Topics

1.) Intl. Capital MKts. with Lack of Commitment
   - Sovereign Risk + Default
   - One-Sided Commitment (Small Country Case)

2.) Caveats
   - Saving
   - Bulow-Rogoff (1989)

3.) 2-sided Lack of Commitment
Last time we saw that non-traded goods and dynamic spanning with bond trading could provide partial resolutions of the portfolio home bias puzzle.

However, the most important explanation likely stems from capital market imperfections.

There are two main sources of imperfection: (1) Lack of commitment, and (2) Asymmetric Info. Today we explore the consequences of lack of commitment.

So far, we have implicitly assumed an infinite penalty for default. This is not realistic, especially for sovereign lending!

Assumptions

1.) A small risk averse country faces risky endowment. The country can't commit to contracts.

2.) There are competitive risk neutral foreign lenders who can commit to deliver resources.

3.) If the country defaults it is shut out of the intl. capital market forever. (lives in autarky).
Preferences and Constraints

\[ U_t = E_t \sum_{s=1}^{S} R_{s,t} U(C_s) \]

\[ C_s = \bar{Y} + \varepsilon_s - P_s(\varepsilon_s) \quad \varepsilon_s \sim i.i.d \quad E[\varepsilon, \varepsilon] \]

\[ E(\varepsilon_s) = 0 \]

\[ P_s(\varepsilon_s) = \text{insurance payment} \]

Zero Profit Condition

\[ \sum_{i=1}^{N} \pi_i(\varepsilon_i) P(\varepsilon_i) = 0 \quad \text{Note: } i.i.d \Rightarrow P_s = P \]

(Note: Period by period contracts)

Arrow-Debreu Contract

\[ P(\varepsilon_s) = \varepsilon_s \Rightarrow C_s = \bar{Y} \quad \forall \ s \quad (\text{full insurance}) \]
Now consider default,

\[ \text{Gain}(\varepsilon_t) = U(\tilde{\gamma} + \varepsilon_t) - U(\tilde{\gamma}) \]

\[ \text{Cost} = \sum_{s=t+1}^{\infty} \beta^s u(\tilde{\gamma}) - \sum_{s=t+1}^{\infty} \beta^s \mathbb{E}_s u(\tilde{\gamma} + \varepsilon_s) \]

\[ \approx \frac{\beta}{1-\beta} \mathbb{E} \left[ u(\tilde{\gamma}) - \mathbb{E} U(\tilde{\gamma} + \varepsilon) \right] \] \text{by stationarity}

\[ \approx \frac{\beta}{1-\beta} \frac{1}{2} \left| U''(\tilde{\gamma}) \right| \text{var}(\varepsilon) \]

Note, gain from default is greatest at \( \tilde{\varepsilon} \).
Sustainability requires \( \text{Cost} > \text{Gain}(\tilde{\varepsilon}) \).

\[ U(\tilde{\gamma} + \varepsilon) - U(\tilde{\gamma}) \leq \frac{\beta}{1-\beta} \left[ \frac{1}{2} \left| U''(\tilde{\gamma}) \right| \text{var}(\varepsilon) \right] \]

Note, this is always true for \( \beta < 1 \).
What if this isn't satisfied? Is partial insurance still possible?
Partial Insurance

\[
\text{Gain}(\varepsilon_+^+) = U(\bar{Y} + \varepsilon_+^+) - U(\bar{Y} + \varepsilon_+^+ - P(\varepsilon_+^+)) \quad \text{(Not as big)}
\]

\[
\text{Cost} = \frac{1}{1-\beta} \left\{ E[U(\bar{Y} + \varepsilon - P(\varepsilon)) - E[U(\bar{Y} + \varepsilon + \varepsilon_+^+ - P(\varepsilon_+^+))]] \right\}
\]

