Topics

1.) International Real Business Cycles
- BKK (JPE, 1992)

2.) Extensions
- Transportation Costs
- Multiple Goods (TOT effects)

3.) Incomplete Markets
- Baxter + Crucini (IER, 1995)

Open-Economy DSGE Models

- We are now going to examine the quantitative implications of 2-country Dynamic Stochastic General Equilibrium models. These are basically 2-country versions of Cass-Koopmans growth models, with random shocks to productivity and endogenous labor supply.

Baseline Assumptions

1.) Complete Markets ⇒ Perfect Risk-Sharing
⇒ Equil. is Pareto Optimal

2.) No Money - Only "real" variables are determined

3.) 2 (symmetric) countries - our earlier reliance on the "separation" between consumption + investment is no longer valid.

4.) 1-good [No Terms of Trade or Real Ex. Rate Issues] (we'll relax this later)

5.) Labor is immobile between countries

6.) Only source of uncertainty is productivity shocks.

International Business Cycle Facts

- The basic business cycle facts are qualitatively similar across countries.

- They can be summarized in reference to 3 properties: (1) Volatility, (2) Persistence, and (3) Comovement

1.) Consumption less volatile than output

2.) Investment more volatile than output

3.) Employment (slightly) less volatile than output

volatility

4.) C, I, N are all procyclical

* 5.) NX is countercyclical

comovement

6.) Y + Z are highly persistent

7.) Productivity shocks are positively correlated across countries

* 8.) $corr(c, c^*) < corr(y, y^*)$

* 9.) I + N are positively correlated across countries

International comovements

Properties of Business Cycles in OECD Economies

	Standard Deviation (%)		Ratio of Standard Deviation to That of *y*					Autocorr.	Correlation with Output					
Country	*y*	*nx*	*c*	*x*	*g*	*n*	*z*	*y*	*c*	*x*	*g*	*nx*	*n*	*z*
Australia	1.45	1.23	0.66	2.78	1.28	0.34	1.00	.60	.46	.67	.15	−.01	.12	.98
Austria	1.28	1.15	1.14	2.92	0.36	1.23	0.84	.57	.65	.75	−.24	−.46	.58	.65
Canada	1.50	0.78	0.85	2.80	0.77	0.86	0.74	.79	.83	.52	−.23	−.26	.69	.84
France	0.90	0.82	0.99	2.96	0.71	0.55	0.76	.78	.61	.79	.25	−.30	.77	.96
Germany	1.51	0.79	0.90	2.93	0.81	0.61	0.83	.65	.66	.84	.26	−.11	.59	.93
italy	1.69	1.33	0.78	1.95	0.42	0.44	0.92	.85	.82	.86	.01	−.68	.42	.96
japan	1.35	0.93	1.09	2.41	0.79	0.36	0.88	.80	.80	.90	−.02	−.22	.60	.98
Switzerland	1.92	1.32	0.74	2.30	0.53	0.71	0.67	.90	.81	.82	.27	−.68	.84	.93
United Kingdom	1.61	1.19	1.15	2.29	0.69	0.68	0.88	.63	.74	.59	.05	−.19	.47	.90
United States	1.92	0.52	0.75	3.27	0.75	0.61	0.68	.86	.82	.94	.12	−.37	.88	.96
Europe	1.01	0.50	0.83	2.09	0.47	0.85	0.98	.75	.81	.89	.10	−.25	.32	.85

Notes: Statistics are based on Hodrick-Prescott–filtered data. Variables are *y*, real output; *c*, real consumption; *x*, real fixed investment; *g* real government purchases; *nx*, ratio of net exports to output, both at current prices; *n*, civilian employment; *z*, Solow residual, defined in text. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Data are quarterly from the OECD's *Quarterly National Accounts*, except employment, which is from the OECD's *Main Economic Indicators*. The sample period is 1970:I to 1990:II.

