SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 | Prof. Kasa
Financial Economics I | Fall 2025

MIDTERM EXAM
( Due October 30, 6pm)

The first four questions are True, False, or Uncertain. Briefly explain. (10 points each).

1. According to the Merton model, investors should hold fewer stocks in their portfolios as they get older.
2. The expected return on a stock does not affect the price of a call option on that stock.
3. There are no benefits to diversification when asset returns are uncorrelated.
4. The expected return on a call option is greater than the expected return on the underlying stock.
5. **Time-Varying Expected Returns**. (20 points). In class we solved the Merton problem when the 'investment opportunity set' was constant (ie., $\mu$ and $\sigma$ we constants). This question asks you to consider the case where the mean return is stochastic. Empirical evidence supports this. Hence, now suppose the risky asset price follows the process

$$\frac{dS}{S} = \mu_t dt + \sigma dB$$

where $\mu_t$ follows a mean-reverting Ornstein-Uhlenbeck process

$$d\mu_t = \alpha(\bar{\mu} - \mu_t)dt + \sigma_\mu dB^\mu$$

For simplicity, suppose $dB$ and $dB^\mu$ are uncorrelated. Finally, continue to assume the investor has time-additive CRRA preferences

$$V(W,\mu) = \max_{c,\pi} E_0 \int_0^\infty \frac{C^{1-\gamma}}{1-\gamma} e^{-\delta t} dt$$

subject to $dW = [(r + \pi(\mu_t - r))W - C]dt + \pi\sigma W dB$.

**(a)** Write down the investor's stationary HJB equation.

**(b)** Verify that a solution is of the form $V(W,\mu) = \frac{1}{1-\gamma} g(\mu) W^{1-\gamma}$. Derive a 2nd-order ODE for $g(\mu)$.

**(c)** Is the investor's optimal portfolio still time invariant? Why or why not?

6. (15 points). Part of the appeal of options is that they can be combined to form very flexible payoff profiles. A couple examples were discussed in class. Here you are asked to consider a few more. For each, illustrate the expiration date payoff and profit from the position.

**(a)** A *bullish vertical spread*, which is created by buying a call option with strike price $K_1$, and simultaneously selling a call option (on the same stock) with strike price $K_2 > K_1$. Why is it called a 'bullish' spread? (Hint: Remember that, all else equal, call options with lower strike prices are more expensive).

**(b)** A *strangle*, which involves buying out-of-the-money call and puts on the same underlying stock (for the same expiration date). That is, if the current stock price is $S$, the call has strike price $K_c > S$ and the put has strike price $K_p < S$. (Hint: This is similar to a straddle, but is cheaper, since the options are purchased out-of-the-money).

**(c)** A *collar*, which involves holding the underlying stock, while simultaneously buying an out-of-the-money put and selling/writing an out-of-the-money call. Why might this strategy be attractive? How does it compare to a bullish vertical spread?

7. (25 points). **Man vs. Machine**. Consider a stock which has a price that follows the following geometric Brownian motion process:

$$\frac{dS}{S} = \mu dt + \sigma dW$$

where $\mu = .12$ and $\sigma = .20$. Suppose the current stock price is \$42, and suppose we are interested in the value of a 6-month (European) call option on this stock. Assume the risk-free rate is constant, and equal to 10%.

**(a)** Suppose the 'strike price' of the option is $K = 40$. Use the Black-Scholes formula derived in class to compute the value of the option. (Hint 1: Note that the time unit here is a year, so that for a 6-month option we have $T - t = 0.5$. Hint 2: Is $\mu$ a relevant parameter? Why, or why not?).

**(b)** Now suppose you trust computers more than math. Write a simple program (using the software of your choice) to numerically calculate the value of the option. Do you get the same answer as in part (a)?