SIMON FRASER UNIVERSITY
Department of Economics

Econ 842
International Monetary Economics

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PROBLEM SET 1 - CURRENT ACCOUNT DYNAMICS
(Solutions)

1. (25 points). Consider a small open economy that can borrow or lend all it wants at a fixed world interest rate, r. Suppose preferences are quadratic, and the rate of time preference equals the interest rate (i.e., $\beta(1+r)=1$). As discussed in class, combining the Euler equation with the budget constraint produces the following consumption function,

$$C_t = \frac{r}{1+r}\left[(1+r)B_t + E_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j (Y_{t+j} - I_{t+j} - G_{t+j})\right]$$

which then produces the following expression for the current account, CA_t,

$$CA_t = Y_t - I_t - G_t - \frac{r}{1+r}E_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j (Y_{t+j} - I_{t+j} - G_{t+j})$$

Suppose the economy produces output with the production function,

$$Y_t = A_t K_t^{\alpha}$$

where productivity, A_t, grows according to the process,

$$A_{t+1} = (1+g)^{1-\alpha} A_t$$

where $0 < g < r$. Finally, suppose government spending, G_t, is a fixed fraction, γ, of output (and that $\gamma < 1 - (\alpha g/r)$.

(a) Calculate the optimal capital stock in this economy. Assume that capital does not depreciate, so that $I_t = K_{t+1} - K_t$. Using your previous answer, and the given process for A_t, calculate I_t in terms of α, r, A_t, and g.

In this small open economy, optimal investment occures up until the point where $MPK = r$. Taking the derivatives and solving for K_t yields,

$$K_t = \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}}$$

Using this in the capital accumulation equation yields the following result for I_t

$$
\begin{aligned}
I_t &= \left(\frac{\alpha A_{t+1}}{r}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}} \\
&= \left(\frac{\alpha(1+g)^{1-\alpha} A_t}{r}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}} \\
&= \left(\frac{\alpha A_t}{r}\right)^{\frac{1}{1-\alpha}} g
\end{aligned}
$$

(b) Given your answer to part (a), what is the economy's level of output?
Substituting the above expression for K_t into the production function yields,

$$Y_t = A_t \left(\frac{\alpha A_t}{r}\right)^{\frac{\alpha}{1-\alpha}} = A_t^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$$

(c) Show that your answers to parts (a) and (b) imply that $I_t = \left(\frac{\alpha g}{r}\right) Y_t$. Given this result, and the assumptions about government spending and the law of motion for A_t, show that

$$Y_{t+j} - I_{t+j} - G_{t+j} = (1+g)^j \left(1 - \frac{\alpha g}{r} - \gamma\right) Y_t$$

Divide the answer to part (a) by the answer in part (b) to get (notice that A_t cancels)

$$\frac{I_t}{Y_t} = \frac{\alpha g}{r}$$

Using the above expression for Y_t, notice that

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{1-\alpha}} \quad \Rightarrow \quad Y_{t+1} = (1+g)Y_t$$

Given this, the fact that $Y_{t+j} - I_{t+j} - G_{t+j} = (1+g)^j \left(1 - \frac{\alpha g}{r} - \gamma\right) Y_t$ follows directly from the fact that I_t and G_t are fixed fractions of Y_t.

(d) Given the result in part (c), derive an expression for CA_t in terms of Y_t, r, g, γ, and α.
Using the answer to part (c) to evaluate the given expression for CA_t yields

$$
\begin{aligned}
CA_t &= \left(1 - \frac{r}{r-g}\right)\left(1 - \frac{\alpha g}{r} - \gamma\right) Y_t \\
&= -\frac{g}{r-g}\left(1 - \frac{\alpha g}{r} - \gamma\right) Y_t
\end{aligned}
$$

(e) Finally, use the fact that $CA_t = B_{t+1} - B_t$ to derive a first-order difference equation for the ratio of net foreign assets to GDP, $b_t = B_t/Y_t$. Characterize the solution of this equation graphically, by plotting b_{t+1} against b_t. Is there a steady state? Is it positive or negative? Is it stable?

Using the fact that $CA_t = B_{t+1} - B_t$, *we have from the previous expression*

$$\frac{B_{t+1}}{Y_t} - \frac{B_t}{Y_t} = -\frac{g}{r-g}\left(1 - \frac{\alpha g}{r} - \gamma\right)$$

Defining $b_t = B_t/Y_t$ *as the net foreign asset/GDP ratio then impies*

$$b_{t+1} = \frac{1}{1+g}b_t - \frac{g}{(1+g)(r-g)}\left(1 - \frac{\alpha g}{r} - \gamma\right)$$

This is a simple linear difference equation that implies the following (stable) steady state net foreign asset/GDP ratio (See pg. 118 in Obstfeld-Rogoff for the graph)

$$\bar{b} = -\left(\frac{1 - (\alpha g)/r - \gamma}{r - g}\right)$$

(f) Suppose $\alpha = .4$, $g = .05$, $r = .08$, and $\gamma = .3$. What is the implied steady state value for b_t? What is the associated steady state value for the trade surplus/GDP ratio? Are these values 'believeable'? What real world features are missing here?

Substituting in the given parameter values

$$\bar{b} = -\left(\frac{1 - .02/.08 - .3}{.03}\right) = -15$$

That is, it is optimal for this country to have a steady state foreign debt that is 15 times its GDP! To service this debt, the country must then run a steady state Trade Surplus of $TB = CA - rB = -(r-g)B$. *Expressed as a share of GDP, the economy must send* $(.08 - .05) \cdot 15 = 45\%$ *of its GDP to foreigners in order to service its foreign debt. No country on earth has ever built up this kind of debt burden. Two main real world considerations are missing here: (1) A country that persistently grows faster than the world average (as reflected in the world interest rate) will eventually become LARGE, and will no longer be a price-taker, and (2) It's going to be awfully tempting for this country to renege on its foreign debt! Of course, lenders know this, so they would not likely lend them this much in the first place.*

2. (25 points). Pick a country, and following the procedure outlined on pages 90-93 of the Obstfeld-Rogoff text, test the Present-Value Model of the current account (i.e., test the model's implied cross-equation restrictions). Plot the model's predicted current account against the actual current account. Comment on the model's fit. (Note: Be sure to express everything in real terms. Although variables should also be expressed in per capita terms as well, don't worry about that. It shouldn't make much of a difference here).