SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 — Prof. Kasa
Financial Economics I — Summer 2019

MIDTERM EXAM
(June 27)

The first four questions are True, False, or Uncertain. Briefly explain. (10 points each).

1. According to the CAPM, stocks with more volatile returns have higher mean returns.
2. Expected Utility theory makes no sense, since often investors do not know the probability distribution of asset returns.
3. There is no trading volume if markets are complete.
4. According to the CAPM, everybody should hold the same portfolio of risky assets.
5. (20 points). Suppose there is a representative agent with time separable utility $U(\cdot)$ over consumption and time discount factor, δ. Assume there are two consumption states, C_H and C_L. Transitions between the two states follow a Markov process with transition probabilities p_{ij} for $i, j \in \{H, L\}$. Denote the four Arrow-Debreu security prices by q_{ij}. That is, q_{ij} is the price of a claim to one unit of consumption in state j tomorrow given state i today. Write down the four optimality (Euler) equations of the agent. Use them to solve for the ratio of marginal utilities, the time discount factor, and the transition probabilities as functions of the state prices, q_{ij}.
6. (20 points). Explain how you would empirically test the CAPM model. What data would you use? What regressions would you estimate? What would be your null hypothesis?
7. **Time-Varying Expected Returns**. (20 points). In class we solved the Merton problem when the 'investment opportunity set' was constant (ie., μ and σ we constants). This question asks you to consider the case where the mean return is stochastic. Empirical evidence supports this. Hence, now suppose the risky asset price follows the process

$$\frac{dS}{S} = \mu_t dt + \sigma dB$$

where μ_t follows a mean-reverting Ornstein-Uhlenbeck process

$$d\mu_t = \alpha(\bar{\mu} - \mu_t)dt + \sigma_\mu dB^\mu$$

For simplicity, suppose dB and dB^μ are uncorrelated. Finally, continue to assume the investor has time-additive CRRA preferences

$$V(W, \mu) = \max_{c,\pi} E_0 \int_0^\infty \frac{C^{1-\gamma}}{1-\gamma} e^{-\delta t} dt$$

subject to $dW = [(r + \pi(\mu_t - r))W - C]dt + \pi\sigma W dB$.

(a) Write down the investor's stationary HJB equation.

(b) Verify that a solution is of the form $V(W, \mu) = \frac{1}{1-\gamma} g(\mu) W^{1-\gamma}$. Derive a 2nd-order ODE for $g(\mu)$.

(c) Is the investor's optimal portfolio still time invariant? Why or why not?