SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 | Prof. Kasa
Financial Economics I | Fall 2021

MIDTERM EXAM
(Due November 8, 6pm)

The first four questions are True, False, or Uncertain. Briefly explain. (5 points each).

1. There is no benefit from diversification if all assets returns are uncorrelated.
2. Someone with constant relative risk aversion has declining absolute risk aversion.
3. If markets are complete then the expected return is the same for all assets.
4. According to the CAPM, more volatile assets have higher expected returns.
5. (25 points). **Habit Persistence**. Some people argue that individuals exhibit 'habit persistence', meaning that current utility depends on past consumption. This question asks you to consider how this would alter the Merton problem. So let's now suppose an individual has the utility function

$$U = \max_c E \int_0^\infty \frac{1}{1-\gamma}(c-H)^{1-\gamma} e^{-\rho t} dt$$

where we can now interpret H as a 'habit stock', which depends on past consumption according to

$$dH = (-\eta H + c)dt$$

where $\eta > 0$ is a parameter that governs how much past consumption influences current utility. As usual, suppose the individual can invest in a risk-free asset and/or a risky asset, with prices that follow the processes

$$\begin{aligned} \frac{dB}{B} &= rdt \\ \frac{dP}{P} &= \mu dt + \sigma dz \end{aligned}$$

where dz is an increment to a Wiener process.

(a) Let W be the agent's current wealth, and α be the share of his wealth invested in the risky asset. Write down the agent's budget constraint.

(b) Write down the stationary Hamilton-Jacobi-Bellman equation. (Hint: Is W the only state variable?).

(c) Without solving the model, explain intuitively how you think this might change the Merton solution.

(For background on this question you could take a look at Cochrane's text (2nd ed.) pages 467-473, or the journal article "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior" by Campbell & Cochrane (JPE, 1999). However, you don't need to do this in order to answer the quesion).

6. (15 points). Suppose a stock is currently worth \$20, and it is known that in 3 months it will be worth either \$22 or \$18. Consider on option on the stock with a strike price of \$21. This option will either be worth \$1 (if the stock price increases) or worth nothing (if the stock price decreases). This question asks you to use no arbitrage reasoning to value this option contract.

   (a) Consider a portfolio consisting of a long position of Δ shares of the stock and a short position of one call option. Find a value of Δ that makes this a riskless portfolio (i.e., its payoff is the same, no matter what the future stock price turns out to be).

   (b) Given this value of Δ, what will be the future value of the portfolio? Assuming the (annual) risk-free interest rate is 12%, what is the present value of this portfolio?

   (c) What must therefore be the current no arbitrage price of the option? If the price deviated from this value, explain how you could make riskless profits.

   (d) Why didn't we need to know the probability that the stock price would increase? Wouldn't this influence your valuation of a call option?

7. (15 points). Part of the appeal of options is that they can be combined to form very flexible payoff profiles. A couple examples were discussed in class. Here you are asked to consider a few more. For each, illustrate the expiration date payoff and profit from the position.

   (a) A *bullish vertical spread*, which is created by buying a call option with strike price K_1, and simultaneously selling a call option (on the same stock) with strike price $K_2 > K_1$. Why is it called a 'bullish' spread? (Hint: Remember that, all else equal, call options with lower strike prices are more expensive).

   (b) A *strangle*, which involves buying out-of-the-money call and puts on the same underlying stock (for the same expiration date). That is, if the current stock price is S, the call has strike price $K_c > S$ and the put has strike price $K_p < S$. (Hint: This is similar to a straddle, but is cheaper, since the options are purchased out-of-the-money).

   (c) A *collar*, which involves holding the underlying stock, while simultaneously buying an out-of-the-money put and selling/writing an out-of-the-money call. Why might this strategy be attractive? How does it compare to a bullish vertical spread?

8. (25 points). **Man vs. Machine**. Consider a stock which has a price that follows the following geometric Brownian motion process:

$$\frac{dS}{S} = \mu dt + \sigma dW$$

   where $\mu = .12$ and $\sigma = .20$. Suppose the current stock price is \$42, and suppose we are interested in the value of a 6-month (European) call option on this stock. Assume the risk-free rate is constant, and equal to 10%.

   (a) Suppose the 'strike price' of the option is $K = 40$. Use the Black-Scholes formula derived in class to compute the value of the option. (Hint 1: Note that the time unit here is a year, so that for a 6-month option we have $T - t = 0.5$. Hint 2: Is μ a relevant parameter? Why, or why not?).

   (b) Now suppose you trust computers more than math. Write a simple program (using the software of your choice) to numerically calculate the value of the option. Do you get the same answer as in part (a)?