International Comovements in OECD Economies

	Correlation of Each Country's Variable with Same U.S. Variable					
Country	*y*	*c*	*x*	*g*	*n*	*z*
Australia	.51	−.19	.16	.23	−.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	−.01	−.01	.53	.75
France	.41	.39	.22	−.20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	−.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	−.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Basic Ingredients

Same as in BKK except : 1.) Ignore Inventories
2.) Ignore distributed lag on leisure

Preferences

$$U_i = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, 1-n_{it})$$

$$u(c, 1-n) = \frac{\left[c^{\theta}(1-n)^{1-\theta}\right]^{1-\rho}}{1-\rho}$$

Technology

$$y_{it} = z_{it} k_{it}^{\alpha} n_{it}^{1-\alpha}$$

$$k_{it+1} = (1-\delta)k_{it} + s_{it}^{1}$$

"Time-to-Build"

s_{it}^{j} = # of investment projects in country i at date t that are j periods from completion

$$s_{i,t+1}^{j} = s_{i,t}^{j+1}$$

x_{it} = fixed capital formation $= \frac{1}{J}\sum_{j=1}^{J} s_{it}^{j}$

It takes J periods of investment to complete a project

Productivity Shocks

$$\begin{pmatrix} z_t \\ z^*_t \end{pmatrix} = \underset{(2 \times 2)}{A} \begin{pmatrix} z_{t-1} \\ z^*_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon^*_t \end{pmatrix}$$

Planner's (Pareto) Problem

$$\max_{c,\,n,\,c^*,\,n^*} \quad \phi\left\{E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)\right\} + (1-\phi)\left\{E_0 \sum_{t=0}^{\infty} \beta^t u(c^*_t, 1-n^*_t)\right\}$$

subject to $\quad (\phi = 1/2)$

1.) $c_t + c^*_t + x_t + x^*_t = z_t k_t^{\alpha} n_t^{1-\alpha} + z^*_t k_t^{*\alpha} n_t^{*\,1-\alpha}$

2.) $k_{t+1} = (1-\delta) k_t + x_t$

3.) $k^*_{t+1} = (1-\delta) k^*_t + x^*_t$

Solution Strategy

1.) Compute steady state

2.) Calibrate parameter values to match key features of the steady state

3.) Take a quadratic approximation of the objective function around the steady state.

4.) Compute (linear) policy functions using standard methods.

Policy Functions

$$c_t = g(k_t, k_t^*, z_t, z_t^*)$$

$$c_t^* = g^*(k_t, k_t^*, z_t, z_t^*)$$

$$n_t = h(k_t, k_t^*, z_t, z_t^*)$$

$$n_t^* = h^*(k_t, k_t^*, z_t, z_t^*)$$

$$k_{t+1} = f(k_t, k_t^*, z_t, z_t^*)$$

$$k_{t+1}^* = f^*(k_t, k_t^*, z_t, z_t^*)$$

Calibration (Time unit = 1/4 year)

$\beta = .99$

$\theta = .34$

$\rho = 2$ $\Longrightarrow$ 1% (quarterly) real interest rate

$\alpha = .36$

$\bar{c}/\bar{y} = .75$

$\delta = .025$

labor share of GDP = .64

$J = 4$

30% of (non-sleeping) time devoted to market etc.

$$A = \begin{bmatrix} .906 & .088 \\ .088 & .906 \end{bmatrix}$$

$$\mathrm{var}(\varepsilon) = \mathrm{var}(\varepsilon^*) = (.00852)^2$$

$$\mathrm{corr}(\varepsilon, \varepsilon^*) = .258$$

Caveats

1.) z, z^* computing from Solow residuals, with labor as the only input. z^* from a composite of 6 European economies.

2.) n^* based on employment not hours.

