Econ 815   Prof. Kasa
Financial Economics, I   Fall 2015

MIDTERM EXAM
(Solutions)

Answer the following questions True, False, or Uncertain. Briefly explain your answers. (8 points each).

1. According to the CAPM, everyone should hold the same portfolio.

   FALSE/UNCERTAIN. *According to the CAPM, everyone should hold the same portfolio of <u>risky</u> assets (i.e, the 'market portfolio'). However, individuals may decide to hold different mixes of risky vs. riskless assets, depending on their wealth and preferences.*

2. There is no benefit to diversification if asset returns are uncorrelated.

   FALSE. *There will no benefit to diversification if assets are <u>perfectly</u> correlated (i.e., have a correlation coefficient equal to one).*

3. Stocks are safer in the long-run (i.e., if you can afford to hold them for a long time).

   FALSE/UNCERTAIN. *Stocks will <u>not</u> be safer in the long-run if their returns are i.i.d. over time. A long holding period just means you are making the same bet over and over again, and so the risk scales with the horizon. However, if returns exhibit <u>mean reversion</u> (i.e., negative autocorrelation), then the variance would be lower over the long run.*

4. Someone with constant absolute risk aversion has increasing relative risk aversion.

   TRUE. *Absolute risk aversion is defined by $-U''(W)/U'(W)$. Relative risk aversion is defined by $-W \cdot U''(W)/U'(W)$. So if absolute risk aversion is constant, relative risk aversion is increasing.*

5. If markets are complete, then everyone has the same utility.

   FALSE. *Complete markets just means there is a market for any date/event that people care about. If this is the case, the marginal rates of substitution across dates/states are equated across individuals. However, this does <u>not</u> mean that the <u>level</u> of wealth/utility is equated across individuals.*

   The following questions are short answer. Briefly explain your answer. Clarity will be rewarded.

6. (15 points). Suppose a stock has a β of 1.5, the market return is 10%, and the risk-free rate is 5%. According to the CAPM, what is the expected rate of return on this stock? If the expected price a year from now is $100, what should the price be today (assuming no dividends will be paid)?

   According to the CAPM, the expected return, r, on an asset is given by the following expression

   $$r = r_f + \beta(r_m - r_f)$$

   where r_f is the riskless rate, and r_m is the expected return on the market portfolio. Plugging in we get

   $$r = .05 + 1.5(.10 - .05) = .125$$

   Hence, according to the CAPM, the expected return on this asset should be 12.5%. The return on an asset that doesn't pay dividends is just $(P_1 - P_0)/P_0$. Therefore, if the future price is expected to be 100, the current price, P_0, satisfies the relationship, $100/P_0 = 1.125$. Therefore, $P_0 = 100/1.125$.

7. (15 points). Assume the expected rate of return on the market portfolio is 22%, and the risk-free rate (eg., the rate of return on government T-bills) is 6%. The standard deviation of the market portfolio is 32%. Assuming the CAPM holds,

(a) What is the equation of the "Capital Market Line"?

The Capital Market Line just gives the relationship between risk and return on all efficient portfolios. It is given by the equation

$$r = r_f + \left(\frac{r_m - r_f}{\sigma_m}\right)\sigma$$

(b) If you desire a 14% expected return, what will be the associated standard deviation of this position (assuming it's efficient)? If you have $1000 to invest, how should you allocate it to achieve this position?

Plugging in the given information, we have

$$.14 = .06 + \left(\frac{.22 - .06}{.32}\right)\sigma \qquad \Rightarrow \qquad \sigma = 2(.14 - .06) = .16$$

Let α be the share invested in the market portfolio. To achieve an expected portfolio return of 14%, we must pick α so that $.14 = \alpha(.22) + (1 - \alpha)(.06)$. This implies $\alpha = 0.5$, so you should invest \$500 in the market and \$500 in the risk-free asset.

8. (30 points). Consider an investor with the following preferences

$$U = \max_c E \int_0^\infty -\frac{1}{2}(\bar{c} - c)^2 e^{-\rho t} dt$$

Suppose he can invest in a risk-free asset and/or a risky asset, with prices that follow the processes

$$\frac{dB}{B} = rdt$$
$$\frac{dP}{P} = \mu dt + \sigma dz$$

where dz is the increment to a Wiener process.

(a) Let W be the agent's current wealth, and α be the share of his wealth invested in the risky asset. Write down the agent's budget constraint.

The budget constraint is

$$dW = [(r + \alpha(\mu - r))W - c]dt + \alpha W \sigma dz$$

(b) Letting $V(W)$ denote his value function, write down the stationary Hamilton-Jabobi-Bellman equation.

The HJB equation is

$$\rho V = \max_{c,\alpha} \left\{ -\frac{1}{2}(\bar{c} - c)^2 + [(r + \alpha(\mu - r))W - c] \cdot V_W + \frac{1}{2}\sigma^2 W^2 \alpha^2 \cdot V_{WW} \right\}$$

(c) Briefly describe the steps you would take to solve for the agent's decision rules. (Note: You do not have to actually compute them).

To find the agent's consumption/portfolio rules we can proceed as follows

(a) *Derive the first-order conditions for c and α.*

(b) *Solve the FOCs for c and α as functions of V_W and V_{WW}.*

(c) *Plug these expressions back into the HJB equation and solve the second-order ODE for V. Since the utility function here is quadratic, a quadratic value function would be a good guess.*

(d) *Finally, once we have the solution for $V(W)$, we can compute the relevant derivatives and substitute them into the FOCs to get the policy functions. Easy!*

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