SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 | Prof. Kasa
Financial Economics I | Fall 2021

PROBLEM SET 1
(Due October 25)

1. (25 points). This question asks you to estimate and test the CAPM. On the course webpage, I've posted two excel files: Fama-French-factors.xls and Fama-French-ports.xls. They contain monthly stock return data from the USA for the period 1926-2019. Column B in Fama-French-factors contains a time-series of market excess returns (Mkt-RF). Columns B-Z of Fama-French-ports contains time-series data on the returns of 25 portfolios sorted by size and book-to-market. (There are 5 categories of size and book-to-market ratios, and Fama & French form 25 portfolios by interacting them with each other).

   (a) Plot the market excess return. What is its mean? What is the Sharpe ratio? (Note: Use whatever software you want).

   (b) Compute the mean returns for the 25 Fama-French portfolios. Which have the highest average return? Which have the lowest?

   (c) Compute (full-sample) β's for the 25 portfolios, by running 25 separate bivariate time-series regressions of portfolio returns on the market excess return. (Be sure to include an intercept). Save the 25 β estimates you get.

   (d) Now do a single cross-sectional regression of the (average) returns of the 25 portfolios onto their β's. (Again, include an intercept). Plot the actual vs. fitted regression line. What is the R^2 (ie, what proportion of the variation in mean returns on $size \times book/market$ sorted portfolios can be explained by the CAPM? Is the estimated slope (approximately) equal to the market excess return? (Note: You don't need to compute a formal test statistic).

2. **Time-Varying Expected Returns**. (25 points). In class we solved the Merton problem when the 'investment opportunity set' was constant (ie., μ and σ we constants). This question asks you to consider the case where the mean return is stochastic. Empirical evidence supports this. Hence, now suppose the risky asset price follows the process

$$\frac{dS}{S} = \mu_t dt + \sigma dB$$

   where μ_t follows a mean-reverting Ornstein-Uhlenbeck process

$$d\mu_t = \alpha(\bar{\mu} - \mu_t)dt + \sigma_\mu dB^\mu$$

   For simplicity, suppose dB and dB^μ are uncorrelated. Finally, continue to assume the investor has time-additive CRRA preferences

$$V(W, \mu) = \max_{c,\pi} E_0 \int_0^\infty \frac{C^{1-\gamma}}{1-\gamma} e^{-\delta t} dt$$

   subject to $dW = [(r + \pi(\mu_t - r))W - C]dt + \pi\sigma W dB$.

   (a) Write down the investor's stationary HJB equation.

   (b) Verify that a solution is of the form $V(W, \mu) = \frac{1}{1-\gamma} g(\mu) W^{1-\gamma}$. Derive a 2nd-order ODE for $g(\mu)$.

   (c) Is the investor's optimal portfolio still time invariant? Why or why not?