Business Cycles in Theoretical Economies

A. Business Cycle Properties

Economy	Standard Deviation (%)		Ratio of Standard Deviation to That of y				Autocorr.	Correlation with Output				
	y	*nx*	*c*	*x*	*n*	*z*	*y*	*c*	*x*	*nx*	*n*	*z*
U.S. data	1.92	0.52	.75	3.27	.61	.68	.86	.82	.94	−.37	.88	.96
Benchmark	1.50	3.77	.42	10.99	.50	.67	.62	.77	.27	.01	.93	.89
Transport cost	1.35	0.37	.47	2.91	.47	.75	.61	.81	.92	.23	.92	.98
Autarky	1.26		.54	2.65	.91	.99	.62	.90	.96		.91	.99

B. International Comovements

Economy	Correlation of Foreign and Domestic Variables				
	y	*c*	*x*	*n*	*z*
U.S. data	.66	.51	.53	.33	.56
Benchmark	−.21	.88	−.94	−.94	.25
Transport cost	−.05	.89	−.48	−.48	.25
Autarky	.08	.56	−.31	−.31	.25

Notes: Statistics are based on Hodrick-Prescott–filtered data. Variables are defined in notes to Table 11.1. Entries are averages over twenty simulations of length 100. The data column refers to the United States in part A and to correlation between the United States and Europe in part B.

Problems with Baseline Model

1.) I + NX are too volatile

2.) C is too highly correlated across countries

3.) Y, I, + N are negatively correlated across countries

4.) NX is (weakly) positively correlated with Y

Extension 1: Trading Frictions

- Subtract a trade cost of $G(nx) = \tau\, nx^2$ from the aggregate resource constraint

$\tau = .1/y^{ss}$ $\Longrightarrow$ marginal shipping cost is 0.58% in each country when $nx/y = .029$

Result: This successfully reduces the volatility of I and NX, but doesn't fix the other problems.

Extension 2: Multiple Goods

- Multiple goods allow us to study the Terms of Trade, and how they transmit shocks between countries. With multiple goods we can also study one of the basic issues in open-economy macro, i.e., the relationship between the Trade Balance and the Terms of Trade.

Assumptions

1.) Each country is specialized in the production of its own (intermediate) good.

2.) Countries trade intermediate goods, but not final goods.

3.) Home country uses inputs of Capital + Labor to produce intermediate good "a"
Foreign country uses Capital + Labor to produce intermediate good "b"

4.) Final goods are produced via a CES "Armington Aggregator"

Preferences

(Same as Before)

Technology

$$a_t + a_t^* = z_t k_t^{\alpha} n_t^{1-\alpha}$$

a^*: H exports of "a" to F

$$b_t + b_t^* = z_t^* k_t^{*\alpha} n_t^{*1-\alpha}$$

b^* = F exports of "b" to H

$$c_t + i_t + g_t = G(a_t, b_t) = \left[\omega a_t^{1-\gamma} + b_t^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

$$c_t^* + i_t^* + g_t^* = G(a_t^*, b_t^*) = \left[\omega a^{*1-\gamma} + b_t^{*1-\gamma}\right]^{\frac{1}{1-\gamma}}$$

$\sigma = 1/\gamma$ = elasticity of substitution between H and F goods

ω = share parameter

With this specification,

$$TOT = \frac{\text{Price of Imports}}{\text{Price of Exports}} = P_t = \frac{\partial G/\partial b}{\partial G/\partial a}$$

$$= \frac{1}{\omega}\left(\frac{a_t}{b_t}\right)^{1/\sigma}$$

Taking logs,

$$\boxed{\ln P_t = -\ln \omega - \frac{1}{\sigma}\ln\left(b_t/a_t\right)}$$

Just a derived demand curve

Low elast. (σ small) $\Longrightarrow$ prices sensitive to quantities

$\Longrightarrow$ isoquants are very curved.

