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Econ 842 International Monetary Economics Prof. Kasa Spring 2011

MIDTERM EXAM (Solutions)

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (10 points each).

1. Positive productivity shocks produce current account surpluses.

UNCERTAIN. It depends on how persistent the shock is. If it's temporary, it will increase saving, with little or no effect on investment. In this case, it will produce a surplus. However, if it's highly persistent, it will reduce saving (because income will be higher in the future relative to the present, due to a higher capital stock), and increase investment. As a result, it would produce a current account deficit.

2. The Balassa-Samuelson theory of real exchange rate determination is not applicable in countries like China, which fix their exchange rates.

FALSE. Rapid rapid productivity growth in Chinese tradeables should lead to a <u>real</u> appreciation of the Chinese currency. However, if China's <u>nominal</u> exchange rate is fixed by the central bank, then a real appreciation will be reflected in a higher Chinese inflation rate (relative to the U.S.). (Remember, the real exchange rate is $q = EP^*/P$, so q can fall either because E falls, <u>or</u> because P^*/P falls). This is one reason why China faces a persistent threat of inflation.

3. Bubbles in nominal exchange rates cannot exist if investors have rational expectations.

FALSE/UNCERTAIN. It is quite possible for there to be <u>rational</u> bubbles in the prices of nominal assets. (Conditions for the existence of rational bubbles in real assets are far more stringent). The bubble will be 'rational', or self-fulfilling, as long as the price rises at the discount rate. However, no one has come up with a convincing story for how bubbles get started in the first place. (Note that the innovation in the bubble must be mean zero. But it can't have a mean of zero when the current value of the bubble is zero, since prices can't be negative!).

4. According to Uncovered Interest Parity, countries with relatively high nominal interest rates should have depreciating currencies.

TRUE. Countries with depreciating currencies must offer higher interest rates as compensation for the capital loss arising from depreciation. (Remember, UIP neglects the possible existence of risk premia. In principle, a country could have both a depreciating currency and a lower interest rate if it were considered to be less risky).

The following questions are short answer. Briefly explain your answer. Clarity will be rewarded.

5. (30 points). Consider a small open economy that can borrow or lend all it wants at a fixed world interest rate, r. Suppose preferences are quadratic, and the rate of time preference equals the interest

rate (i.e., $\beta(1+r) = 1$). As discussed in class, combining the Euler equation with the budget constraint produces the following consumption function,

$$C_t = \frac{r}{1+r} \left[(1+r)B_t + E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j (Y_{t+j} - I_{t+j} - G_{t+j}) \right]$$

which then produces the following expression for the current account, CA_t ,

$$CA_{t} = Y_{t} - I_{t} - G_{t} - \frac{r}{1+r} E_{t} \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^{j} \left(Y_{t+j} - I_{t+j} - G_{t+j}\right)$$

Suppose the economy produces output with the production function,

$$Y_t = A_t K_t^{\alpha}$$

where productivity, A_t , grows according to the process,

$$A_{t+1} = (1+g)^{1-\alpha} A_t$$

where 0 < g < r. Finally, suppose government spending, G_t , is a fixed fraction, γ , of output (and that $\gamma < 1 - (\alpha g/r)$.

(a) Calculate the optimal capital stock in this economy. Assume that capital does not depreciate, so that $I_t = K_{t+1} - K_t$. Using your previous answer, and the given process for A_t , calculate I_t in terms of α , r, A_t , and g.

In this small open economy, optimal investment occures up until the point where MPK = r. Taking the derivatives and solving for K_t yields,

$$K_t = \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}}$$

Using this in the capital accumulation equation yields the following result for I_t

$$I_t = \left(\frac{\alpha A_{t+1}}{r}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}}$$
$$= \left(\frac{\alpha (1+g)^{1-\alpha} A_t}{r}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}}$$
$$= \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}} g$$

(b) Given your answer to part (a), what is the economy's level of output? Substituting the above expression for K_t into the production function yields,

$$Y_t = A_t \left(\frac{\alpha A_t}{r}\right)^{\frac{\alpha}{1-\alpha}} = A_t^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$$

(c) Show that your answers to parts (a) and (b) imply that $I_t = \left(\frac{\alpha g}{r}\right) Y_t$. Given this result, and the assumptions about government spending and the law of motion for A_t , show that

$$Y_{t+j} - I_{t+j} - G_{t+j} = (1+g)^j \left(1 - \frac{\alpha g}{r} - \gamma\right) Y_{t+j}$$

Divide the answer to part (a) by the answer in part (b) to get (notice that A_t cancels)

$$\frac{I_t}{Y_t} = \frac{\alpha g}{r}$$

Using the above expression for Y_t , notice that

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{1-\alpha}} \qquad \Rightarrow \qquad Y_{t+1} = (1+g)Y_t$$

Given this, the fact that $Y_{t+j} - I_{t+j} - G_{t+j} = (1+g)^j \left(1 - \frac{\alpha g}{r} - \gamma\right) Y_t$ follows directly from the fact that I_t and G_t are fixed fractions of Y_t .

