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# Exchange rates and fundamentals under adaptive learning

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ABSTRACT

This paper studies a monetary model that is standard in all respects except that market participants have incomplete knowledge about the economic structure and employ adaptive learning rules to learn about the economic environment. Market participants also must contend with unannounced regime shifts. Simulation results suggest that the models under adaptive learning, especially constant-gain learning combined with a structural change, dominate the alternative specifications of expectations in their ability to account for why fundamentals predict exchange-rate returns over long horizons but not over short horizons, and for generating excessively volatile returns and persistent deviations of the exchange rate from the monetary fundamentals.



## 1. Introduction

This paper studies a monetary model of nominal exchange rates where market participants, who have incomplete knowledge about the structure of the economy, learn about the economic environment using adaptive learning (AL) rules. The introduction of adaptive learning into an otherwise standard monetary model is motivated by the well-documented fact that when the underlying economic environment is known and is common knowledge to market participants, the monetary model under rational expectations (RE) cannot account for such basic features of the data as the relative volatility between the exchange rate and economic fundamentals or the predictability of future exchange rate changes by current deviations of the exchange rate from the fundamentals.[1]

A serious shortcoming of RE models of exchange rate determination is that movements of exchange rates have been too volatile to be justified by the fundamentals, although numerous RE approaches to exchange rates have provided some attractive explanations for highly volatile exchange rate returns.[2] For example, MacDonald and Taylor (1994) adopted the tests of the present-value model, which were proposed by Campbell and Shiller (1987) and showed that the theoretically

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[1] For survey papers see Frankel and Rose (1995), Taylor (1995), and Frydman and Goldberg (2003).

[2] The following have been suggested as potential sources of highly volatile returns: agents' revisions of expectations about the future in the response to unexpected changes in future values of exogenous variables in flexible-price monetary models (Frenkel, 1976; Mussa, 1976; Frenkel and Mussa, 1980); overshooting effects in sticky-price monetary models (Dornbusch, 1976); and heterogeneous information and higher-order belief dynamics (Bacchetta and van Wincoop, 2006), among others.

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implied error-correction term in the monetary model is much too smooth. Second, despite their theoretical appeal as a link between the nominal exchange rate and a simple set of economic fundamentals, the RE monetary models have failed to explain why the fundamentals predict exchange-rate returns over long horizons but not over short horizons. Ever since Meese and Rogoff (1983) conducted a postsample fit analysis, which has been a standard way of evaluating exchange-rate models, the difficulty in predicting the logarithm of the exchange rate in an RE model has been a longstanding problem in international economics.[3]

The RE hypothesis has become the benchmark paradigm in macroeconomics. In a standard RE approach, economic agents often are assumed to have a great deal of knowledge about the economy and use available information optimally. Even in the simplest form of the monetary model, computing the time path of the exchange rate requires complete knowledge about the model structure, including parameter values in the stochastic process of the fundamentals. This informational assumption underlying the RE model appears to be fairly strong in the sense that, in practice, economists who postulate RE do not themselves know the true parameter values and need to estimate them econometrically (Mark, 2001). In the hypothesis of adaptive expectations (AE), on the other hand, market participants in the foreign exchange market are assumed to have extremely limited knowledge about the economic environment. The market participants forecast future monetary fundamentals using their past observations of the fundamentals without the need to specify the process. This specification of expectations might be a reasonable way to form expectations when market participants are placed in a very complex environment (Evans and Ramey, 2006) or if it is almost impossible to know the economic structure due to insufficient data. However, as the data are accumulated over time, it is difficult to reconcile this suboptimal use of available information with the idea of optimality, which is the foundation of most economic analysis. The AL approach can accommodate those specifications of expectations. Market participants initially face some limited knowledge about the economic structure, and thus have incentives to learn the true economic environment over time. For instance, in this study, I assume market participants know the functional form of the model structure, but they do not know the parameter values, which they assess by employing least squares learning.[4] Therefore, a more plausible view of rationality might be that market participants who have incomplete knowledge about the model structure act like an applied econometrician when forecasting the future state of the economy. This insight is the starting point of the adaptive learning approach based on a specific form of bounded rationality to macroeconomics, as discussed in Sargent (1993) and in Evans and Honkapohja (2001).

To evaluate predictions of the monetary model under those specifications of expectations, I consider the following aspects of model structure. First, I specify monetary fundamentals as a trend-stationary process. This is not only empirically plausible, but it also is appropriate for evaluating models in multiple dimensions. Second, the stability of monetary fundamentals over the post-Bretton Woods period is investigated. The monetary-model fundamentals consist of the relative money supply and relative real income between two countries. There are many good reasons, such as oil shocks in the 1970s, German reunification in the late 1980s, and several important regime changes in U.S. monetary policy over the post-Bretton Woods period, to doubt that the parameter values of the process of fundamentals have remained constant over this period. In addition, market participants, who set the exchange rates that become our data, may face unanticipated occasional regime shifts or gradual and random changes in the economic environment. Thus, I model structural changes in the fundamentals so that market participants can contend with them. Finally, a variety of AL rules are considered. Depending on the variables being forecasted and the types of gain sequences in the learning algorithm, I employ (i) present-value learning with decreasing gain, (ii) present-value learning with constant gain, (iii) self-referential learning with decreasing gain, and finally (iv) self-referential learning with constant gain. I found that AL models dominate models with alternative specifications of expectations, namely RE and AE, in that they produce results consistent with some important empirical aspects of the data in the foreign exchange market. In addition, constant-gain learning combined with a small number of structural breaks in the fundamentals performs better than any other specifications of expectations or alternative learning algorithms in terms of volatility and persistence.

The remainder of the paper is organized as follows. The next section reports stylized facts about exchange-rate returns, monetary fundamentals, and their dynamic relationships. Section 3 introduces the monetary model of exchange-rate determination and the nominal exchange rate solutions are presented. Possible structural shifts in the fundamentals also are discussed. In Section 4, I compare the predictions of the models under AL with that of the alternative specifications of

[3] Mark (1995) rejuvenated the monetary model by showing that long-horizon changes in log nominal exchange rates contain economically significant predictable components, and further, much favorable evidence has been provided from a panel or a century of data (Groen, 2000; Mark and Sul, 2001; Rapach and Wohar, 2002, 2004).

[4] In this paper, I focus on parameter uncertainty, but not model uncertainty. A recent paper by Branch and Evans (2007) introduces model uncertainty into a Lucas-type monetary model in which agents must decide among multiple models, and shows switching between models generates more volatility. Lewis and Whiteman (2007) also employ model uncertainty in a present-value asset pricing model and find if agents are robust to the most misspecification possible, the resulting stock prices may be too volatile than those found in the data. Note that, to deal with model misspecification, a constant-gain stochastic gradient algorithm by Evans et al. (2008) may be a useful alternative. Next, I consider identical market participants. Heterogeneous agents may be another important source of failure of a standard RE model of exchange rates (De Grauwe and Grimaldi, 2006). Hommes (2006) provides an extensive review of reinforcement learning in heterogeneous agents models with agents switching between fundamentalists and chartists strategies. De Grauwe and Markiewicz (2006) compare the empirical performance of adaptive learning and reinforce learning. Finally, Bacchetta and van Wincoop (2004) also study investors who have heterogeneous information on some structural parameters of the economy and they show, in their scapegoat model, that the impact of macroeconomic variables on exchange rate changes over time.

**Table 1**
Stylized facts I: volatility and persistence.

| | GBP | DEM | JPY | CHF |
|---|---|---|---|---|
| *A. Exchange-rate returns* | | | | |
| Std. Dev. | 5.079 | 6.357 | 5.993 | 6.699 |
| VR(1) | 1.000 | 1.000 | 1.000 | 1.000 |
| VR(8) | 1.264 | 1.057 | 1.156 | 1.012 |
| VR(16) | 1.022 | 0.838 | 0.885 | 0.814 |
| VR(32) | 0.312 | 0.312 | 0.447 | 0.250 |
| *B. Deviations from fundamentals* | | | | |
| Std. Dev. | 0.222 | 0.298 | 0.394 | 0.251 |
| VR(1) | 1.000 | 1.000 | 1.000 | 1.000 |
| VR(8) | 1.674 | 1.350 | 1.472 | 1.133 |
| VR(16) | 1.573 | 1.306 | 1.270 | 0.893 |
| VR(32) | 0.800 | 0.775 | 0.704 | 0.265 |

*Note*: The U.S. dollar is the numeraire currency. GBP is the U.K. pound, DEM is the Deutsche mark, JPY is the Japanese yen, and CHF is the Swiss franc. VR($k$) refers to the variance ratio for $k = 1, 8, 16$, and 32.

expectations in terms of their ability to account for some important empirical aspects of the data in the foreign exchange market. Concluding remarks are contained in Section 5.

## 2. Stylized facts of monetary fundamentals and nominal exchange rate

Tables 1–3 present descriptive statistics for U.S. dollar returns on the U.K. pound, the Deutsche mark, the Japanese yen, and the Swiss franc. The data contain quarterly observations from 1973:Q1 to 1997:Q4 for the Deutsche mark and 1973:Q1 to 2005:Q4 for the other currencies. These series were obtained from the *International Financial Statistics* CD-ROM. The nominal exchange rate is the end-of-quarter U.S. dollar price of one unit of foreign currency. For all countries except the United Kingdom, M2 is the monetary variable used to construct the fundamental value of the exchange rate (M0 is used for the U.K.). The quarterly industrial production index is used as a proxy for real income.