H's Trade Balance,

$$\boxed{NX_t = a_t^* - P_t \cdot b_t}$$

Calibration

$\sigma = 1.5$ (small elast. case $\sigma = 0.5$)

ω set so that $\frac{\text{imports}}{\text{GDP}} = .15$ in steady state

Properties of the Terms of Trade in OECD Economies

Country	SD p(%)	Autocorr. p	Correlation of: (p, nx)	(p, y)
Australia	5.78	.82	−.10	−.27
Austria	1.73	.46	−.24	.04
Canada	2.99	.85	.05	−.05
France	3.52	.75	−.50	−.13
Germany	2.66	.85	−.08	−.11
Italy	3.50	.78	−.66	.38
Japan	7.24	.86	−.56	−.22
Switzerland	2.85	.88	−.61	.41
United Kingdom	3.14	.80	−.58	.09
Europe	3.68	.83	.30	−.20

Notes: Statistics are based on Hodrick-Prescott–filtered data. Variables are p, terms of trade, relative price of imports to exports; y, real output; nx, ratio of net exports to output, both at current prices. Except for the ratio of net exports to output, statistics refer to logarithms of variables. Most variables are from the OECD's *Quarterly National Accounts*. The sample period is 1970:I to 1990:II.

Properties of the Terms of Trade in Theoretical Economics

Country	SD p(%)	Autocorr. p	Correlation of: (p, nx)	(p, y)
U.S. data	3.68	.83	.30	−.20
Benchmark	0.48	.83	−.41	.49
Two shocks (technology and government spending)	0.57	.67	−.05	.39
Large import share	0.66	.83	−.41	.55
Small elasticity	0.76	.77	−.80	.51

Notes: Statistics are based on Hodrick-Prescott–filtered data. Variables are defined in the notes to Table 11.5. Entries are averages over 20 simulations of length 100. The data column refers to the United States.

Results

1.) Model can match persistence of p

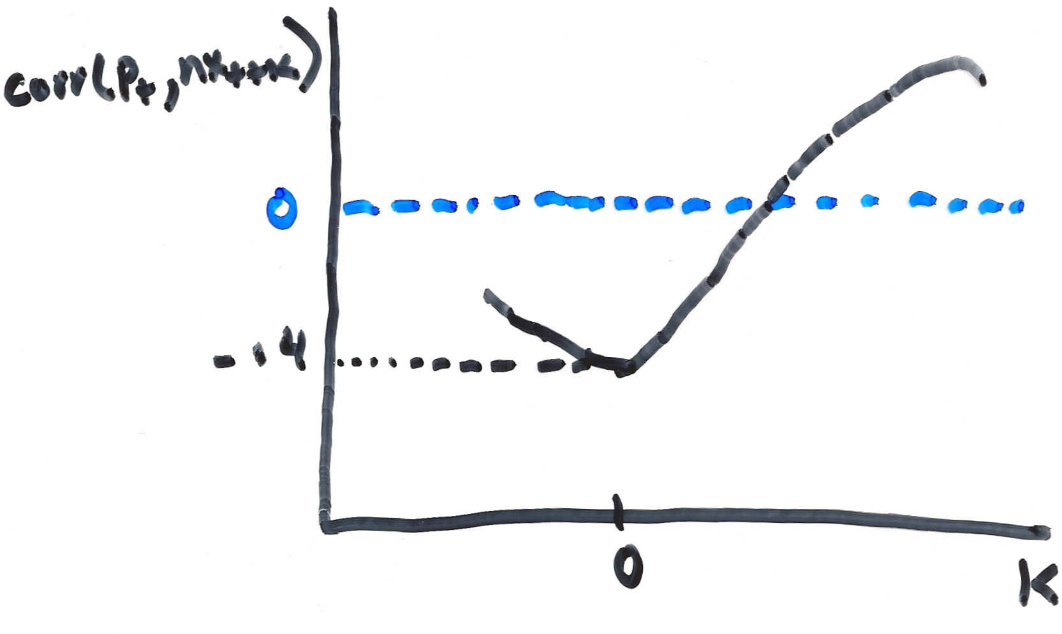
2.) Model can't match volatility of p
(model p is too smooth).

