SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 | Prof. Kasa
Financial Economics I | Fall 2020

MIDTERM EXAM
(October 30 – Due November 2, 6pm)

The first four questions are True, False, or Uncertain. Briefly explain. (10 points each).

1. In the CAPM, the market portfolio is defined as an equally-weighted average of all stocks.
2. Holding stocks for a long time makes them less risky. Good and bad luck average out over time.
3. There is no trading volume if markets are complete.
4. In the Merton model, the share of wealth invested in risky assets does not depend on a person's age.
5. (30 points). This question asks you to estimate and test the CAPM. On the course webpage, I've posted two excel files: Fama-French-factors.xls and Fama-French-ports.xls. They contain monthly stock return data from the USA for the period 1926-2019. Column B in Fama-French-factors contains a time-series of market excess returns (Mkt-RF). Columns B-Z of Fama-French-ports contains time-series data on the returns of 25 portfolios sorted by size and book-to-market. (There are 5 categories of size and book-to-market ratios, and Fama & French form 25 portfolios by interacting them with each other).
   (a) Plot the market excess return. What is its mean? What is the Sharpe ratio? (Note: Use whatever software you want).
   (b) Compute the mean returns for the 25 Fama-French portfolios. Which have the highest average return? Which have the lowest?
   (c) Compute (full-sample) β's for the 25 portfolios, by running 25 separate bivariate time-series regressions of portfolio returns on the market excess return. (Be sure to include an intercept). Save the 25 β estimates you get.
   (d) Now do a single cross-sectional regression of the (average) returns of the 25 portfolios onto their β's. (Again, include an intercept). Plot the actual vs. fitted regression line. What is the R^2 (ie, what proportion of the variation in mean returns on $size \times book/market$ sorted portfolios can be explained by the CAPM? Is the estimated slope (approximately) equal to the market excess return? (Note: You don't need to compute a formal test statistic).
6. **Time-Varying Expected Returns**. (30 points). In class we solved the Merton problem when the 'investment opportunity set' was constant (ie., μ and σ we constants). This question asks you to consider the case where the mean return is stochastic. Empirical evidence supports this. Hence, now suppose the risky asset price follows the process

$$\frac{dS}{S} = \mu_t dt + \sigma dB$$

where μ_t follows a mean-reverting Ornstein-Uhlenbeck process

$$d\mu_t = \alpha(\bar{\mu} - \mu_t)dt + \sigma_\mu dB^\mu$$

For simplicity, suppose dB and dB^{μ} are uncorrelated. Finally, continue to assume the investor has time-additive CRRA preferences

$$V(W,\mu) = \max_{c,\pi} E_0 \int_0^{\infty} \frac{C^{1-\gamma}}{1-\gamma} e^{-\delta t} dt$$

subject to $dW = [(r + \pi(\mu_t - r))W - C]dt + \pi\sigma W dB$.

(a) Write down the investor's stationary HJB equation.

(b) Verify that a solution is of the form $V(W,\mu) = \frac{1}{1-\gamma} g(\mu) W^{1-\gamma}$. Derive a 2nd-order ODE for $g(\mu)$.

(c) Is the investor's optimal portfolio still time invariant? Why or why not?