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| International Business Cycles |
| * replication of Backus-Kehoe-Kydland
 |
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 All countries experience business cycles, yet their responses and behaviours are different. Particular countries are more susceptible to technology shocks evidenced by increased variations of macroeconomic aggregates. Discovering and modeling consistent international co-movements between countries would greatly improve a models relevance and practicality. Canada and the United States are the countries currently being examined. The initial process is to evaluate the current aggregate situations of these countries and then to discover how accurate the model is relative to the real world circumstances. In this model the countries experience stochastic yet correlated technology shocks and access to international capital markets. It is hoped that the combination of these assumption will enable precise estimates. The purpose is to explore the interactions of macroeconomic aggregates with output fluctuations.

 The first objective was to collect data which was accomplished via CANSIM and BEA. The macroeconomic aggregates of interest were consumption, investment, gross national product (GNP), hours worked, capital stock, inventories and net exports. GNP, hours worked and capital stock for both countries was easily gathered but some of the other categories involved discretion. Consumption includes only non-durables and services. Durables were not included as these could be viewed as a type of investment since the flow of utility is over a long period. Investment is meant for new capital formation and recovering depreciated stock (fixed capital investment) while investment in inventories is seen as a separate category. Net exports were computed by subtracting imports of goods and services for exports of goods and services. Net exports (NX) will always be analyzed as$\frac{NX}{Y}$ because the ratio indicates the relative importance of NX to country income. The final pieces of data gathered were savings rates ($S=Y-C-G$) and interest rates. The minimum sample period was 1990-2007, but the maximum range of 1961-2008 was used were both countries had entries.

 All figures were computed in real terms but some of the information was gathered in nominal. For this reason a deflator was created for both countries via$\frac{nGDP}{rGDP}$. Furthermore, some information was not given in quarterly data. For these entries the geometric growth rate was calculated:

$$r= \frac{ln⁡(\frac{K\_{t+1}}{K\_{t}})}{4}$$

Then each year was turned quarterly so as could be viewed with other categories. Logarithms were used for every category except NX. The data was then hpfiltered in Matlab and differences taken. Standard deviations and correlations of the de-trended data (centered on zero) were reported for comparison. The standard deviations were reported in percent form to evaluate the volatility of the macroeconomic aggregates, and then the ratio was taken against output to establish relative instability.

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|  | **Cyclical Properties of the U.S Economy** |
| **B-K-K** |  | *Standard Deviation:* |  |
|  |  | % | Relative to Y |
| Output | 1.71 | 1.00 |
| Consumption | 0.84 | 0.49 |
| Investment | 5.38 | 3.15 |
| Hours | 1.47 | 0.86 |
| Capital Stock | 0.63 | 0.37 |
| Inventory | 1.65 | 0.96 |
| NX/Y | 0.45 | N/A |

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|  | **Cyclical Properties of the U.S Economy** |
| **Callum** |  | *Standard Deviation:* |  |
|  |  | % | *Relative to Y* |
| Output | 1.7129 | 1.00 |
| Consumption | 0.6216 | 0.3629 |
| Investment | 4.2743 | 2.4954 |
| Hours | 1.5791 | 0.9219 |
| Capital Stock | 0.8421 | 0.4916 |
| Inventory | 1.4122 | 0.8245 |
| NX/Y | 0.3322 | N/A |

The above charts depict the U.S situation given by B-K-K from 1954-1985, while Callum is updated until 2007. It appears that the macroeconomic aggregates are similar to those found by Backus-Kehoe-Kydland. U.S data exhibits consumption being less volatile, investment that is more volatile and hours which is slightly less volatile than output. These empirical ‘facts of business cycles’ taught in class are clearly exemplified here. The ratio of consumption to output variation has decreased over the new period. This is indicative of increased consumption risk sharing through completer markets. The U.S appears to be more able hedge idiosyncratic risk than they could in the earlier B-K-K period.