3.) Model can't match $corr(p, nx)$ for U.S.,
but can for other countries

$corr(p, nx) > 0$ for U.S.

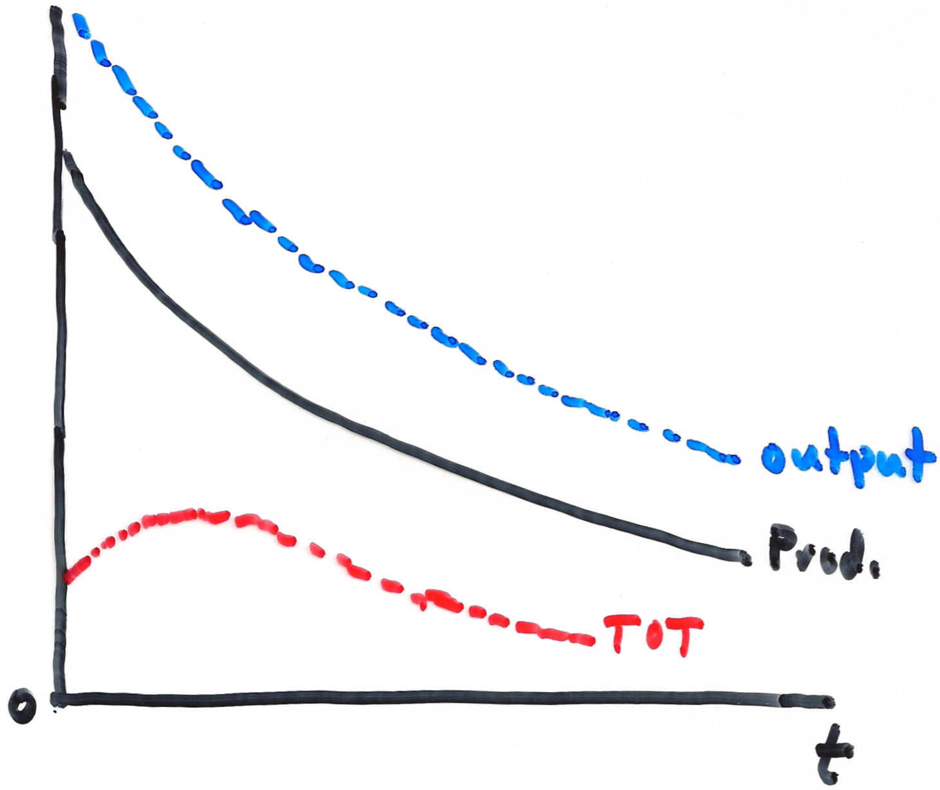
$corr(p, nx) < 0$ for other countries

Dynamics

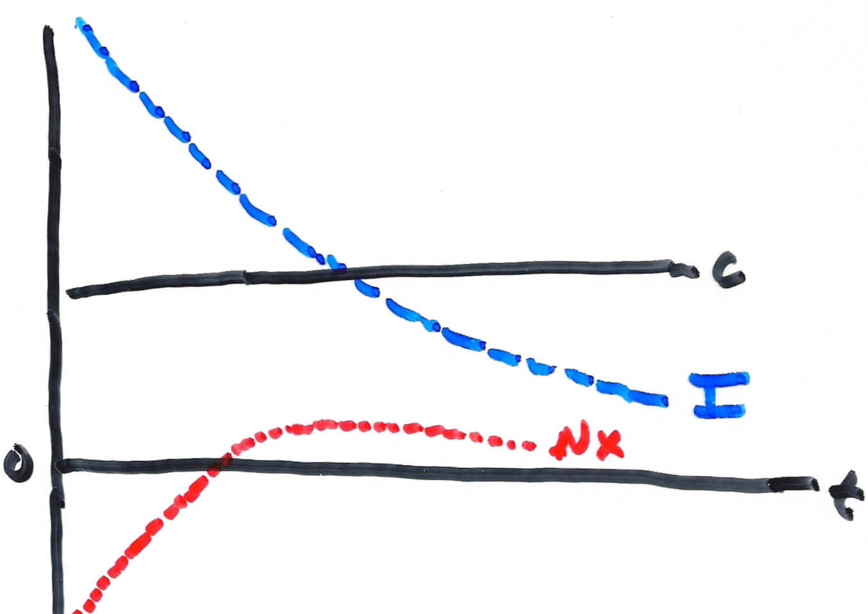


This pattern resembles a "J-curve", but has nothing to do with import elasticities or the "Marshall-Lerner" Condition. It reflects investment dynamics

Impulse Response Functions (to an H productivity shock)