Planner’s Lagrangian

\[
L = \sum_{i=1}^{N} \pi(\varepsilon_i) U(\bar{Y} + \varepsilon_i - P(\varepsilon_i)) + \sum_{i=1}^{N} \lambda(\varepsilon_i) \left\{ \frac{\beta}{1-\beta} \sum_{i=0}^{\varepsilon_i} \pi(\varepsilon_i) [U(\bar{Y} + \varepsilon_i - P(\varepsilon_i)) - U(\bar{Y} + \varepsilon_i - P(\varepsilon_i))] - [U(\bar{Y} + \varepsilon_i - P(\varepsilon_i)) - U(\bar{Y} + \varepsilon_i - P(\varepsilon_i))] \right\} + \mu \sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i)
\]

FOCs (differentiate w.r.t. \( P(\varepsilon_i) \), \( i=1, 2, \ldots, N \))

1. \[ \left\{ \pi(\varepsilon) + \lambda(\varepsilon) + \frac{\beta}{1-\beta} \sum_{i=0}^{\varepsilon} \lambda(\varepsilon_i) \right\} U[C(\varepsilon)] = \mu \pi(\varepsilon) \]

2. \[ \lambda(\varepsilon) \cdot PC(\varepsilon) = 0 \quad \text{complementary slackness} \]

Participation Constraint
2 cases

1) Slack PC ($\lambda = 0$)

$$u'[c(n)] = \frac{M}{1 + \frac{M}{Y} \sum \lambda (n_i)}$$

$\Rightarrow$ Consumption is state independent

$\Rightarrow P(n) = P_0 + \epsilon, \quad c(n) = Y - P_0$

($P_0 > 0$ by zero profit constraint)

2) PC binds ($\lambda > 0$)

In this case, the constraint defines an implicit function, $P(n)$.

$$\frac{dp}{d\epsilon} = \frac{u'[\bar{Y} + \epsilon - P(n)] - u'(\bar{Y} + \epsilon)}{u'[\bar{Y} + \epsilon - P(n)]]} < 1$$

Since $P(n) > 0$ and $u$ is concave

Inability to commit implies country must get some of the upside. The cost is that its consumption is lower than $Y$ in the insured states.
Graphically

\[ P(\varepsilon) \]

\[ \varepsilon^* \]

\[ \text{constraint binds} \]

\[ C(\varepsilon) \]

\[ \hat{\varepsilon} + \varepsilon - P(\varepsilon) \]
Caveats

1.) Saving. If the country can save, then these savings can serve as collateral and enhance insurability (especially if they are deposited in foreign banks!). If \( \beta(1+r) = 1 \), then the country gradually saves enough to eventually obtain full insurance.

2.) Bulow-Rogoff (1989). To be sustainable, lenders must be committed and coordinated. That is, being cut-off from the int'l. capital market means you can't lend/deposit either. Otherwise, the above contract can be broken by the following strategy: Default at \( \bar{E} \), and then invest \( P(\bar{E}) \) in some other bank. Then, sign a new collateralized insurance contract with another lender. You avoid paying \( P(\bar{E}) \) + get to consume the interest!

3.) Renegotiation [sub-game perfect vs. Renegotiation Proofness]
Now consider a symmetric treatment of borrowers and lenders (2-sided lack of commitment).

Assume a large number of countries, who want to ensure each other

\[ U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s^j) \]

\[ Y_t^j = \bar{Y} + \sum_{i,j} \varepsilon_t^i + \omega_t \]

\[ \varepsilon_t^i \sim \text{i.d.} \]

\[ \sum_{i} \varepsilon_t^i = 0 \]

Arrow-Debreu

\[ C_t^j = \bar{Y} + \omega_t \quad \forall j,t \]

Can this be supported without commitment?
2 conditions

1.) If country j defaults, it is permanently shut-out.

2.) Other countries also lose their reputations for repaying j. [This prevents j from buying collateralized insurance contracts in good states, as in Bulow-Rogoff].

\[
\text{Gain}(\tilde{\epsilon}, \omega) = U(\tilde{\gamma} + \tilde{\epsilon} + \omega) - U(\tilde{\gamma} + \omega)
\]

\[
\text{Cost} = \beta \left[ E[U(\tilde{\gamma} + \omega)] - E[U(\tilde{\gamma} + \tilde{\epsilon} + \omega)] \right]
\]

Sustainable if, \( \text{Gain}(\tilde{\epsilon}, \omega) \leq \text{Cost} \)