Table 1 presents estimates for the volatility of exchange-rate returns. The sample standard deviations of quarterly returns range from 5.1 percent to 6.7 percent. I also report variance ratio statistics that provide a measure of the serial correlation properties of the data.[5] The variance ratio statistics for the returns presented in panel A of Table 1 are consistently greater than one over a short horizon, but are less than one over a long horizon, and their magnitudes decrease, in all cases, as the length of horizon increases. This suggests that returns are negatively serially correlated over long horizons and thus exhibit mean reverting behavior. This is because a positive change today is expected to be reversed in the future. Next, the empirical specification of the long-run equilibrium exchange rate used in this paper is the monetary-model fundamentals, a linear combination of relative money stock and relative real income between two countries. Let $f_t$ be the monetary fundamentals at time $t$,

$$f_t \equiv (m_t - m_t^*) - (y_t - y_t^*), \tag{1}$$

where $m_t$ ($m_t^*$) is the log of the nominal money stock of the domestic (foreign) country and $y_t$ ($y_t^*$) is real income of the domestic (foreign) country.[6] In the monetary approach, changes in relative money supply and relative income cause prices to adjust to maintain equilibrium in the money market. This leads to changes in exchange rates. Let $\xi_t$ denote the current deviation of the log spot rate from its fundamental value, or the error-correction term in the monetary model,

$$\xi_t \equiv f_t - s_t, \tag{2}$$

where $s_t$ is the log of the nominal exchange rate. It is well known that exchange-rate deviations from the fundamentals display substantial persistence, and much less volatility than returns, as we clearly can see in Table 1. The variance ratio statistics presented in panel B of Table 1 exhibit the same pattern as those in panel A. First differences of deviations are positively serially correlated over a short horizon, but apparent evidence of mean reversion of the spread is inferred from the variance ratios being less than one over a long horizon. It is worth noting that the variance ratios for deviations from the fundamentals take on values that are less than one at horizons that are longer than what we observe in the variance ratios for exchange-rate returns. That is, changes in the error-correction terms are positively serially correlated over 8

[5] The variance ratio statistic is a widely used nonparametric measure of the relative size of the random walk component in a time-series. The variance ratio statistic for a time-series, $s_t$, at horizon $k$ is represented as $\mathrm{VR}(k) = \mathrm{Var}(s_t - s_{t-k})/k \cdot \mathrm{Var}(\Delta s_t) = \mathrm{Var}(\Delta s_t + \cdots + \Delta s_{t-k+1})/k \cdot \mathrm{Var}(\Delta s_t)$. VR($k$) exceeds one if $\Delta s_t$ is positively serially correlated, and thus the variance grow faster than linearly. Similarly, if $\Delta s_t$ is negatively serially correlated, VR($k$) is less than one and the variance grow slower than linearly. Under the null hypothesis of random walk, the population value of variance ratio statistic VR($k$) is one for all $k$ (Campbell et al., 1997).

[6] The domestic country is the United States.

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**Table 2**
Stylized facts II: long-horizon U.S. dollar return regression.

| Horizon | GBP | DEM | JPY | CHF |
|---|---|---|---|---|
| $\hat{\beta}_k$ | | | | |
| 1 | 0.045 [0.045] | 0.036 [0.032] | 0.019 [0.017] | 0.062 [0.061] |
| 4 | 0.188 [0.186] | 0.152 [0.135] | 0.086 [0.072] | 0.245 [0.243] |
| 8 | 0.420 [0.417] | 0.335 [0.302] | 0.193 [0.174] | 0.474 [0.473] |
| 16 | 0.708 [0.692] | 0.875 [0.817] | 0.376 [0.350] | 0.734 [0.726] |
| $t_k$ | | | | |
| 1 | 1.932 [0.040] | 1.703 [0.050] | 1.374 [0.090] | 2.952 [0.003] |
| 4 | 3.666 [0.008] | 2.173 [0.060] | 1.787 [0.088] | 3.870 [0.005] |
| 8 | 5.864 [0.002] | 2.750 [0.057] | 2.664 [0.047] | 6.337 [0.001] |
| 16 | 4.595 [0.022] | 3.733 [0.057] | 4.369 [0.027] | 8.034 [0.001] |
| $R^2$ | | | | |
| 1 | 0.039 [0.035] | 0.029 [0.024] | 0.016 [0.013] | 0.053 [0.049] |
| 4 | 0.137 [0.122] | 0.119 [0.098] | 0.069 [0.055] | 0.215 [0.200] |
| 8 | 0.289 [0.257] | 0.220 [0.177] | 0.166 [0.136] | 0.367 [0.340] |
| 16 | 0.508 [0.443] | 0.544 [0.458] | 0.393 [0.337] | 0.536 [0.487] |

*Note*: The long-horizon U.S. dollar return regression is the regression of the $k$-period-ahead change in the log of the nominal exchange rate ($s_{t+k} - s_t$) on its current spread ($\xi_t \equiv f_t - s_t$), where $\beta_k$ is the regression slope for $k = 1, 4, 8$, and 16. The numbers in brackets are bias-adjusted slope coefficients and $R^2$'s and the bootstrap marginal significance level.

**Table 3**
Stylized facts III: out-of-sample forecasts of U.S. dollar returns.

| Horizon | GBP | DEM | JPY | CHF |
|---|---|---|---|---|
| *U statistic* | | | | |
| 1 | 0.993 | 0.999 | 1.009 | 0.983 |
| 4 | 1.055 | 1.068 | 1.047 | 0.938 |
| 8 | 1.185 | 1.141 | 1.026 | 0.904 |
| 16 | 1.399 | 1.034 | 0.959 | 0.581 |
| *DM statistic* | | | | |
| 1 | 0.403 | 0.340 | 0.819 | −0.695 |
| 4 | 1.181 | 1.462 | 1.294 | −0.709 |
| 8 | 1.202 | 1.206 | 0.340 | −0.566 |
| 16 | 1.694 | 0.310 | 0.062 | −1.175 |

*Note*: Table entries are Theil's $U$ statistics and Diebold–Mariano ($DM$) statistics. The monetary fundamentals regression outperforms the random walk model in prediction accuracy when $U<1$ or $DM<0$.

quarters for the Swiss franc and 16 quarters for the others. This suggests that nominal exchange rates hardly converge towards their theoretically implied fundamental determinants, once they deviate from their long-run equilibrium values.

I also study in-sample predictive power at long horizons in the error-correction framework.[7] This can be examined by running regressions of the $k$-period-ahead change in the log spot exchange rate on its current deviation for $k \geqslant 1$,

$$s_{t+k} - s_t = \alpha_k + \beta_k \xi_t + \upsilon_{t+k}, \tag{3}$$

where $\beta_k$ is slope coefficient of the linear least-squares projection and $\upsilon_{t+k}$ is the projection error. When the exchange rate is below its long-run equilibrium value ($f_t > s_t$) or the spread is positive ($\xi_t > 0$), it is expected to rise over time. This implies that the slope coefficients of the return regression should be "positive." Furthermore, in the presence of mean reversion of the deviations from fundamentals, the slope coefficient should increase with return horizon. When the current spot rate deviates from its mean but tends to move towards its mean value over time, the statistically predictable component must increase with return horizon. To investigate this issue, I ran U.S. dollar return regressions at horizons of 1, 4, 8, and 16 quarters.[8] Table 2 presents the familiar pattern where slope coefficient estimates, $t$ ratios, $R^2$'s, and aspects of their finite sample distributions increase with the return horizon for all cases, except the U.K. pound.

[7] Mark and Sul (2004) provide the asymptotic justification for long-horizon predictive regression of testing the null hypothesis that the return is unpredictable.

[8] For serial correlation in the disturbances of the regression, I employ the heteroskedasticity and autocorrelation consistent standard errors of Newey and West (1987) together with the data-dependent bandwidth selection method of Andrews (1991). In order to account for small sample bias and size distortions in asymptotic tests, I generate nonparametric bootstrap distributions under the null hypothesis of random walk, as in Mark (1995). Bias-adjusted slope coefficients and $R^2$'s, and the bootstrap marginal significance levels shown in Table 2 exhibit qualitatively similar results. Note that small sample bias and size distortions appear to be much less severe when using an update of the data set than what has been found in previous studies.

Finally, I conduct postsample fit analysis by utilizing regression (3) to generate out-of-sample forecasts of the depreciation.[9] Mark (1995) documented that out-of-sample forecast accuracy of monetary fundamentals relative to the random walk tends to improve with prediction horizon using a data set consisting of quarterly observations spanning 1973:Q2 to 1991:Q4. Groen (1999) and Faust et al. (2003) show that this pattern breaks down when the time series are extended over 1990s.[10] To evaluate the forecasting accuracy of the two competing models, I employ both Theil's $U$ statistic and the Diebold–Mariano (DM) statistic. Table 3 contains the results of the out-of-sample prediction exercise. I found that, even over long horizons, the monetary model does not outperform the random walk model from the standpoint of out-of-sample forecasting accuracy for all currencies but the Swiss franc.[11] The $U$ statistics are generally greater than unity and the $DM$ statistics are consistently positive for the U.K. pound, the Deutsche mark, and the Japanese yen. For the Swiss franc, out-of-sample prediction power is significantly better than the random walk as the $DM$ statistics are negative and the $U$ statistics are less than one and decline with forecasting horizons.

## 3. Monetary fundamentals and nominal exchange rate

### 3.1. The monetary model

Consider the monetary model of exchange rates consisting of a pair of stable money demand functions in which the log of real money balances depends linearly and contemporaneously on the log of real income and the nominal interest rate, a continuous stock equilibrium in the money market, the uncovered interest parity (UIP) condition, and, finally, the purchasing power parity (PPP) condition. The nominal exchange rate implied by the monetary model is given by the first-order stochastic difference equation,[12]

$$s_t = \frac{\lambda}{1+\lambda} s^e_{t+1} + \frac{1}{1+\lambda} f_t, \tag{4}$$

where $s^e$ is the expected nominal exchange rate and $\lambda$ is the interest semi-elasticity of money demand, which is assumed to be identical across countries. Solving Eq. (4) forward and imposing the transversality condition, $\lim_{j\to\infty}(\lambda 1+\lambda)^j s^e_{t+j} = 0$,[13] yields the no-bubble log nominal exchange-rate solution,

$$s_t = \frac{1}{1+\lambda} \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^j f^e_{t+j}. \tag{5}$$

This solution relates the exchange rate to the sum of expected future fundamentals discounted to the present using a constant discount rate.

To complete the model structure, the empirical specification of monetary fundamentals and market participants' knowledge about the process must be specified. In this study, a trend stationary AR(1) process for the monetary fundamentals is considered. That is,

$$f_t = \mu + \delta t + \rho f_{t-1} + \varepsilon_t, \quad \varepsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2_\varepsilon). \tag{6}$$

I employ this process on the following grounds. First, it is challenging to explain some salient features of the data in the foreign exchange market, such as excess volatility and substantial persistence of exchange rates, when the economic fundamentals follow a stationary process.[14] Second, there is no single robust means of evaluating the monetary models, as pointed out by Engel et al. (2007). As shown in the previous section, I consider some important ways to compare models such as examining volatility, persistence, in-sample fit, and out-of-sample fit, that most empirical studies have employed. Therefore, a process that is especially in favor of or contrary to a certain model may not be appropriate in this study.[15]

[9] Out-of-sample forecast accuracy of the predictive regression implied by the monetary model is compared to the predictions of a random walk with drift. Forecasting begins with 1984:Q1. To test if one model predicts better than another, the $U$ statistic and the $DM$ statistic are based on the ratio of root-mean-square prediction errors and the sample mean-square loss differential, respectively. When the monetary model outperforms the random walk in prediction accuracy, $U<1$ and $DM$ is set up to be negative.