TOT↑ due to increased supply of domestic goods which lowers its price



nx↓ because investment rises more than saving

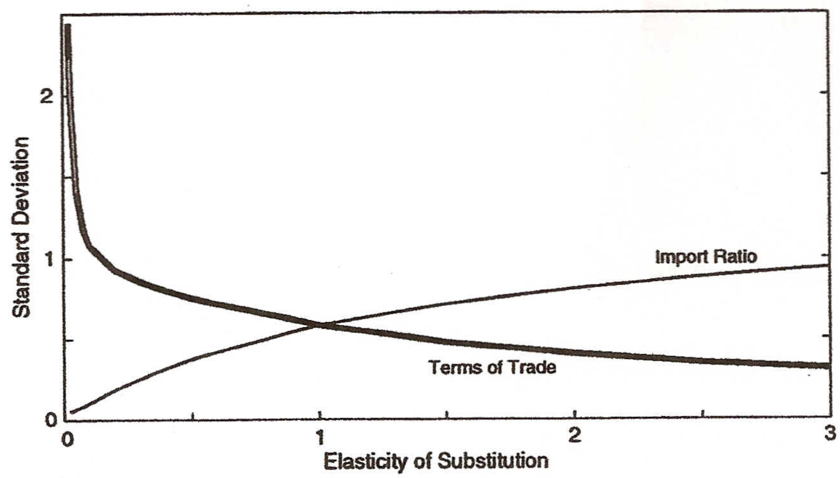


Figure 11.4 Effects of Varying the Elasticity of Substitution between Foreign and Domestic Goods on the Volatility of the Terms of Trade and the Import Ratio

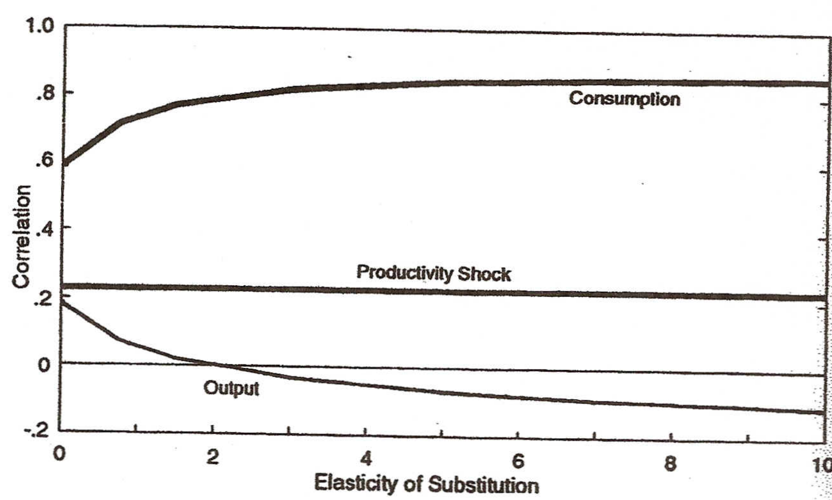


Figure 11.5 Effects of Varying the Elasticity of Substitution between Foreign and Domestic Goods on the Cross-Country Correlations of Selected Variables