - c_i)\cdot dt + \sigma dZ_i$$

   where r is the market-clearing interest rate.

   (a) Write down the agent's (stationary) HJB equation.

   (b) Use a guess-and-verify strategy to solve the HJB equation. (Hint: Try the guess $V(B) = -\gamma^{-1}e^{-\gamma A_1 B + A_2}$, where A_1 and A_2 are undetermined coefficients. Compute them.

   (c) Given your answer to part (b), write down the agent's optimal consumption/savings policy. Interpret your answer in terms of intertemporal substitution and precautionary saving.

   (d) Remembering that the saving of agent-i is $dB_i = (rB_i + \bar{y} - c_i)\cdot dt + \sigma dZ_i$, impose market-clearing and compute the equilibrium value of r. (Hint 1: Note the law of large numbers implies $\int_0^1 dZ_i di = 0$. Hint 2: You should get a quadratic equation for the *constant* equilibrium value of r).

   (e) Using comparative statics, verify that $\frac{dr}{d\gamma} < 0$ and $\frac{dr}{d\sigma^2} < 0$. Explain the economic intuition behind these results.

2. (10 points). Suppose there are two states of the world, s_1 and s_2. Also suppose there are two assets: (1) A risky asset that pays 1 unit in s_1 and 3 units in s_2, and (2) A riskless asset that pays 1 unit in both states. Assume the riskless asset is in zero net supply. There are two agents: (1) A risk neutral agent with utility function $U = E(c)$, and (2) A risk averse agent with utility function $U = E\sqrt{c}$. Both agents assign equal probabilities to s_1 and s_2, and each is endowed with half the shares of the risky asset. Solve for the competitive equilibrium (relative) price of the risky asset, and compute the equilibrium allocation of the two securities. Explain your results intuitively.