[10] Engel and West (2005) also show exchange rate changes should not be forecastable out of sample when the fundamentals are I(1) and the discount factor is close to one. Thus, their theorem suggests out-of-sample prediction power relative to a random walk is not a reliable means for evaluating exchange-rate models.

[11] Note that Engel et al. (2007) show this result can be reversed by using panel data. They find the monetary model significantly outforecasts the random walk model over long horizons for all currencies studied in this analysis.

[12] For the derivation of the nominal exchange-rate solution, see Mark (2001).

[13] By imposing the condition, this model falls in line with recent studies, Engel and West (2005) and Chakraborty and Evans (2009), among others, that rule out bubble solutions. Note that Campbell et al. (1997, pp. 259–260) provide theoretical and empirical arguments against bubble solutions in present-value relations.

[14] In fact, a nonstationary process performs well in terms of matching exchange-rate volatility. For example, if the growth rate of fundamentals is a persistent stationary process, the monetary model can be consistent with highly volatile exchange rates (Engel and West, 2004).

[15] For example, for models under rational expectations, specifying the fundamentals as a random walk closely matches the observed excess exchange-rate volatility, but this is unlikely to help our understanding of the predictability of changes in exchange rates and mean-reverting behavior of error-correction terms, because it implies constant deviations of exchange rates.

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Third, in the presence of apparent structural breaks in the monetary fundamentals, market participants may believe the monetary fundamentals follow a piecewise stationary process and a segmented time trend may play an important role in explaining large swings of exchange rates (Engel and Hamilton, 1990). Finally, this process has been employed in other present-value models and thus can facilitate comparisons of the results in this paper.[16] Since the assumed properties of an exogenous process can be important for the outcome, the results in this study must be interpreted conditional on the trend-stationary specification of the fundamentals being correct. Next, a more important element of this analysis is the assumption regarding how much information about the model structure market participants have in each time period. I consider three main versions of monetary models that differ according to information availability and to how efficiently market participants utilize available information.

### 3.2. Nominal exchange rates under RE

In standard RE monetary models, market participants are often assumed to have complete information on economic fundamentals, the functional form of the stochastic fundamental process, and the parameter values. Under rational expectations, the forecast of $f_{t+1}$ formed at time $t$ is given by the mathematical expectation of $f_{t+1}$ conditional on variables observable at time $t$, including its past values—that is, $f^e_{t+1} = \mathrm{E}_t f_{t+1}$. In this approach, the log of the nominal exchange rate is

$$\begin{aligned} s_t^{\mathrm{RE}} &= \frac{1}{1+\lambda}\mathrm{E}_t \sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^j f_{t+j} \\ &= \frac{\lambda}{1+\lambda-\lambda\rho}[\mu + (1+\lambda)\delta + \delta t] + \frac{1}{1+\lambda-\lambda\rho} f_t. \end{aligned} \tag{7}$$

This RE solution shows that the log of nominal exchange rate is expressed in terms of the current fundamentals value $f_t$ and the true parameter values known to market participants.[17]

Some useful model implications immediately are found from the analytical solution. The variability of nominal exchange rates implied by RE depends solely on the innovation of fundamentals with a fixed scaling factor, $1/(1+\lambda-\lambda\rho)$, which is the only uncertainty in this case. Since the scaling factor is less than unity in a stationary environment, exchange-rate returns are expected to be less volatile than changes in monetary fundamentals. Specifically, one-period returns implied by the model are

$$\Delta s_t^{\mathrm{RE}} = \frac{\lambda}{1+\lambda-\lambda\rho}\delta + \frac{1}{1+\lambda-\lambda\rho}\Delta f_t, \tag{8}$$

and, thus, the variance of returns is

$$\mathrm{Var}(\Delta s_t^{\mathrm{RE}}) = \left(\frac{1}{1+\lambda-\lambda\rho}\right)^2 \mathrm{Var}(\Delta f_t) < \mathrm{Var}(\Delta f_t). \tag{9}$$

This is one of the well-known aspects of the RE monetary model: the variance of depreciation does not exceed the variation of fundamentals growth, which is not consistent with the data. Notice that, in the RE model, variations in returns are greater the lower $\lambda$ and the higher $\rho$. Next, another important property about the relationship between nominal exchange rates and fundamentals found in the data is that the exchange rate is expected to rise when it lies below its fundamental or long-run equilibrium value. That is, the slopes of regressions of exchange-rate returns on current error-correction term are positive, as I showed in the previous section. However, this empirical regularity may not be found in the RE model in which exchange rates are too smooth compared to the fundamentals. For instance, the exchange rate could be expected to fall even when it is below its fundamental value. Therefore, I conjecture that deviations of exchange rates from the fundamentals predict returns with a negative sign in the RE monetary model. Finally, RE forecast errors do not contain systematic components and are serially uncorrelated. This is because market participants are assumed to know with complete certainty the model and are assumed to use available information efficiently.

### 3.3. Nominal exchange rates under AE

The AE monetary model can be considered as another important benchmark case in which market participants have a very small set of data and do not use the available data efficiently. Under this approach, market participants set exchange rates according to the present-value formula, as before, but they are assumed to have extremely limited knowledge about nature of the stochastic process of monetary fundamentals. I consider market participants who do not even know the

[16] Chakraborty and Evans (2009) employ a stationary AR(1) process for the monetary fundamentals to investigate whether the forward premium anomaly can be generated by perpetual learning. Timmermann (1996) also used a trend-stationary dividend process to examine volatility and predictability in the U.S. stock market.

[17] Note that this present-value solution agrees with the solution to the reduced form model of Eq. (4), which is used in the self-referential learning approach.

functional form of the process and, thus, forecast future fundamentals based on their past forecast errors. In terms of monetary fundamentals, this hypothesis takes the form:

$$f^e_{t+1} - f^e_t = \eta(f_t - f^e_t), \tag{10}$$

where $\eta$ is the AE adjustment parameter and $0<\eta<1$.[18] That is, having some forecast $f^e_t$ based on information available at the end of time $t-1$, market participants examine *ex post* how well that forecast predicted the actual value $f_t$ and then revise their forecast for the fundamentals one period later by some fraction of the forecasting error at time $t$. Alternatively, we can write the form as

$$f^e_{t+1} = \eta f_t + (1-\eta) f^e_t = \eta \sum_{j=0}^{\infty} (1-\eta)^j f_{t-j}, \tag{11}$$

which is a distributed lag model with exponentially declining weights.

Using the fact that, under AE, the one-step ahead forecast also is the $k$-step ahead forecast $f^e_{t+k} = f^e_{t+1}$ for all $k \geqslant 1$, the log nominal exchange-rate solution is given by

$$s^{\mathrm{AE}}_t = \frac{1+\lambda\eta}{1+\lambda} f_t + \frac{\lambda\eta(1-\eta)}{1+\lambda} \sum_{j=0}^{\infty} (1-\eta)^j f_{t-1-j}. \tag{12}$$

The AE nominal exchange rate depends on both current fundamentals and a weighted average of past observations of fundamentals with some scaling factors. The variability of the model-implied nominal exchange rate is now determined by both innovations of fundamentals and the magnitude of the adaptive expectations adjustment parameter value. From the model solution above, one-period exchange-rate returns are

$$\Delta s^{\mathrm{AE}}_t = \frac{1+\lambda\eta}{1+\lambda} \Delta f_t + \frac{\lambda\eta(1-\eta)}{1+\lambda} \sum_{j=0}^{\infty} (1-\eta)^j \Delta f_{t-1-j}. \tag{13}$$

The returns still are less volatile than the growth of the fundamentals, as in the RE model. That is,

$$\mathrm{Var}(\Delta s^{\mathrm{AE}}_t) = \left[\omega_1^2 + \frac{\omega_2^2}{1-\omega_3^2} + \frac{\omega_1\omega_2(\rho-1)}{(1-\omega_3\rho)} + \frac{\omega_2^2\omega_3(\rho-1)}{(1-\omega_3\rho)(1-\omega_3^2)}\right] \mathrm{Var}(\Delta f_t) < \mathrm{Var}(\Delta f_t), \tag{14}$$

where $\omega_1 = 1-\omega_3\omega_4$, $\omega_2 = \omega_3(1-\omega_3)\omega_4$, $\omega_3 = 1-\eta$, and $\omega_4 = \lambda/(1+\lambda)$. Although the variance of returns does not exceed the variation of fundamentals growth, we easily can show that AE model-implied returns are more volatile than those in the RE model, unless the serial correlation parameter $\rho$ is very close to unity.[19] Notice that the variance of returns is larger as market participants adopt a larger value of the AE adjustment parameter, $\eta$, placing a greater weight on relatively more recent events in the monetary fundamentals.

### 3.4. Nominal exchange rates under AL

Thus far, I have considered two benchmark cases: RE and AE. In the rational expectations monetary model, market participants are assumed to have complete knowledge about the underlying exogenous process, and they efficiently forecast future values of monetary fundamentals. On the other hand, if market participants have little information about the process, forecasts may be formed by their past forecast errors. In this approach, the market participants are not able or willing to understand the true economic structure, and this fixed forecasting rule allows the possibility of persistent systematic mistakes. Next, I consider a more plausible case where market participants initially face uncertainty about the economic environment, but they have an incentive to learn the true model structure over time. That is, the market participants do not know the true value of parameters and, like economists in empirical work, they assess the parameters by estimating them and by adjusting forecast rules as new data points become available. When market participants are placed in this type of learning environment, two natural approaches can be considered depending on whether they use long-horizon forecasts of the monetary fundamentals or they forecast next period's exchange rate to set today's exchange rate.

#### 3.4.1. Present-value learning

First, market participants are assumed to use the present-value form of exchange-rate determination of Eq. (5) and know the functional form of the stochastic process of monetary fundamentals, but not the parameter values in the

[18] In this paper, I assume $\eta$ to be constant. Note that AE can be rational when monetary fundamentals follow a random walk plus a noise process, with $\eta$ depending on the signal-to-noise ratio.

