1. This question is based on the monetary model of exchange rate determination. Equilibrium in the domestic and foreign money markets is given by (with all variables in logs, except the interest rate).

\[ m_t - p_t = \phi y_t - \lambda i_t \]

\[ m_t^* - p_t^* = \phi y_t^* - \lambda i_t^* \]

where \( \phi \) is the income elasticity of money demand and \( \lambda \) is the interest rate semi-elasticity of money demand. Money demand parameters are identical across countries. International capital market equilibrium is given by uncovered interest parity:

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]

where \( E_t s_{t+1} \) is the expectation at time-\( t \) of the exchange rate in period \( t + 1 \).

Price levels and the exchange rate are related through purchasing-power parity:

\[ s_t = p_t - p_t^* \]

Define \( f_t = (m_t - m_t^*) - \phi(y_t - y_t^*) \) as the economic fundamentals.

(a) Derive a first-order stochastic difference equation for the equilibrium exchange rate, \( s_t \).

(b) Find the fundamentals (no bubbles) solution. What is the condition for this solution to hold?

(c) Consider the effect of an unanticipated announcement at date \( t = 0 \) that the money supply is going to permanently rise on a future date \( T \), i.e., \( f_t = \tilde{f} \) when \( t < T \), and then \( f_t = \tilde{f} + \Delta \) for \( t \geq T \). Derive the path of exchange rate and show the path in a graph.

(d) Suppose that the fundamentals are governed by a stationary AR(1) process, \( f_t = \rho f_{t-1} + \epsilon_t \), where \( \epsilon_t \) is an i.i.d. shock. Show and discuss how the persistence of fundamentals affect the volatility of the exchange rate.
2. Consider the following present value model of exchange rate determination:

\[ s_t = (1 - \beta) \sum_{j=0}^{\infty} \beta^j E(f_{t+j} | \Omega_t) \quad 0 < \beta < 1 \]

where \( s_t \) is the log exchange rate, \( f_t \) is the log of fundamentals, and \( \Omega_t \) is the information set at time-\( t \).

Assume fundamentals follow a random walk,

\[ f_t = f_{t-1} + \varepsilon_t \]

and assume \( \text{var}(\varepsilon_t) = 1 \).

Clearly, if \( \Omega_t \) contains only \( f_t \) and its lags, the solution for the exchange rate is just \( s_t = f_t \). Suppose, however, that \( \Omega_t \) contains \( f_{t+1} \) as well as \( (f_t, f_{t-1}, \ldots) \). In other words, agents get a noiseless, one-period ahead signal of the fundamentals.

(a) Solve for \( s_t \) in terms of \( f_t \) and \( f_{t+1} \).

(b) Calculate the variance of \( s_t - s_{t-1} \). Is the variance bigger or smaller than in the case where \( \Omega_t \) only contains \( f_t \) and its lags?

(c) Calculate the covariance of \( s_t - s_{t-1} \) with \( f_t - f_{t-1} \).

(d) Now square the answer in part (c), and divide by your answer in part (b). That is, compute \[ \frac{\text{cov}(\Delta s_t, \Delta f_t)^2}{\text{var}(\Delta s_t)} \]. Note, that this is just the squared correlation between \( \Delta s_t \) and \( \Delta f_t \), since by assumption the variance of \( \Delta f_t = 1 \). How does this model help to explain the observation that exchange rates are ‘disconnected’ from fundamentals?

(e) Engel and West (JPE, 2005) prove a theorem that says exchange rates are unforecastable under certain circumstances, even though fundamentals are forecastable. How does their theorem apply to this model?

3. Pick a country, and using the Campbell-Shiller methodology, test the monetary model of exchange rate determination. Define the fundamentals to be \( f_t = m_t - m^*_t - (y_t - y^*_t) \), where \( m \) and \( y \) are logs of the money supply and (real) GDP. (Notice that you can just set the income elasticity of money demand to one). First, plot \( f_t \) against \( s_t \). Next, test whether \( f_t \) has a unit root, and based on the results, estimate a VAR in either \( (s_t, f_t) \) or \( (s_t - f_t, \Delta f_t) \). Following the discussion in Engel and West (JPE, 2005), set the discount factor, \( \beta \), to 0.96. Report tests of the implied cross-equation restrictions, and then plot the actual exchange rate (or the spread, \( s_t - f_t \), if fundamentals have a unit root) against the predicted exchange rate (or spread). Finally, check whether exchange rates Granger Cause fundamentals. How do your results here compare to Engel and West’s?