SIMON FRASER UNIVERSITY
Department of Economics

Econ 815   Prof. Kasa
Financial Economics, I   Fall 2015

PROBLEM SET 3 - Lucas (1978)
The Consumption-Based Capital Asset Pricing Model
(Solutions)

1. (10 points). Consider an economy with just two states: "normal" and "disaster". There is a single
   risky asset in the economy that yields a nonstorable 'dividend', which represents the only consumption
   good in the economy. In the normal state dividend (and consumption) growth is 3.0%, and in the
   disaster state it is -20%. Suppose agents have identical constant relative risk aversion preferences,
   with coefficient of relative risk aversion of 2.0, and a time discount factor of $\beta = .99$. Finally, assume
   that the state is i.i.d. over time, and that the probability of the normal state is .90 and the probability
   of the disaster state is .10.

   (a) What is the implied risk-free rate in this economy? (Hint: Use the agent's Euler equation).
       The investors Euler equation is

       $$1 = \beta E \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-2} R_{t+1} \right\}$$

       *For the riskless rate we have $R_{t+1} = R_f$ in both states. Using given probabilities and dividend
       growth data, we get*
       $$1 = .99 \left\{ .9(1.03)^{-2} + .1(.80)^{-2} \right\} R_f$$

       Solving this gives $R_f \approx 1.0055$.

   (b) Suppose the rate of return on the risky asset is 17% in the normal state. What must be the rate
       of return in the disaster state? What is the implied equity premium?
       *Let x be the disaster state rate of return. Substituting in the given information gives us the
       following equation for x*

       $$1 = .99 \left\{ .9(1.03)^{-2}(1.17) + .1(.80)^{-2}(x) \right\}$$

       Solving for x gives $x \approx .1127$, implying a rate of return of about -88.7%!

2. (10 points). One way to resolve the equity premium puzzle is to argue that experimental evidence
   on risk aversion is not applicable to the real world of stock market investment, and to conclude that
   the coefficient of relative risk aversion is in fact quite large. What's the problem with this argument?
   (Bonus: How would your answer change if we considered a broader class of preferences than time-
   additive CRRA preferences?)

   See class notes