[19] Since $\rho$ is nearly one in one-regime case in Table 4, it is hard to expect a substantial difference in model-implied volatility of the returns between RE and AE. However, this picture somewhat changes for multiple-regime case as $\rho$ becomes significantly lower.

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process.[20] A sensible strategy for market participants would be to use the following two-step procedure. Each period in time, market participants fit the process, use estimated parameter values to set the nominal exchange rate, and periodically update their estimates. Market participants engage in updating of the parameter values in the assumed process of (6) by running a least squares regression of $f_t$ on $z_{t-1}$, where $z'_{t-1} = (1tf_{t-1})$. Let the vector of parameters in the fundamentals process be $\theta'_t = (\mu\delta\rho)$. In order to incorporate revisions of the estimated parameters into the monetary model, it is useful to apply the recursive least-squares (RLS) algorithm with appropriate initial values,[21]

$$S_t = S_{t-1} + \gamma_t(z_{t-1}z'_{t-1} - S_{t-1}), \tag{15}$$

$$\theta_t = \theta_{t-1} + \gamma_t S_t^{-1} z_{t-1}(f_t - z'_{t-1}\theta_{t-1}), \tag{16}$$

where $S_t$ is an estimate of the second moment of the data, and $\gamma_t$ is a deterministic sequence of gains that measures the responsiveness of estimate revisions with respect to new data. In decreasing gain least-squares learning, $\gamma_t = t^{-1}$ and as $t \to \infty$, market participants are able to obtain true parameter values. Next, under discounted least-squares or constant-gain learning, $\gamma_t$ is replaced by a small positive constant $\gamma_t = \gamma$. The main difference between this type of learning and ordinary RLS learning is that a constant-gain algorithm places a higher weight on recent forecast errors, which may be more reasonable when market participants believe the economic environment is continually changing but they do not know when a structural change occurs or they are concerned about possible model misspecification. See, for example, Sargent (1999), Kasa (2004), Orphanides and Williams (2005), and Branch and Evans (2006), among others.

The recursive formulas above show that the updated estimator $\theta_t$ is equal to the estimator used in the previous learning period plus an adjustment factor, which is proportional to the forecast error or innovation under the present-value learning. Since market participants know the functional form of the process, they forecast future values of fundamentals as if they were in a rational world, except that their estimates might differ from the true values. Therefore, the functional form of the nominal exchange-rate solution under present-value learning equals the RE solution, but here parameter values are replaced by market participants' estimates. Specifically, the nominal exchange rate under this type of learning now is obtained by

$$s_t^{\mathrm{PV}} = \frac{\lambda}{1+\lambda-\lambda\rho_t}[\mu_t + (1+\lambda)\delta_t + \delta_t t] + \frac{1}{1+\lambda-\lambda\rho_t} f_t. \tag{17}$$

The volatility of exchange rates implied by this model depends not only on unobservable innovations of fundamentals but also on parameter estimates that evolve over time. For example, the greater $\rho_t$ is, the stronger is the expected increase in $f_t$ and, hence, the higher the nominal exchange rate $s_t$. An expected sharp increase in $f_t$ is, therefore, immediately translated into a correspondingly high depreciation. By comparing analytical exchange-rate solutions, we can find other sources rendering more volatile exchange rates under learning compared to the RE case. First, innovations in the fundamentals now are magnified by even a small change in the time-varying persistence parameter $\rho_t$, which is in the denominator of the scaling factor in Eq. (17). Second, gradual movements of constant parameter estimate $\mu_t$ result in a bubble-type effect and thus create extra volatility during transition periods. Highly volatile exchange rates also help to explain why exchange-rate deviations from the fundamental value predict future depreciations, which might not be possible in the rational expectations approach. In addition, if parameter estimates move slowly to the true parameter values, the model-implied exchange rates exhibit substantial deviations from the long-run equilibrium value as shocks impinge on the economic fundamentals.[22] Therefore, AL combined with a changing economic environment may be an important source of highly persistent exchange-rate behavior.

### *3.4.2. Self-referential learning*

I next employ a less-bounded approach, self-referential learning. In this case, market participants fit a model to the exchange rate using the one-period-ahead forecast of the exchange rate instead of estimating the process of economic fundamentals itself and iterating the uncertain model forward into the infinite future.[23] That is, under self-referential learning, the monetary model is given by the first-order stochastic difference equation of (4) while maintaining the empirical specification of $f_t$. Let $\psi \equiv \lambda/(1+\lambda)$ and $\kappa \equiv 1/(1+\lambda)$. Then a rational expectations equilibrium (REE) is given by

$$s_t = \bar{a} + \bar{b}t + \bar{c}f_t, \tag{18}$$

where $\bar{a} = \psi/(1-\psi)[\bar{b} + (\mu+\delta)\bar{c}]$, $\bar{b} = \psi\delta/(1-\psi)\bar{c}$, and $\bar{c} = \kappa/(1-\psi\rho)$. I assume that market participants believe that the exchange rate is being generated by the process, but that they do not know the REE. Therefore, perceived law of motion that

[20] This assumption may be fairly strong when analyzing effects of incomplete knowledge about the economic environment. Timmermann (1996) and Evans and Honkapohja (2001), among others, however, show that the present-value model with an exogenous autoregressive forcing variable is strongly stable with respect to the inclusion of superfluous lags in the learning algorithm.

[21] There are two popular ways of setting initial values–initial conditions from randomly generated data and ad hoc initial conditions (Carceles-Poveda and Giannitsarou, 2007). I use initial values obtained from randomly generated data for the simulation exercise.

[22] In their recent paper, Boswijk et al. (2007) estimate an endogenous switching model, driven by reinforcement learning, of stock prices to explain large swings around the fundamentals and mean reverting behavior of stock prices.

[23] For a more detailed discussion of these two different specifications of learning, see, for example, Preston (2005) who argues, in multiperiod decision problems, agents' optimal decision rule depends on long-horizon forecasts, not just on one-period-ahead forecasts.

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they use to make forecasts of the next period's nominal exchange rate is $s_t = a + bt + cf_t$. The map from the perceived law of motion to the actual law of motion is given by

$$T_a(\phi) = \psi[a + b + c(\mu + \delta)], \tag{19}$$
$$T_b(\phi) = \psi(b + c\delta), \tag{20}$$
$$T_c(\phi) = \psi\rho c + \kappa, \tag{21}$$

where $\phi' = (a\ b\ c)$. Through continuous updating of the estimates, market participants are engaged in a learning process during which they try to derive the REE in a rational way. Specifically, market participants estimate $\boldsymbol{\phi}$ by using the following recursive algorithm:

$$R_t = R_{t-1} + \gamma_t(z_{t-1}z'_{t-1} - R_{t-1}), \tag{22}$$
$$\phi_t = \phi_{t-1} + \gamma_t R_t^{-1} z_{t-1}(s_t - z'_{t-1}\phi_{t-1}). \tag{23}$$

The behavior of exchange rates and the relation between exchange rates and monetary fundamentals implied by the model under self-referential learning are expected to be qualitatively similar to those generated under present-value learning. Time-varying parameter estimates under self-referential learning along with exogenous shocks can generate more volatile returns than under RE, and this may help to explain the correlation between exchange rates and fundamentals.

Having introduced a variety of learning specifications, it is important to examine the performance of the monetary model across the learning algorithms. First, in regard to variability, present-value learning is expected to generate relatively more volatile exchange-rate returns than self-referential learning because present-value learning, which deals directly with uncertainty about parameters in the process of fundamentals, is regarded as a more bounded case. That is, parameter estimates in the perceived law of motion under present-value learning, especially the constant and the time-trend coefficient, tend to converge to the REE more slowly than those under self-referential learning. Therefore, after a large number of learning periods, the contribution of the learning component to variations in returns will be relatively higher in the present-value learning model. On the other hand, given a reasonable value of constant gain, it is not easy to see how well the present-value learning with decreasing gain performs in comparison to constant-gain self-referential learning. Second, the introduction of constant-gain learning into the model is likely to aid much in explaining the persistence of the data. Since market participants stay alert for possible regime changes under the learning, a small number of potential structural breaks combined with constant-gain learning can generate escape-type dynamics of exchange rates, even if the underlying process for fundamentals does not undergo regime shifts.[24]

### 3.5. *Structural changes in monetary fundamentals*

When market participants set exchange rates in the monetary model, their forecasts of the variables of interest are based on the assumption of a stable process of fundamentals. This assumption, however, becomes less plausible in the presence of market participants' beliefs regarding structural changes. The monetary fundamentals, like many other economic variables, always are subject to change for unexpected events such as war, policy changes, and changes in exchange-rate regime. Therefore, market participants must contend with unannounced structural changes in the fundamentals. In order to gain further insight into this issue, I implement the testing methodology for multiple structural breaks by Bai and Perron (2003). Following their guideline, I assume the break does not occur during the initial 15 percent nor the final 15 percent of the sample period. Since both the *UD max* test and the *WD max* test indicate the presence of at least one break, I use a sequential procedure to estimate the location of the breakpoints. It is worth noting several features of the estimated break dates. First, as shown in Fig. 1, break points are identified around major macroeconomic events such as the second oil shock in the late 1970s, German reunification in 1990, the Plaza agreement in 1985, U.S. monetary policy regime changes in the early 1990s and in the early 2000s, and the creation of the EMU and the introduction of the euro from the late 1990s until the early 2000s. Second, in Table 4, the sign of the trend parameter estimate in both the U.K. pound and the Japanese yen changes signs across regimes. This ties in with the "long swings" result of Engel and Hamilton (1990), who developed a statistical model of exchange rates as a segmented time trend and rejected the random walk model in favor of their model of long swings. Third, the estimated serial correlation parameters are very close to one in the one-regime case, especially for the U.K. pound and the Deutsche mark, whereas those in the multiple-regime case are substantially lower. Finally, the fact that regime 1 has relatively higher volatility of innovations of fundamentals than the subsequent regimes suggests that the economic environment has been perturbed by less severe unexpected disturbances.

To incorporate changes in the economic environment into the monetary model, I begin with the special case that market participants know whether a break had in fact occurred right after the break. This may be reasonable if structural shifts are infrequent and their expected impacts are large enough for market participants to immediately recognize significant changes in the underlying process.[25] Under RE, market participants who can access a complete set of information on the

[24] For examples of escape dynamics, see Sargent (1999), Cho et al. (2002), and Bullard and Cho (2005), among others.