 The next figures analyzed were cross correlations both between and within Canada and the U.S. The charts below show that updated data sets still portray similar results as the time period examined by B-K-K. It is clear correlation of consumption across countries has diminished. Possible explanations are increase of incomplete markets or growth in service sectors. Increase in incomplete markets is unlikely, as the previous results show otherwise. The service sector justification seems more probable, as the relative size of this segment of the economy has drastically increased. The service sector is also non-traded (for the most part); therefore consumption risk between countries cannot be fully shared. The two inferences about consumption point to the U.S being more able to diversify idiosyncratic risk in tradable goods, yet still unable to deal with risk in their growing service sector. Net exports seem to be analogous in both periods for standard deviations and correlation with output. And finally, a noticeable correlation difference within country saving rate and interest rate for Canada exists. The correlation goes from positive 0.06 in B-K-K to negative -0.6 in the updated period. The standard deviation % of the savings rate in Canada is 3.7919, opposed to the 1.7113 in the U.S. Clearly savings is very volatile in Canada and it is possible there is spurious correlation over the sample period.

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| **BKK** | **International Co-movements** |
|  | Contemporaneous Cross Correlations |  |  |  |
|  | *Vs U.S.* | *Within Country* |  | *Standard deviations %:* |
| Country | Output | Consumption | Save Vs. r | NX/Y Vs. Y |  | NX/Y |
| Canada: | 0.77 | 0.65 | 0.06 | -0.29 |  | 0.79 |
| U.S: | 1.00 | 1.00 | 0.68 | -0.36 |  | 0.42 |

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| **Callum** | **International Co-movements** |
|  | Contemporaneous Cross Correlations |  |  |  |
|  | *Vs U.S.* | *Within Country* |  | *Standard deviations %:* |
| Country | Output | Consumption | Save Vs. r | NX/Y Vs. Y |  | NX/Y |
| Canada: | 0.6545 | 0.5661 | -0.6378 | -0.2935 |  | 0.8360 |
| U.S: | 1.00 | 1.00 | 0.7233 | -0.4210 |  | 0.3322 |

The model is of two countries involved in international trade. The model exhibits a homogenous product and complete markets for state contingent claims. Yet technology shocks and capital market changes are added in an attempt to improve the results. The addition of international capital markets allows countries to smooth idiosyncratic risk. The technology shocks are meant to resemble the real world, where shocks are correlated across countries (aggregate in nature) and display spillover.

$$Z\_{t+1}=AZ\_{t}+ \in \_{t+1}$$

Where$ Z\_{t}≡\left(Z\_{t}^{h} Z\_{t}^{f}\right) \in \_{t}≡\left(\in \_{t}^{h} \in \_{t}^{f}\right)$

The spillover is when another country’s technology shock last period affects your technology shock this period through the dissemination of technology. The correlation is through the Є’s commoving, while the spillover is propagated through the off diagonal of A.

$$A = \left[\begin{matrix}0.906&0.088\\0.088&0.906\end{matrix}\right] A\_{2} = \left[\begin{matrix}0.796&0.131\\0.00&0.989\end{matrix}\right]$$

The benchmark calibration used A, because symmetry between the countries was imposed. However B-K-K did calculate the matrix for U.S and Canada during their period. It appears from $A\_{2}$that American technology shocks have little or no impact on Canadian technological growth. The model will be run with both calibrations to see if the shock propagation mechanism changes the results.

$$E\_{0}\sum\_{t=0}^{\infty }β^{t}U(c\_{t},l\_{t})$$

 The objective was to maximize expected utility subject to resource constraints. The utility function chosen was: $U= \frac{(C^{μ}l^{1-μ})^{γ}}{γ}$ when 0 < μ < 1 & γ = 0 [log utility]

The parameter values were chosen to resemble steady-state conditions in the U.S economy post World War 2. The parameter α = 1 meant that there is no lagged effects of leisure or the model imposed ‘separability’ of leisure.

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|  |  | Benchmark Parameter Values |  |  |
| Preferences: | β = 0.99 | μ = 0.34 | α = 1 | γ = 0 |  |
| Technology: | δ = 0.25 | σ=0.00852  | var(Є) = σ2 |  |  |
|  |  | corr(Є,Є\*) = 0.258 |  |  |  |

The model was ran with the above parameters and with the A matrix used. The results were than reported to assess their proximity to the true values. With the updated data the U.S consumption variation was accurately replicated. The standard deviation of hours worked is roughly half, this could be caused since work is divisible. With divisible labour people may work slightly more/less when faced by shocks, but the variation will be smaller than in reality where people are unable to do this. Usually people cannot change labour by this amount so when labour does shift, it clearly must be more than in a divisible case.

