SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 — Prof. Kasa
Financial Economics, I — Fall 2016

MIDTERM EXAM
(Solutions)

Answer the following questions True, False, or Uncertain. Briefly explain your answers. (8 points each).

1. According to the CAPM, stocks with a higher variance of returns will have a higher mean return.

   FALSE. *The CAPM implies that stocks with high (positive)* <u>*covariance*</u> *with the market will have relatively high returns. It's even possible that a high variance stock has a relatively* <u>*low*</u> *return, if it moves inversely with the market.*

2. The Merton consumption/portfolio model predicts that asset portfolios remain constant over time.

   UNCERTAIN. *This was true in the simple examples considered in class, but is not true in general. Of course, if you* <u>*define*</u> *the Merton model as one with i.i.d. Gaussian returns and investors with CRRA preferences, then I suppose you could say it was true.*

3. There are no benefits from diversification if everyone has the same preferences.

   FALSE. *Diversification benefits depend on return covariances, not preferences. (Of course, there's a trivial sense in which this could be true, i.e., if everyone had 'risk-loving' preferences, since only risk averse individuals care about risk).*

4. In financial markets there are always winners and losers, so there is no possibility of Pareto improving trade.

   FALSE. *In theory at least, financial markets are not casinos. They are more like insurance markets, which facilitate the transfer of risks in the economy. Like all trade, trade in state-contingent claims can make both agents better off. Remember, we are looking at this from an* ex ante *perspective!*

5. If markets are complete, then no trading occurs.

   UNCERTAIN. *It depends on the asset market structure. If a full menu of date/state contingent claims is initially available, then trading need never occur in the future. This is because the original allocation is Pareto Optimal. Even if some agent would like to trade in the future, there won't be anyone on the other side of the market. However, as Arrow (1964) showed, dynamic trading over time in a greatly reduced set of one-step ahead markets can reproduce the same outcome (under the assumption that agents know the equilibrium pricing functions!)*

   The following questions are short answer. Briefly explain your answer. Clarity will be rewarded.

6. (15 points). What is the difference between Absolute Risk Aversion and Relative Risk Aversion? Explain why the assumption of Constant Relative Risk Aversion is empirically more plausible than the assumption of Constant Absolute Risk Aversion.

   By definition

$$
\begin{aligned}
CARA &= -\frac{U''(c)}{U'(c)} \\
CRRA &= -\frac{c \cdot U''(c)}{U'(c)}
\end{aligned}
$$

CARA is derived as a local approximation of the Certainty Equivalent of an additive bet. CRRA is derived as a local approximation of the Certainty Equivalent of a multiplicative bet. In financial markets, most risks concern rate of return, and so relative risk aversion is more relevant. More importantly, CARA preferences imply that wealthier individuals invest a smaller fraction of their wealth in risky assets, because their relative risk aversion increases. This is not consistent with the data. Also, if absolute risk aversion were constant, we should have witnessed declines in market risk premia over time, as economies have developed and become wealthier. There is no evidence for this.

7. (15 points). Suppose the expected return on the market portfolio is 12%, and the risk-free rate is 2%. The standard deviation of the market portfolio is 30%. Assuming the CAPM holds,

   (a) What is the equation of the "Capital Market Line"?

   The Capital Market Line gives the relationship between risk and return on all efficient portfolios. It is given by the equation

$$r = r_f + \left(\frac{r_m - r_f}{\sigma_m}\right)\sigma$$

   (b) If you desire a 10% expected return, what will be the associated standard deviation of this position (assuming it's efficient)? If you have $1000 to invest, how should you allocate it to achieve this position?

   Plugging in the given information, we have

$$.10 = .02 + \left(\frac{.12 - .02}{.30}\right)\sigma \qquad \Rightarrow \qquad \sigma = 3(.10 - .02) = .24$$

   Let α be the share invested in the market portfolio. To achieve an expected portfolio return of 10%, we must pick α so that $.10 = \alpha(.12) + (1-\alpha)(.02)$. This implies $\alpha = 0.8$, so you should invest $800 in the market and $200 in the risk-free asset.

8. (30 points). Some people argue that individuals exhibit 'habit persistence', meaning that current utility depends on past consumption. This question asks you to consider how this would alter the Merton problem. So let's now suppose an individual has the utility function

$$U = \max_c E \int_0^\infty \frac{1}{1-\gamma}(c - H)^{1-\gamma} e^{-\rho t} dt$$

where we can now interpret H as a 'habit stock', which depends on past consumption according to

$$dH = (-\eta H + c)dt$$

where $\eta > 0$ is a parameter that governs how much past consumption influences current utility. As usual, suppose the individual can invest in a risk-free asset and/or a risky asset, with prices that follow the processes

$$\frac{dB}{B} = rdt$$
$$\frac{dP}{P} = \mu dt + \sigma dz$$

where dz is an increment to a Wiener process.

   (a) Let W be the agent's current wealth, and α be the share of his wealth invested in the risky asset. Write down the agent's budget constraint.

   Note that habit accumulation is a matter of preferences. There is no (direct) financial cost. Hence, the budget constraint is the usual one

$$dW = [(r + \alpha(\mu - r))W - c]dt + \alpha\sigma W \cdot dz$$

   (b) Write down the stationary Hamilton-Jabobi-Bellman equation. (Hint: Is W the only state variable?)

The key point in this question is that now we have a new state variable, H. It conditions our current marginal utility of consumption, and it depends on past consumption choices. Note, however, that it evolves deterministically, so no 2nd-order Ito term enters the HJB equation. We just get a drift term

$$\rho V(W,H) = \max_{c,\alpha}\left\{\frac{1}{1-\gamma}(c-H)^{1-\gamma} + [(r+\alpha(\mu-r))W - c]\cdot V_W + [-\eta H + c]\cdot V_H + \frac{1}{2}\sigma^2 W^2 \alpha^2 \cdot V_{WW}\right\}$$

where subscripts denote partial derivatives.