SIMON FRASER UNIVERSITY
Department of Economics

Econ 842 | Prof. Kasa
International Monetary Economics | Spring 2016

PROBLEM SET 1 - CURRENT ACCOUNT DYNAMICS
(Due February 4)

1. (25 points). Consider a small open economy that can borrow or lend all it wants at a fixed world interest rate, r. Suppose preferences are quadratic, and the rate of time preference equals the interest rate (i.e., $\beta(1+r)=1$). As discussed in class, combining the Euler equation with the budget constraint produces the following consumption function,

$$C_t = \frac{r}{1+r}\left[(1+r)B_t + E_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j (Y_{t+j} - I_{t+j} - G_{t+j})\right]$$

which then produces the following expression for the current account, CA_t,

$$CA_t = Y_t - I_t - G_t - \frac{r}{1+r}E_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j (Y_{t+j} - I_{t+j} - G_{t+j})$$

Suppose the economy produces output with the production function,

$$Y_t = A_t K_t^{\alpha}$$

where productivity, A_t, grows according to the process,

$$A_{t+1} = (1+g)^{1-\alpha}A_t$$

where $0 < g < r$. Finally, suppose government spending, G_t, is a fixed fraction, γ, of output (and that $\gamma < 1 - (\alpha g/r)$.

(a) Calculate the optimal capital stock in this economy. Assume that capital does not depreciate, so that $I_t = K_{t+1} - K_t$. Using your previous answer, and the given process for A_t, calculate I_t in terms of α, r, A_t, and g.

(b) Given your answer to part (a), what is the economy's level of output?

(c) Show that your answers to parts (a) and (b) imply that $I_t = \left(\frac{\alpha g}{r}\right)Y_t$. Given this result, and the assumptions about government spending and the law of motion for A_t, show that

$$Y_{t+j} - I_{t+j} - G_{t+j} = (1+g)^j\left(1 - \frac{\alpha g}{r} - \gamma\right)Y_t$$

(d) Given the result in part (c), derive an expression for CA_t in terms of Y_t, r, g, γ, and α.

(e) Finally, use the fact that $CA_t = B_{t+1} - B_t$ to derive a first-order difference equation for the ratio of net foreign assets to GDP, $b_t = B_t/Y_t$. Characterize the solution of this equation graphically, by plotting b_{t+1} against b_t. Is there a steady state? Is it positive or negative? Is it stable?

(f) Suppose $\alpha = .4$, $g = .05$, $r = .08$, and $\gamma = .3$. What is the implied steady state value for b_t? What is the associated steady state value for the trade surplus/GDP ratio? Are these values 'believeable'? What real world features are missing here?

2. (25 points). Pick a country, and following the procedure outlined on pages 90-93 of the Obstfeld-Rogoff text, test the Present-Value Model of the current account (i.e., test the model's implied cross-equation restrictions). Plot the model's predicted current account against the actual current account. Comment on the model's fit. (Note: Be sure to express everything in real terms. Although variables should also be expressed in per capita terms as well, don't worry about that. It shouldn't make much of a difference here).