[25] There may exist some situation in which agents' beliefs about the stability of the process do not change instantaneously. For example, business cycle dating committees usually determine the dates of peaks (troughs) some time after a peak (trough) of economic activity has occurred. Benveniste et al. (1990) incorporate sequential testing for structural breaks into the standard stochastic recursive algorithm.

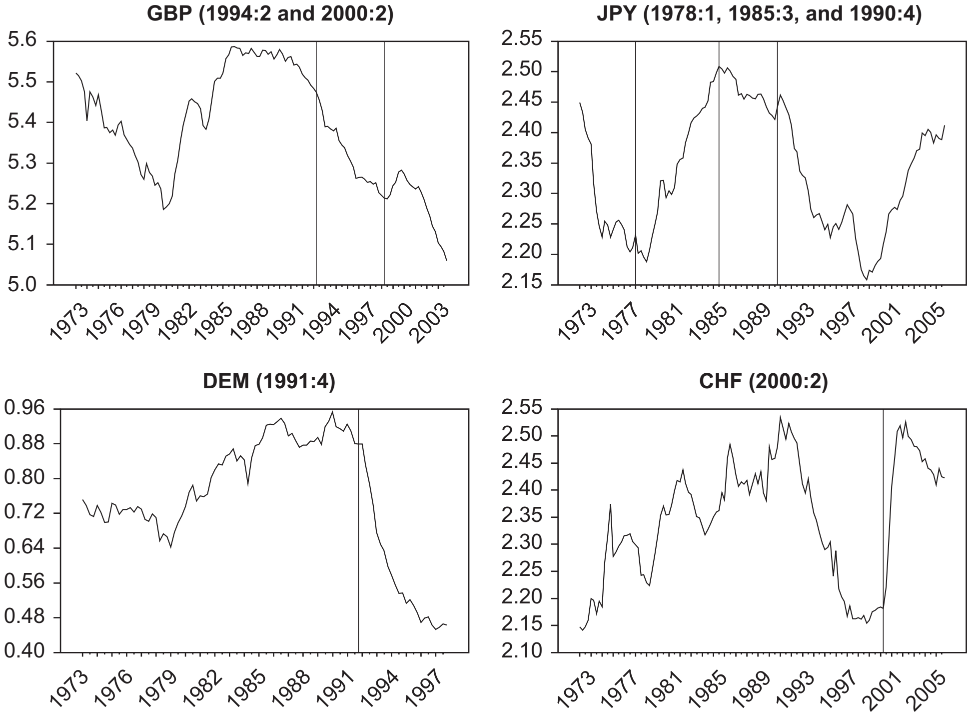


**Fig. 1.** Structural break dates in monetary fundamentals.

new structure of the economy can replace old parameters with new ones as soon as they realize that there has been a regime shift. On the other hand, if market participants were not able to obtain parameters for the new regime, which appears to be a more realistic situation, they must learn the new structure of the economy. I assume that market participants discard past observations of monetary fundamentals that are no longer useful and forecast the variables of interest with post-break data points.[26] Next and more importantly, when market participants believe the structure of the economy is changing over time, they are concerned about continuing structural shifts rather than infrequent discrete changes and have incentives to track any possible structural break by placing a relatively higher weight on recent data. In this case, as suggested by Sargent (1999), Sargent and Williams (2005), Bullard and Eusepi (2005), and Orphanides and Williams (2005), a constant-gain algorithm may perform better at picking up a slow change in parameters.

Introducing structural shifts is likely to help to understand mainly the following two issues. First, the model-implied volatility of exchange-rate returns is expected to be higher. Even in the RE monetary model, a one-time jump in the process would generate more volatile returns, albeit slightly, but it is difficult to imagine that a structural break substantially improves its performance in matching predictability. In the multiple-regime case, since the serial correlation parameter estimates are significantly lower than unity, the volatility of returns implied by the AE model will be higher than in the RE model. In AL models, changes in parameters account for a great deal of the excess volatility. In the case of slow changes in the model structure, market participants are likely to employ a constant-gain sequence and their parameter estimates gradually fluctuate the REE, even in the long run. Even when the structural changes are discrete and occasional, market participants struggle with a small sample problem around a structural break point, which generates largely biased time-varying parameter estimates. Therefore, any type of learning algorithm may result in more volatile model-implied returns than those under the rational expectations approach. Second, even for the decreasing gain learning approach, a structural change may help to explain highly persistent error-correction terms or large swings of exchange rates. This is because unanticipated regime shifts in monetary fundamentals, and, thus, changes in the REE, prevent market participants from quickly learning a new REE, resulting in much slower adjustments of the parameter estimates.

## 4. Simulation results

The predictions of the monetary models under AL are compared to those generated under standard RE and under AE. In addition, the differences in empirical performance across specifications of learning also are examined. I evaluate how the

[26] With regard to gain reinitialization, for decreasing gain learning, market participants reset the gain at the time of the perceived break point so that the gain is increased to a lager value. I use a fixed value of the constant-gain parameter for both regimes. For endogenous constant-gain parameters, see Chakraborty and Evans (2009), for example. Finally, I use initial values obtained from randomly generated data in the new regime.

**Table 4**
Estimated parameter values of monetary fundamentals process.

| $f_t = \mu + \delta t + \rho f_{t-1} + \varepsilon_t$ | $\hat{\mu}$ | $\hat{\delta}$ | $\hat{\rho}$ | $\hat{\sigma}_\varepsilon$ | $R^2$ |
|---|---|---|---|---|---|
| Case I: one-regime case | | | | | |
| GBP | 0.0184 | −0.00008 | 0.9970 | 0.0214 | 0.974 |
| | (0.0843) | (0.00006) | (0.0154) | | |
| DEM | 0.0100 | −0.00021 | 0.9984 | 0.0217 | 0.977 |
| | (0.0144) | (0.00008) | (0.0165) | | |
| JPY | 0.0290 | 0.00005 | 0.9858 | 0.0174 | 0.970 |
| | (0.0367) | (0.00004) | (0.0156) | | |
| CHF | 0.1186 | −0.00002 | 0.9509 | 0.0311 | 0.922 |
| | (0.0592) | (0.00007) | (0.0255) | | |
| Case II: multiple-regime case | | | | | |
| *Regime* 1 | | | | | |
| GBP | 0.2707 | 0.00023 | 0.9484 | 0.0237 | 0.960 |
| | (0.1602) | (0.00015) | (0.0302) | | |
| DEM | 0.0844 | 0.00043 | 0.8773 | 0.0208 | 0.946 |
| | (0.0407) | (0.00026) | (0.0615) | | |
| JPY | 0.6561 | −0.00011 | 0.7063 | 0.0176 | 0.827 |
| | (0.3282) | (0.00147) | (0.1385) | | |
| CHF | 0.1152 | −0.00018 | 0.9551 | 0.0282 | 0.925 |
| | (0.0632) | (0.00009) | (0.0269) | | |
| *Regime* 2 | | | | | |
| GBP | 2.0579 | −0.00326 | 0.6189 | 0.0103 | 0.975 |
| | (0.8069) | (0.00151) | (0.1482) | | |
| DEM | 0.0494 | 0.00022 | 0.8784 | 0.0109 | 0.990 |
| | (0.0400) | (0.00092) | (0.0495) | | |
| JPY | 0.9621 | 0.00506 | 0.5593 | 0.0134 | 0.981 |
| | (0.3417) | (0.00180) | (0.1580) | | |
| CHF | 0.8615 | −0.00273 | 0.6651 | 0.0159 | 0.897 |
| | (0.1306) | (0.00062) | (0.0539) | | |
| *Regime* 3 | | | | | |
| GBP | 0.7500 | −0.00296 | 0.8614 | 0.0082 | 0.986 |
| | (0.2876) | (0.00054) | (0.0542) | | |
| JPY | 1.1117 | −0.00150 | 0.5548 | 0.0085 | 0.874 |
| | (0.5668) | (0.00096) | (0.2257) | | |
| *Regime* 4 | | | | | |
| JPY | 0.0784 | 0.00049 | 0.9590 | 0.0138 | 0.971 |
| | (0.0519) | (0.00011) | (0.0225) | | |

*Note*: Table entries are OLS estimates and standard errors are in parentheses.

simulated data are capable of matching some important features of the actual data in the foreign exchange market, as described in Section 2.[27] A set of reasonable model parameter values is employed. First, I consider an interest semi-elasticity of money demand that is constructed with a fixed value of $\lambda = 8$ (0.02 for annual data with the interest rate expressed in percent per annum) which is taken from Mark and Sul (2003). Second, I take the OLS estimates of Eq. (6), presented in Table 4, as parameters in the data generating process of the monetary fundamentals. Finally, I use adaptive expectations adjustment parameter of $\eta = 0.7$ from Kim and Mark (2006) and a constant-gain parameter of $\gamma = 0.02$ from Orphanides and Williams (2005).[28]

I simulate 5,000 replications of artificial data and calculate median values of the model-implied statistics from this data.[29] I use a sample size of 100 for the Deutsche mark and 130 for the other currencies, which is identical to the sizes of the historical data, and also use a large sample consisting of 1,000 observations in order to examine the long-run implications of the models. Both one-regime and multiple-regime cases are considered. Although statistical tests suggest

[27] Alternatively, Ki et al. (2008) focuses more on actual historical sample paths than on matching moments to examine to what extent learning can account for the actually observed fluctuations in exchange rates.

[28] I performed an extensive sensitivity analysis. I found that the basic conclusion of this paper is robust to variations in $\lambda$, $\eta$, and $\gamma$.

[29] I also compute simulated distributions of a variety of estimates and statistics to check whether the median value can serve as a measure of comparing the models in this paper. I found significant distributional shifts in model-generated statistics across models. To conserve on space, I did not report percentile values of each individual statistic (available from the author upon request). To overcome potential issues related to this, I will employ a statistic as a way of conveying the overall fit of the monetary models later in this section.