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|  | **Benchmark Economy** |  | **U.S Economy (Callum)** |
| **BKK** |  | *Standard Deviation:* |  | *Standard Deviation:* |
|  |  | % | Relative to Y | % | *Relative to Y* |
| Output |   | 1.55 | 1.00 | 1.7129 | 1.00 |
| Consumption | 0.62 | 0.40 | 0.6216 | 0.3629 |
| Investment | 16.91 | 10.94 | 4.2743 | 2.4954 |
| Hours |   | 0.76 | 0.49 | 1.5791 | 0.9219 |
| Capital Stock | 1.16 | 0.74 | 0.8421 | 0.4916 |
| Inventory | 3.70 | 2.39 | 1.4122 | 0.8245 |
| NX/Y |   | 2.90 | N/A | 0.3322 | N/A |

The model further over estimates the variance of investment (x4), inventory (x2) and capital stock. These three categories are all investment saving type classifications. In the model complete markets were assumed. Under this assumption people should invest more than expected when a positive technology shock appears. This is because they invest more and also borrow to further invest against the future increase in output. With incomplete markets these countries may be unable to take advantage of their positive shocks to the full extent. Thus variation in the data would be lower than in the model.

 Next, B-K-K tried the model with asymmetric spillovers [$A\_{2}$]. The asymmetric spillover has marginal effects on all aggregates except$\frac{NX}{Y}$. Contemporaneous cross correlations within a country increase to 0.30. With the countries being hit with more drastically different shocks, the net exports are more highly correlated with output. Countries use imports and exports to smooth the shocks.

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| Statistics |  | U.S | Benchmark | Asymmetric A  | Autarky |
| *Standard Deviations %:* |  |  |  |  |
| Output |  | 1.7129 | 1.55 | 1.54 | 1.33 |
| Net exports / Output | 0.3322 | 2.90 | 3.08 | N/A |
| *Relative to Output:* |  |  |  |  |
| Consumption | 0.3629 | 0.40 | 0.42 | 0.48 |
| Investment | 2.4954 | 10.94 | 10.60 | 2.31 |
| *Cross Correlations:* |  |  |  |  |
| Consumption | 0.7876 | 0.79 | 0.76 | 0.91 |
| Net exports / Output | -0.4210 | -0.02 | 0.30 | N/A |
| *Country Cross [CAN]:* |  |  |  |  |
| Output |  | 0.6545 | -0.18 | -0.26 | 0.11 |
| Consumption | 0.5661 | 0.88 | 0.85 | 0.73 |
|  |  |  |  |  |  |

 To replicate the B-K-K model Dynare was utilized. A two country set up was created with ‘star’ denoting foreign. The countries were set to have the same utility functions. The first step was to create correlated technology shocks that exhibited spillover effects. This was accomplished by modeling the current period error terms between the two countries to be correlated [corr e, estar = 0.258]. The spillover effect was imposed differently. Each country’s technology shock $Z\_{t+1}=AZ\_{t}+ \in \_{t+1}$comprises of the now correlated Є and the previous technology shocks from each country. The size of the coefficient on the opposing countries $Z\_{t}$ dictates the spillover effect. Unfortunately I was unable to get the two countries trading properly, so a look at the benchmark case is impossible. But I was able to replicate the autarky situation with both asymmetric and symmetric A matrix. The two cases look similar except in consumption. The variation of consumption relative to output is much lower in the asymmetric case and the correlation between countries is much higher.

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|  |  |  | **Autarky** |
| Statistics |  | U.S | Asymmetric | Symmetric |
| *Standard Deviations %:* |  |  |  |
| Output |  | 1.7129 | 1.2396 | 1.2660 |
| Net exports / Output | 0.3322 | N/A | N/A |
| *Relative to Output:* |  |  |  |
| Consumption | 0.3629 | 0.1530 | 0.3352 |
| Investment | 2.4954 | 0.8738 | 0.7342 |
| *Cross Correlations:* |  |  |  |
| Consumption | 0.7876 | 0.8494 | 0.8519 |
| Net exports / Output | -0.4210 | N/A | N/A |
| Country Cross: [CAN] |  |  |  |
| Output |  | 0.6545 | 0.4061 | 0.1118 |
| Consumption | 0.5661 | 0.9484 | 0.8225 |

The lower variation follows from their not being any feedback in the foreign technology shock to home technology shocks. With this not occurring, the technology shocks hitting home will not be as variable since one of the exogenous shock terms is less influential. Consumption correlation between the two countries is then increased since overall variation has decreased and the Є’s are still correlated.

