Decentralization

- In the real world, AD securities are not traded. However, it may be possible that actually traded securities, like stocks, bonds, & derivatives, produce the same outcomes.

- This will always be true if there are as many (linearly independent) traded securities as there are states of nature. A more interesting (and relevant) possibility is that spanning occurs even when the number of securities is far less than the number of states. We will now see an example of this.

- More generally, we want to ask how a given Pareto Optimal allocation of risk can be decentralized or supported by trade in financial assets.

Assumptions

1.) 2 dates

2.) S states in period 2

3.) N countries

4.) Endowment Economy

5.) Only equities are traded (bonds too, but they turn out to be redundant).

V_1^n = date-1 market value of country-n's date-2 output.

= price of an asset that pays $Y_2^n(s)$ in state s.

Let x_m^n = fraction of shares in country-m's output held by residents of country-n

Budget Constraints $(n = 1, 2, \cdots N)$

$$C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m = Y_1^n + V_1^n$$

$\uparrow$ net bond purchases

Note, countries are initially endowed with all claims to its own output.

$$C_2^n(s) = (1+r) B_2^n + \sum_{m=1}^{N} x_m^n Y_2^m(s)$$

First-Order Conditions

$$B_2^n : \quad u'(C_1^n) = \beta(1+r) \sum_{s=1}^{S} \pi(s)\, u'[C_2^n(s)]$$

$$x_m^n : \quad V_1^m u'(C_1^n) = \beta \sum_{s=1}^{S} \pi(s)\, u'[C_2^n(s)]\, Y_2^m(s)$$

To solve, assume all countries have identical CRRA preferences

It turns out to be easier to pursue a guess & verify solution strategy, which involves the following 3 steps:

1.) Guess an allocation

2.) Given this allocation, compute equilibrium prices

3.) Go back and check that at these prices each country's allocation satifies its Euler eq. & budget constraint.

To begin, let

$$\mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{N}(Y_1^m + V_1^m)} = \text{country·n's share of initial world wealth}$$

Now guess the following allocations,

(a.) $C_1^n = \mu^n \sum_{m=1}^{N} Y_1^m = \mu^n Y_1^w$

(b.) $C_2^n(s) = \mu^n \sum_{m=1}^{N} Y_2^m(s) = \mu^n Y_2^w(s)$

Note that (b.) is consistent with n's period·2 budget constraint when $x_m^n = \mu^n$ $m = 1, 2, \ldots N$ and $B_2^n = 0$.

It can also be easily verified that the conjectured allocation in (a) satisfies n's period-1 budget constraint. Suppose $B_2^n = 0$ and $x_m^n = \mu^n$ $\forall$ $m = 1, 2, \dots N$. Then the period-1 budget constraint becomes:

$$Y_1^n + V_1^n = C_1^n + \sum_{m=1}^{N} x_m^n V_1^m = \mu^n \sum_{m=1}^{N} Y_1^m + \mu^n \sum_{m=1}^{N} V_1^m$$

which is identically true by defn. of μ^n.

What about optimality? To verify optimality we just neet to make sure the conjectured allocation satisfies each country's Euler eqs.

It can be readily verified that the Euler eqs. are satisfied at the conjectured allocations when:

$$1+r = \frac{(Y_1^w)^{-\rho}}{\beta \sum_{s=1}^{S} \pi(s)(Y_2^w(s))^{-\rho}}$$

$$V_1^m = \beta \sum_{s=1}^{S} \pi(s) \left[\frac{Y_2^w(s)}{Y_1^w} \right]^{-\rho} Y_2^m(s)$$

(To see this, not that the μ^n terms cancel out).

- Now, the punch-line is that we get the same complete markets, Pareto Optimal, allocation as before (when there were AD securities), even though it could well be that $N \ll S$!

- To see why, note that the equilibrium V_1^n turns out to be the same as the value of each country's uncertain future output evaluated at the date 1 AD prices.

- This is not a general result, however. It hinges crucially on the fact that people care about multiplicative risk (CRRA preferences) and the fact that equities provide a share, or fraction, of each country's endowment. With CRRA preferences, everyone holds the same portfolio of risky assets, regardless of the initial wealth levels. If this isn't the case, then things get much more complicated.

To summarize, the above decentralization makes two predictions:

1.) Everyone holds the same portfolio of risky assets (a world "mutual fund").

2.) Home equity share = Home country's share of world wealth.

Both predictions are counterfactual. For large countries like the U.S. + Japan, home equity share is more like 90%.

So countries seem to exhibit a portfolio home bias.

For countries like the U.S. & Japan, home bias might actually be getting worse, since the share of foreign equities in their portfolios has probably increased by less than the reductions in their shares of world GDP or stock market capitalization.

Potential Explanations of Portfolio Home Bias

1.) Incomplete Asset Markets
- Asymmetric Info (ability to repay)
- Lack of Enforcement/Commitment (willingness to repay)
- Govt. regulations / taxes

2.) Goods / Factor Market Distortions (even with complete asset markets)
- Non-tradeable Goods
- Non-tradeable Labor Income

3.) Small Gains
- Dynamic Spanning (with bonds)
- Endogenous Terms of Trade (Cole & Obstfeld '91)
- Lucas '87

4.) Multinationals / Indirect Intl. Diversification

For now, we will focus on (2) & (3). Later we will discuss (1) in the context of open-economy DSGE models.