**Table 5**
Out-of-sample forecasts of U.S. dollar returns.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| *A. U statistic* | | | | | | | | | | | | | |
| *GBP* | | | | | | | | | | | | | |
| 1 | 0.99 | 1.02 | 0.05 | 1.00 | 1.00 | 1.01 | 1.01 | 1.11 | 1.82 | 1.35 | 1.30 | 1.01 | 1.02 |
| 16 | 1.40 | 1.57 | 0.94 | 1.00 | 1.05 | 1.17 | 1.05 | 1.04 | 1.07 | 1.20 | 1.13 | 1.14 | 1.16 |
| *DEM* | | | | | | | | | | | | | |
| 1 | 1.00 | 0.99 | 0.04 | 0.99 | 0.93 | 0.99 | 0.99 | 1.27 | 2.74 | 1.21 | 1.04 | 1.09 | 1.07 |
| 16 | 1.03 | 1.24 | 0.94 | 1.15 | 0.90 | 1.03 | 0.94 | 0.99 | 1.12 | 1.07 | 1.11 | 1.02 | 1.00 |
| *JPY* | | | | | | | | | | | | | |
| 1 | 1.01 | 1.02 | 0.04 | 1.01 | 1.02 | 1.01 | 1.01 | 0.90 | 0.84 | 1.11 | 1.00 | 1.01 | 1.05 |
| 16 | 0.96 | 1.26 | 0.99 | 1.03 | 1.21 | 1.14 | 1.10 | 1.17 | 1.12 | 1.21 | 0.72 | 1.09 | 1.39 |
| *CHF* | | | | | | | | | | | | | |
| 1 | 0.98 | 1.01 | 0.04 | 1.02 | 1.00 | 1.01 | 1.01 | 1.00 | 2.16 | 1.22 | 1.05 | 1.02 | 1.02 |
| 16 | 0.58 | 0.95 | 0.94 | 0.99 | 0.93 | 1.02 | 0.89 | 1.03 | 1.02 | 1.07 | 1.17 | 1.03 | 1.02 |
| *B. DM statistic* | | | | | | | | | | | | | |
| *GBP* | | | | | | | | | | | | | |
| 1 | 0.40 | 0.47 | −6.57 | 0.38 | −0.06 | 0.39 | 0.73 | 1.27 | 0.94 | 1.22 | 1.28 | 0.91 | 0.97 |
| 16 | 1.69 | 1.13 | −0.46 | 0.48 | 0.89 | 1.06 | 1.24 | 0.85 | 1.66 | 1.81 | 1.78 | 1.79 | 1.89 |
| *DEM* | | | | | | | | | | | | | |
| 1 | 0.34 | −0.50 | −5.44 | −0.55 | −0.84 | −0.29 | 0.46 | 1.18 | 1.02 | 1.43 | 1.02 | 1.22 | 1.25 |
| 16 | 0.31 | 0.75 | 1.11 | −0.20 | 0.06 | 1.58 | 1.89 | 1.19 | 1.67 | 1.69 | 1.97 | 1.02 | 1.12 |
| *JPY* | | | | | | | | | | | | | |
| 1 | 0.82 | 1.53 | −6.68 | 0.90 | 1.21 | 0.56 | 0.66 | −1.85 | −0.96 | 1.39 | 1.23 | 1.59 | 1.44 |
| 16 | 0.06 | 1.56 | 1.49 | 1.53 | 1.59 | 1.31 | 1.54 | 1.41 | 3.05 | 1.18 | −0.26 | 1.38 | 2.03 |
| *CHF* | | | | | | | | | | | | | |
| 1 | −0.69 | 0.74 | −6.75 | 1.52 | 1.65 | 0.50 | 0.86 | 0.86 | 0.91 | 1.49 | 0.81 | 1.20 | 1.08 |
| 16 | −1.18 | 1.15 | 1.42 | 1.48 | 1.73 | 0.77 | 1.04 | 1.11 | 1.60 | 1.62 | 1.10 | 1.65 | 1.72 |

that there is at least one structural change in the monetary fundamentals, a one-regime case is useful to examine the possibility that market participants believe there has not been a structural shift, or the economy is continually changing. More importantly, a one-regime environment allows us to separate the ability of models from the presence of structural beaks.

### 4.1. *Statistics from simulated data*

First, I simulate the monetary model under RE, where market participants know the parameter values of the assumed process of monetary fundamentals and they set the nominal exchange rate according to the RE solution given by Eq. (7). Next, as another benchmark case, the AE model is considered. Since market participants know neither the functional form of the process nor the parameter values in the AE approach, nominal exchange rates are set in terms of past observations of the monetary fundamentals, as in Eq. (12). Finally, I introduce adaptive learning into the monetary model where market participants have limited knowledge about the model structure in various dimensions and learn the model structure in accordance with the least squares principle. Depending on the variables being forecasted and the types of gain sequences in the learning procedure, I consider four specifications of the learning model, (i) present-value learning with decreasing gain, (ii) present-value learning with constant gain, (iii) self-referential learning with decreasing gain, and (iv) self-referential learning with constant gain. I compare the predictions of the aforementioned monetary models and evaluate which model has a better ability to account for the following aspects of the data: (1) standard deviations of quarterly exchange-rate returns that are around 5–7 percent, which are excessively volatile compared to those of monetary fundamentals, (2) variance ratio statistics that decline from greater than one over short horizons to less than one over long horizons, (3) slope coefficients, $t$ ratios, and $R^2$'s of long-horizon exchange-rate return regressions that increase with horizon,[30] and (4) no

[30] To evaluate in-sample fit of the models, it may be more appropriate to use estimates and test statistics corrected for size distortions and small sample bias, although such an exercise leaves the basic conclusion intact as in Table 2. However, for example, generating finite sample critical values for the large number of cases and simulations I deal with would be computationally infeasible. Note that the large sample analysis shown in Table 13 may mitigate this potential issue.

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**Table 6**
Volatility of historical and simulated data.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| *Returns* | | | | | | | | | | | | | |
| GBP | 5.08 | 2.09 | 1.87 | 3.60 | 4.06 | 3.49 | 3.70 | 2.83 | 3.84 | 4.13 | 4.88 | 4.15 | 4.38 |
| DEM | 6.36 | 2.23 | 1.85 | 2.90 | 3.04 | 2.72 | 2.79 | 3.40 | 3.64 | 4.92 | 5.97 | 4.64 | 5.32 |
| JPY | 5.99 | 1.58 | 1.80 | 2.37 | 2.81 | 2.14 | 2.71 | 1.81 | 3.87 | 3.84 | 4.36 | 2.95 | 4.34 |
| CHF | 6.70 | 2.20 | 2.61 | 4.03 | 4.50 | 3.28 | 4.16 | 3.12 | 3.28 | 5.06 | 5.82 | 4.45 | 5.39 |
| *Deviation from fundamentals* | | | | | | | | | | | | | |
| GBP | 0.22 | 0.02 | 0.01 | 0.07 | 0.16 | 0.23 | 0.26 | 0.07 | 0.02 | 0.08 | 0.11 | 0.12 | 0.11 |
| DEM | 0.30 | 0.05 | 0.01 | 0.05 | 0.14 | 0.08 | 0.08 | 0.05 | 0.02 | 0.08 | 0.13 | 0.25 | 0.25 |
| JPY | 0.39 | 0.01 | 0.01 | 0.03 | 0.06 | 0.08 | 0.11 | 0.05 | 0.03 | 0.16 | 0.16 | 0.11 | 0.13 |
| CHF | 0.25 | 0.03 | 0.01 | 0.07 | 0.15 | 0.24 | 0.36 | 0.04 | 0.02 | 0.13 | 0.37 | 0.14 | 0.23 |

*Note*: Table entries are standard deviations and medium values of model-implied statistics of 5,000 simulations. RE and AE refer to rational expectations and adaptive expectations, respectively. Specifications of adaptive learning are (i) present-value learning with decreasing gain, (ii) present-value learning with constant gain, (iii) self-referential learning with decreasing gain, and (iv) self-referential learning with constant gain.

systematic pattern of *U* statistics and *DM* statistics, except for the Swiss franc, that measure out-of-sample forecasting power of the monetary model relative to the random walk model. I present results for the competing monetary models in Table 6 through Table 7. To facilitate a comparison with the historical data, I also report corresponding historical statistics.

#### *4.1.1. The RE model*

The implied behavior of exchange rates and monetary fundamentals when market participants have RE are seen to perform poorly. The volatilities of one-quarter nominal exchange rate returns and deviations of the exchange rate from the fundamentals are far below their sample values, and the returns and deviations of the exchange rate from the fundamentals do not exhibit substantial persistence, except in the case of the Deutsche mark. Long-horizon regressions of the exchange-rate returns on their current deviations at horizons of 1, 4, 8, and 16 quarters are presented in Tables 9–12. The statistics obtained from the historical data show that the slope coefficients $\beta_k$ all have positive signs and increase linearly with horizon, and the $R^2$ statistics start low but then rise to impressive values. The RE model yields almost the same patterns as those found in data, but with the "wrong" sign. That is, the proportion of the total variation in $k$-period returns explained by the regression of $k$-period returns on the current deviation from the fundamentals increases, but the fitted values move in the opposite direction of the actual changes. A possible explanation for this comes from the well known property of the RE monetary models with stationary fundamentals which suggests that the fundamentals are more volatile than the exchange rates. When the variability of the exchange rate is relatively low compared to that of the fundamentals, the exchange rate could be expected to fall, even if today's exchange rate is below its long-run equilibrium value. In Table 5, I also report the *U* statistics and *DM* statistics measuring the accuracy of out-of-sample forecast of U.S. dollar returns for horizons $k = 1$ and 16. With an exception for one-quarter dollar return on the Deutsche mark, the RE-model simulated data suggest that the random walk model consistently dominates the monetary model in the context of out-of-sample forecastability, even for 16-quarter return on the Swiss franc. Finally, I investigate the long-run implications of the RE model and present some selected results for the Swiss franc in Table 13.[31] I found that there is no significant difference in the model implications between two sample sizes. Given the fact that in the RE model the economic environment is assumed to be stable over time and market participants fully understand the structure of the economy from the beginning of sample, this finding does not come as surprise.

Introducing structural shifts into the fundamentals generates a bit more volatile exchange-rate returns, but still much lower than their sample values. Returns now become more persistent, but the exchange-rate deviations from the fundamentals are still negatively serially correlated, even over short horizons. Furthermore, the model with breaks generates error-correction terms that predict depreciations in the wrong direction and its out-of-sample fit performance is virtually indistinguishable from the one-regime case. Therefore, the introduction of a small number of structural changes into the monetary approach under rational expectations is unlikely to help much to understand the observed exchange-rate dynamics.