 It is clear the macroeconomic aggregates display certain characteristics. These trends can be fashioned into real business cycle theory so model predictions are more accurate. The addition of technological spillovers, correlated shocks and participation in international capital markets definitely increases precision. Crucial in using models effectively is to understand how the assumptions of the model cause differences between realized ratios and those predicted. Assuming complete markets and perfect free trade can explain why models over predict consumption correlations. The model proposed by B-K-K is fairly good at estimating real world macroeconomic aggregates.

Appendix

This was the code used for transforming the information from annual to quarterly.

for i=1:1000

 for j=1:1000

 capital(i,1)=cap(j,1)

 capital(i+1,1)=cap(j,1)\*r(j,1)

 capital(i+2,1)=cap(j,1)\*(r(j,1))^2

 capital(i+3,1)=cap(j,1)\*(r(j,1))^3

 i=i+4

 j=j+1

 end

end

% Basic RBC Model in Dynare

%----------------------------------------------------------------

% 1. Defining variables

%----------------------------------------------------------------

var y ystar c cstar k kstar i istar l lstar z zstar;

varexo e estar;

parameters beta psi delta alpha rho;

%----------------------------------------------------------------

% 2. Calibration

%----------------------------------------------------------------

alpha = 0.34;

beta = 0.99;

delta = 0.025;

psi = 1.75;

rho = 0.95;

sigma = .00852;

%----------------------------------------------------------------

% 3. Model

%----------------------------------------------------------------

model;

 (1/c) = beta\*(1/c(+1))\*(1+alpha\*(k^(alpha-1))\*(exp(z(+1))\*l(+1))^(1-alpha)-delta);

 psi\*c/(1-l) = (1-alpha)\*(k(-1)^alpha)\*(exp(z)^(1-alpha))\*(l^(-alpha));

 c+i = y;

 y = (k(-1)^alpha)\*(exp(z)\*l)^(1-alpha);

 i = k-(1-delta)\*k(-1);

 z = 0.796\*z(-1)+0.131\*zstar(-1)+e;

 (1/cstar) = beta\*(1/cstar(+1))\*(1+alpha\*(kstar^(alpha-1))\*(exp(zstar(+1))\*lstar(+1))^(1-alpha)-delta);

 psi\*cstar/(1-lstar) = (1-alpha)\*(kstar(-1)^alpha)\*(exp(zstar)^(1-alpha))\*(lstar^(-alpha));

 cstar+istar = ystar;

 ystar = (kstar(-1)^alpha)\*(exp(zstar)\*lstar)^(1-alpha);

 istar = kstar-(1-delta)\*kstar(-1);

 zstar = 0.0\*zstar(-1)+0.989\*z(-1)+estar;

end;

%----------------------------------------------------------------

% 4. Computation

%----------------------------------------------------------------

initval;

 k = 12.44579408;

 c = 0.579697213;

 l = 0.3;

 z = 0;

 e = 0;

 kstar = 12.44579408;

 cstar = 0.579697213;

 lstar = 0.3;

 zstar = 0;

 estar = 0;

end;

shocks;

var e = sigma^2;

var estar = sigma^2;

corr e, estar = 0.258;

end;

steady;

stoch\_simul(hp\_filter = 1600, order = 1, periods=125);

Works Cited

Backus, David; Kehoe, Patrick and Kydland, Finn. International Real Business Cycles. *The Journal of Political Economy*. vol.100 issue 4.

CANSIM: GNP – v499688; consumption – v1992047, 8; savings – v41707154, 5, 6;

 net exports – v41707154

BEA: GNP – 1.7.5; consumption – 2.3.6; investment – 5.1.5; hours – 6.9B-D; capital – 9.1 fixed; inventories – 5.7.6A-B; net exports – 1.1.6; savings – 5.1