(d) Given the result in part (c), derive an expression for CA_t in terms of Y_t, r, g, γ, and α.
Using the answer to part (c) to evaluate the given expression for CA_t yields

$$CA_t = \left(1 - \frac{r}{r-g}\right) \left(1 - \frac{\alpha g}{r} - \gamma\right) Y_t$$
$$= -\frac{g}{r-g} \left(1 - \frac{\alpha g}{r} - \gamma\right) Y_t$$

(e) Finally, use the fact that $CA_t = B_{t+1} - B_t$ to derive a first-order difference equation for the ratio of net foreign assets to GDP, $b_t = B_t/Y_t$. Characterize the solution of this equation graphically, by plotting b_{t+1} against b_t . Is there a steady state? Is it positive or negative? Is it stable? Using the fact that $CA_t = B_{t+1} - B_t$, we have from the previous expression

$$\frac{B_{t+1}}{Y_t} - \frac{B_t}{Y_t} = -\frac{g}{r-g} \left(1 - \frac{\alpha g}{r} - \gamma\right)$$

Defining $b_t = B_t/Y_t$ as the net foreign asset/GDP ratio then imples

$$b_{t+1} = \frac{1}{1+g}b_t - \frac{g}{(1+g)(r-g)}\left(1 - \frac{\alpha g}{r} - \gamma\right)$$

This is a simple linear difference equation that implies the following (stable) steady state net foreign asset/GDP ratio (See pg. 118 in Obstfeld-Rogoff for the graph)

$$\bar{b} = -\left(\frac{1-(\alpha g)/r-\gamma}{r-g}\right)$$

(f) Suppose $\alpha = .4$, g = .05, r = .08, and $\gamma = .3$. What is the implied steady state value for b_t ? What is the associated steady state value for the trade surplus/GDP ratio? Are these values 'believeable'? What real world features are missing here?

Substituting in the given parameter values

$$\bar{b} = -\left(\frac{1 - .02/.08 - .3}{.03}\right) = -15$$

That is, it is optimal for this country to have a steady state foreign debt that is 15 times its GDP! To service this debt, the country must then run a steady state Trade Surplus of TB = CA - rB = -(r-g)B. Expressed as a share of GDP, the economy must send $(.08 - .05) \cdot 15 = 45\%$ of its GDP to foreigners in order to service its foreign debt. No country on earth has ever built up this kind of debt burden. Two main real world considerations are missing here: (1) A country that persistently grows faster than the world average (as reflected in the world interest rate) will eventually become LARGE, and will no longer be a price-taker, and (2) It's going to be awfully tempting for this country to renege on its foreign debt! Of course, lenders know this, so they would not likely lend them this much in the first place.

6. (30 points). The Monetary Model with Endogenous Monetary Policy. In class we developed a simple model of flexible exchange rate dynamics that assumed monetary policy was exogenous. It was

assumed the Central Bank simply set the money supply according to some process, and the exchange rate, price level, and interest rate had to adjust so that money supply equals money demand. Now let's try something different. Suppose instead the Central bank targets the price level, and does so by raising the nominal interest rate when the price level exceeds some fixed target. Now the money supply becomes endogenous, and we simply assume the Central Bank adjusts the money supply as needed to hit its target. In particular, suppose the Central Bank's policy function is

$$i_t = \lambda (P_t - \bar{P})$$

where \bar{P} is a fixed target price level.

The rest of the economy is (almost) the same as before. To simplify, let's normalize the foreign interest rate and price level to be zero. (Note, as far as exchange rate determination is concerned, we don't need to worry about outputs and money supplies, since money is endogenous here). In this case, Uncovered Interest Parity takes the form,

$$i_t = E_t s_{t+1} - s_t$$

where s_t is the log of the spot exchange rate (defined as the price of foreign currency). Finally, suppose that PPP holds only in the long run, so that (given that P^* is assumed to be zero)

$$P_t = s_t + u_t$$

where u_t is a stationary, autoregressive process that captures temporary deviations from PPP. Assume it follows the process

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad 0 < \rho < 1$$

where ε_t is mean zero i.i.d., and can be interpreted as a domestic price level shock.

(a) Combine the PPP and Uncovered Interest Parity relationships with the Policy Reaction function to derive an exchange rate equation that looks just like the one we had in class:

$$s_t = (1 - \beta)f_t + \beta E_t s_{t+1}$$

What is β and f_t now?

From UIP we know $s_t = E_t s_{t+1} - i_t$. Using the policy function and PPP to sub out for i_t yields,

$$s_t = E_t s_{t+1} - \lambda [(s_t + u_t) - \overline{P}]$$

Collecting terms then gives

$$s_t = (1 - \beta)f_t + \beta E_t s_{t+1}$$

where $\beta = 1/(1+\lambda)$ and $f_t = (\bar{P} - u_t)$. This is the same basic form as before!

(b) Use the given law of motion for u_t to solve the above difference equation.

Iterating the above difference equation forward and using the law of iterated expectations (and ruling out bubbles) yields,

$$s_t = (1 - \beta)E_t \sum_{j=0}^{\infty} \beta^j [\bar{P} - u_{t+j}]$$

The given process for u_t implies $E_t u_{t+j} = \rho^j u_t$. Substituting this result into the previous equation, and using the formula for a geometric sum yields,

$$s_t = \bar{P} - \frac{1 - \beta}{1 - \rho\beta} u_t$$

(c) How does a positive inflation shock affect the exchange rate? Does the exchange rate appreciate or depreciate? Explain intuitively.

Notice that a positive shock to inflation causes the currency to <u>appreciate</u>. This is kind of surprising from the perspective of PPP. What's going on here is that the exchange rate is a <u>forward-looking</u> variable, and market participants know that the central bank will respond to higher inflation by <u>raising</u> the interest rate and tightening the money supply. The anticipated higher future interest rates actually make the domestic currency more attractive, and cause it to appreciate. Sound far fetched? See Richard Clarida, "Is Bad News on Inflation Good News for the Dollar?", for some empirical evidence.