[31] Results for other currencies are similar and are suppressed.

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**Table 7**
Persistence of historical and simulated data (I): returns.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| *GBP* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.26 | 0.92 | 1.22 | 0.71 | 1.00 | 1.29 | 1.15 | 1.35 | 0.57 | 1.06 | 1.16 | 1.16 | 1.18 |
| 16 | 1.02 | 0.82 | 1.10 | 0.54 | 0.85 | 1.35 | 1.11 | 1.43 | 0.46 | 0.87 | 0.92 | 1.04 | 1.03 |
| 32 | 0.31 | 0.59 | 0.76 | 0.30 | 0.50 | 1.16 | 0.86 | 1.42 | 0.39 | 0.60 | 0.61 | 0.86 | 0.76 |
| *DEM* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.06 | 1.21 | 1.79 | 1.10 | 1.96 | 1.53 | 1.78 | 1.43 | 0.81 | 1.04 | 1.33 | 1.00 | 0.94 |
| 16 | 0.84 | 1.22 | 1.91 | 0.96 | 1.91 | 1.80 | 2.17 | 1.61 | 1.01 | 0.96 | 1.05 | 0.68 | 0.66 |
| 32 | 0.31 | 0.95 | 1.48 | 0.60 | 1.11 | 1.66 | 1.89 | 1.07 | 0.66 | 0.62 | 0.64 | 0.26 | 0.27 |
| *JPY* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.16 | 0.90 | 0.87 | 0.66 | 0.70 | 0.99 | 1.07 | 1.92 | 0.45 | 0.92 | 1.01 | 1.25 | 1.29 |
| 16 | 0.88 | 0.76 | 0.72 | 0.44 | 0.48 | 0.90 | 1.01 | 2.62 | 0.53 | 0.86 | 0.93 | 1.25 | 0.91 |
| 32 | 0.45 | 0.50 | 0.49 | 0.22 | 0.22 | 0.64 | 0.79 | 3.05 | 0.53 | 0.86 | 0.66 | 0.80 | 0.42 |
| *CHF* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.01 | 0.80 | 1.11 | 0.68 | 0.91 | 0.95 | 1.12 | 1.03 | 1.25 | 0.64 | 0.92 | 1.19 | 1.32 |
| 16 | 0.81 | 0.60 | 0.88 | 0.48 | 0.63 | 0.80 | 1.09 | 0.83 | 1.03 | 0.43 | 0.83 | 1.10 | 1.23 |
| 32 | 0.25 | 0.35 | 0.51 | 0.24 | 0.33 | 0.54 | 0.91 | 0.42 | 0.56 | 0.22 | 0.55 | 0.68 | 0.81 |

*Note*: Table entries are variance ratio statistics.

### 4.1.2. *The AE model*

Predictions of the monetary model under AE appear to match the data more closely than the RE model. The AE model without a structural change generates more volatile one-quarter dollar returns for the Japanese yen and the Swiss franc than those in the RE model, while the volatility of the returns on the U.K. pound and the Deutsche mark are somewhat smaller than in the RE model as the serial correlation coefficients in the process of fundamentals are nearly unity. However, in the multiple-regime case, the returns become consistently more volatile than in the RE model. I found that the more volatile returns in the AE model with structural changes in the fundamentals are mainly due to the serial correlation parameter values that are significantly lower, not because of the temporarily excessive volatility of returns during the transition between two regimes. The variance ratio statistics reported in Tables 7 and 8 indicate that, in the one-regime case, the AE model generates positively serially correlated returns over short horizons, except for the Japanese yen. However, the variance ratios for deviations from the fundamentals are less than one at all horizons and the model-implied returns are more persistent than the deviations, even with the structural shifts. This suggests that the AE model fails to substantially explain the persistent error-correction terms in the monetary model.

In Tables 9–12, I present results of long-horizon return regressions. The AE model now is able to generate "positive" slope coefficients, but not in such a way that the estimates and regression statistics increase with return horizon. In general, the slope coefficients, $t$ ratios, and $R^2$ tend to have larger values in short return horizon.[32] This reflects, under the AE hypothesis, market participants may place too much weight on recent data when they revise their expectations. I found the same implications of the model when computing out-of-sample fit statistics. As measured by the $U$ statistic and the $DM$ statistic, the performance of out-of-sample forecasts with the AE model is much better for short return horizons, for most cases. Overall, in terms of predictability, the AE model comes closer to the actual data than the RE model as the monetary fundamentals predict future depreciations with the correct sign, but beyond that there is little systematic relationship. Finally, in large samples, the AE model generates volatility of returns on the Swiss franc that is lower than in the small

[32] Note that the AE model predictions on the in-sample fit analysis are sensitive to the choice of $\eta$. The AE model with a small value of $\eta$ tends to produce the slope coefficients that increase with return horizon. For a large value of $\eta$, such as $\eta = 0.9$, the slope coefficients are negative, as in the RE model.

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**Table 8**
Persistence of historical and simulated data (II): deviation from fundamentals.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| *GBP* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.67 | 0.92 | 0.18 | 0.50 | 0.76 | 1.40 | 1.19 | 0.61 | 0.19 | 0.93 | 1.08 | 1.09 | 1.07 |
| 16 | 1.57 | 0.82 | 0.08 | 0.31 | 0.55 | 1.52 | 1.19 | 0.49 | 0.10 | 0.66 | 0.79 | 0.92 | 0.85 |
| 32 | 0.80 | 0.59 | 0.04 | 0.17 | 0.29 | 1.29 | 0.98 | 0.28 | 0.04 | 0.32 | 0.39 | 0.61 | 0.48 |
| *DEM* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.35 | 1.21 | 0.17 | 0.50 | 0.65 | 2.04 | 2.35 | 0.66 | 0.17 | 0.75 | 1.13 | 1.66 | 1.33 |
| 16 | 1.31 | 1.22 | 0.08 | 0.29 | 0.42 | 2.55 | 3.12 | 0.42 | 0.08 | 0.47 | 0.69 | 1.73 | 1.36 |
| 32 | 0.78 | 0.94 | 0.03 | 0.14 | 0.19 | 2.32 | 2.96 | 0.11 | 0.02 | 0.14 | 0.22 | 0.91 | 0.73 |
| *JPY* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.47 | 0.89 | 0.19 | 0.70 | 0.84 | 1.00 | 1.09 | 0.90 | 0.18 | 1.02 | 1.02 | 1.03 | 1.23 |
| 16 | 1.27 | 0.76 | 0.09 | 0.48 | 0.62 | 0.91 | 1.03 | 0.89 | 0.09 | 0.91 | 1.04 | 0.93 | 0.70 |
| 32 | 0.70 | 0.50 | 0.04 | 0.28 | 0.34 | 0.65 | 0.80 | 0.86 | 0.04 | 0.79 | 0.83 | 0.62 | 0.19 |
| *CHF* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.13 | 0.79 | 0.18 | 0.48 | 0.49 | 0.96 | 1.07 | 0.46 | 0.21 | 0.90 | 1.03 | 1.02 | 1.10 |
| 16 | 0.89 | 0.60 | 0.08 | 0.29 | 0.29 | 0.83 | 1.05 | 0.30 | 0.11 | 0.71 | 0.96 | 0.99 | 1.13 |
| 32 | 0.27 | 0.34 | 0.04 | 0.14 | 0.13 | 0.57 | 0.83 | 0.13 | 0.03 | 0.41 | 0.62 | 0.68 | 0.86 |

*Note*: Table entries are variance ratio statistics.

**Table 9**
Long-horizon U.S. dollar return regression: U.K. pound.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| $\hat{\beta}_k$ | | | | | | | | | | | | | |
| 1 | 0.05 | −0.14 | 0.73 | 0.13 | 0.11 | 0.03 | 0.02 | −0.10 | 0.70 | 0.09 | 0.02 | 0.03 | 0.04 |
| 4 | 0.19 | −0.54 | 1.13 | 0.44 | 0.40 | 0.11 | 0.08 | −0.35 | 0.94 | 0.33 | 0.15 | 0.14 | 0.18 |
| 8 | 0.42 | −1.09 | 1.24 | 0.76 | 0.72 | 0.22 | 0.16 | −0.65 | 0.73 | 0.56 | 0.35 | 0.29 | 0.37 |
| 16 | 0.71 | −2.17 | 1.41 | 1.17 | 0.99 | 0.43 | 0.29 | −0.90 | 0.04 | 0.89 | 0.56 | 0.61 | 0.76 |
| $t_k$ | | | | | | | | | | | | | |
| 1 | 1.93 | −1.54 | 4.01 | 1.74 | 3.44 | 1.33 | 1.62 | −3.24 | 7.65 | 2.16 | 0.66 | 0.82 | 1.05 |
| 4 | 3.67 | −1.80 | 2.01 | 2.23 | 4.22 | 1.57 | 1.95 | −2.94 | 3.44 | 3.23 | 1.52 | 1.20 | 1.43 |
| 8 | 5.86 | −2.29 | 1.49 | 2.95 | 5.12 | 2.04 | 2.50 | −2.71 | 2.07 | 4.65 | 2.22 | 1.69 | 1.89 |
| 16 | 4.60 | −3.25 | 1.21 | 3.94 | 7.72 | 2.92 | 3.69 | −1.95 | 0.12 | 6.26 | 2.94 | 2.94 | 3.03 |
| $R^2$ | | | | | | | | | | | | | |
| 1 | 0.04 | 0.01 | 0.09 | 0.03 | 0.08 | 0.04 | 0.01 | 0.04 | 0.26 | 0.06 | −0.01 | 0.00 | 0.00 |
| 4 | 0.14 | 0.05 | 0.03 | 0.11 | 0.23 | 0.16 | 0.07 | 0.12 | 0.12 | 0.21 | 0.01 | 0.04 | 0.05 |
| 8 | 0.29 | 0.11 | 0.02 | 0.18 | 0.39 | 0.28 | 0.13 | 0.14 | 0.03 | 0.31 | 0.05 | 0.09 | 0.10 |
| 16 | 0.51 | 0.23 | 0.01 | 0.28 | 0.50 | 0.46 | 0.26 | 0.08 | −0.01 | 0.41 | 0.09 | 0.21 | 0.22 |

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**Table 10**
Long-horizon U.S. dollar return regression: Deutsche mark.

| | Historical data | Simulated data | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) | |
| $\hat{\beta}_k$ | | | | | | | | | | | | | | |
| 1 | 0.04 | −0.13 | 0.92 | 0.28 | 0.16 | 0.01 | 0.03 | −0.10 | 0.54 | 0.01 | 0.03 | 0.00 | 0.00 | |
| 4 | 0.15 | −0.51 | 1.99 | 0.89 | 0.60 | 0.13 | 0.19 | −0.31 | 0.32 | 0.15 | 0.31 | 0.01 | 0.00 | |
| 8 | 0.34 | −1.01 | 2.89 | 1.02 | 0.97 | 0.35 | 0.48 | −0.43 | −0.12 | 0.33 | 0.73 | 0.05 | 0.04 | |
| 16 | 0.87 | −2.00 | 4.28 | 1.11 | 1.02 | 0.91 | 1.24 | −0.43 | −0.53 | 0.64 | 1.10 | 0.34 | 0.50 | |
| $t_k$ | | | | | | | | | | | | | | |
| 1 | 1.70 | −3.05 | 3.70 | 4.23 | 5.34 | 0.29 | 0.84 | −3.47 | 16.73 | 0.40 | 0.64 | −0.23 | −0.11 | |
| 4 | 2.17 | −3.57 | 2.39 | 5.10 | 6.47 | 1.11 | 2.47 | −2.76 | 2.61 | 1.14 | 1.76 | 0.29 | 0.07 | |
| 8 | 2.75 | −4.41 | 2.06 | 6.53 | 7.76 | 2.80 | 4.64 | −2.25 | −0.60 | 1.35 | 2.41 | 0.54 | 0.32 | |
| 16 | 3.73 | −6.03 | 1.85 | 8.63 | 10.91 | 6.27 | 9.21 | −1.41 | −2.01 | 1.63 | 2.94 | 0.81 | 1.17 | |
| $R^2$ | | | | | | | | | | | | | | |
| 1 | 0.03 | 0.06 | 0.12 | 0.13 | 0.29 | 0.02 | 0.02 | 0.02 | 0.13 | −0.01 | −0.01 | −0.01 | −0.01 | |
| 4 | 0.11 | 0.23 | 0.08 | 0.36 | 0.47 | 0.10 | 0.09 | 0.04 | 0.01 | 0.00 | 0.07 | 0.00 | 0.00 | |
| 8 | 0.22 | 0.39 | 0.07 | 0.47 | 0.50 | 0.16 | 0.16 | 0.03 | −0.01 | 0.02 | 0.19 | 0.00 | 0.01 | |
| 16 | 0.53 | 0.59 | 0.06 | 0.58 | 0.61 | 0.30 | 0.36 | 0.01 | 0.00 | 0.05 | 0.31 | 0.04 | 0.08 | |

**Table 11**
Long-horizon U.S. dollar return regression: Japanese yen.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| $\hat{\beta}_k$ | | | | | | | | | | | | | |
| 1 | 0.02 | −0.18 | 0.66 | 0.10 | 0.03 | 0.01 | 0.02 | −0.11 | 0.69 | 0.08 | 0.07 | 0.06 | 0.06 |
| 4 | 0.08 | −0.71 | 0.75 | 0.36 | 0.14 | 0.03 | 0.09 | −0.39 | 0.91 | 0.26 | 0.26 | 0.25 | 0.28 |
| 8 | 0.18 | −1.38 | 0.55 | 0.60 | 0.28 | 0.07 | 0.18 | −0.68 | 0.89 | 0.44 | 0.49 | 0.47 | 0.53 |
| 16 | 0.36 | −2.57 | 0.14 | 0.86 | 0.55 | 0.14 | 0.34 | −0.91 | 0.91 | 0.63 | 0.87 | 0.82 | 0.73 |
| $t_k$ | | | | | | | | | | | | | |
| 1 | 1.37 | −1.98 | 3.09 | 1.48 | 0.75 | 0.81 | 1.15 | −3.52 | 21.36 | 2.29 | 2.02 | 1.16 | 0.63 |
| 4 | 1.79 | −2.33 | 1.32 | 1.96 | 1.01 | 1.03 | 1.50 | −3.23 | 7.50 | 3.13 | 3.05 | 1.48 | 1.05 |
| 8 | 2.66 | −2.90 | 0.69 | 2.55 | 1.34 | 1.38 | 1.99 | −2.92 | 3.59 | 4.02 | 4.93 | 2.22 | 2.23 |
| 16 | 4.37 | −4.04 | 0.18 | 3.24 | 2.33 | 1.93 | 2.90 | −2.11 | 1.96 | 4.79 | 9.32 | 6.29 | 10.70 |
| $R^2$ | | | | | | | | | | | | | |
| 1 | 0.01 | 0.02 | 0.11 | 0.02 | 0.01 | 0.01 | 0.01 | 0.09 | 0.27 | 0.05 | 0.06 | 0.04 | 0.02 |
| 4 | 0.06 | 0.09 | 0.02 | 0.08 | 0.05 | 0.04 | 0.05 | 0.22 | 0.24 | 0.18 | 0.25 | 0.17 | 0.13 |
| 8 | 0.15 | 0.18 | 0.00 | 0.14 | 0.10 | 0.09 | 0.11 | 0.24 | 0.12 | 0.31 | 0.43 | 0.30 | 0.24 |
| 16 | 0.37 | 0.34 | 0.00 | 0.20 | 0.21 | 0.18 | 0.20 | 0.15 | 0.05 | 0.34 | 0.62 | 0.43 | 0.31 |

sample case, but still is higher than that of the RE model. For other features of the data, I found no significant changes in the model implications between the sample sizes.

### *4.1.3. The AL model*

I now examine predictions of the monetary model under adaptive learning and compare the model implications to those generated in the benchmark models. I find that any of the AL models (i)–(iv) dominates the alternative specifications of expectations in its ability to account for the stylized facts found in the foreign exchange market. First, the AL model-implied volatilities of one-quarter nominal exchange-rate returns and error-correction terms are much higher than the RE and the AE model, as we see clearly in Table 6. Even when parameter estimates under decreasing gain learning appear to

**Table 12**
Long-horizon U.S. dollar return regression: Swiss franc.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| $\hat{\beta}_k$ | | | | | | | | | | | | | |
| 1 | 0.06 | −0.18 | 0.69 | 0.20 | 0.18 | 0.01 | 0.02 | −0.12 | 0.97 | 0.10 | 0.03 | 0.01 | 0.01 |
| 4 | 0.24 | −0.67 | 0.95 | 0.64 | 0.60 | 0.06 | 0.08 | −0.41 | 1.74 | 0.39 | 0.12 | 0.07 | 0.08 |
| 8 | 0.47 | −1.19 | 0.94 | 0.92 | 0.90 | 0.11 | 0.16 | −0.64 | 1.79 | 0.68 | 0.22 | 0.16 | 0.18 |
| 16 | 0.72 | −1.97 | 0.98 | 1.00 | 1.00 | 0.22 | 0.30 | −0.74 | 1.37 | 1.05 | 0.61 | 0.37 | 0.42 |
| $t_k$ | | | | | | | | | | | | | |
| 1 | 2.95 | −2.57 | 2.90 | 3.20 | 4.18 | 1.69 | 2.13 | −2.78 | 19.63 | 1.10 | 1.30 | 0.64 | 0.88 |
| 4 | 3.87 | −3.20 | 1.13 | 4.35 | 5.47 | 2.07 | 2.65 | −2.45 | 12.08 | 1.58 | 1.44 | 0.99 | 1.27 |
| 8 | 6.34 | −4.21 | 0.52 | 5.70 | 7.18 | 2.68 | 3.52 | −2.17 | 7.04 | 2.16 | 1.50 | 1.30 | 1.67 |
| 16 | 8.03 | −5.96 | 0.25 | 7.62 | 11.05 | 3.81 | 5.28 | −1.44 | 3.51 | 2.62 | 2.09 | 1.91 | 2.40 |
| $R^2$ | | | | | | | | | | | | | |
| 1 | 0.05 | 0.04 | 0.08 | 0.08 | 0.11 | 0.01 | 0.03 | 0.02 | 0.49 | 0.01 | 0.15 | 0.00 | 0.00 |
| 4 | 0.21 | 0.15 | 0.02 | 0.23 | 0.34 | 0.08 | 0.14 | 0.05 | 0.29 | 0.04 | 0.30 | 0.02 | 0.02 |
| 8 | 0.37 | 0.28 | 0.01 | 0.34 | 0.47 | 0.15 | 0.26 | 0.07 | 0.14 | 0.07 | 0.26 | 0.04 | 0.06 |
| 16 | 0.53 | 0.46 | 0.00 | 0.44 | 0.53 | 0.29 | 0.45 | 0.05 | 0.05 | 0.09 | 0.17 | 0.11 | 0.14 |

approach to their true values after a number of learning periods, the returns are still more volatile than those generated in the benchmark models. This suggests that even a small deviation from the REE plays an important role in generating highly volatile movements of the exchange rates. If the economic environment is expected to be stable and market participants have a sufficient number of data points from which to understand the true structure of the economy, variations in exchange rates and their deviations from the fundamentals would be lower over time and would converge to those in the RE model. That is, the contribution of the AL component to the volatility of returns falls as sample size increases. The long-run implications of the AL models in Table 13 confirms this insight. In particular, the standard deviation of returns on the Swiss franc generated by model (iii), self-referential learning with decreasing gain, comes very close to the RE specification and is even lower than under the AE approach. On the other hand, if market participants are assumed to use more complex forecasting rules, as in present-value learning, or to weight recent data more heavily due to their beliefs about structural shifts, as in constant-gain learning, there are increased variations in returns that do not completely disappear even after 1,000 learning periods. I also compare volatilities across learning specifications. Table 6 shows that present-value learning models, (i) and (ii), generate more volatile returns than self-referential learning models, (iii) and (iv), whereas self-referential learning has a better ability to fit variations in deviations from the fundamentals, except the Deutsche mark.[33] Not surprisingly, constant-gain learning models, (ii) and (iv), create additional volatility for both exchange-rate returns and deviations that do not decline substantially over time. In the one-regime case, returns generated by decreasing gain learning tend to be less volatile as market participants accumulate more data points, which is not consistent with the data over the post-Bretton Woods period. Therefore, structural changes in the economic fundamentals may be necessary for decreasing gain models to sustain the learning process and thus to maintain the performance of the model. For all AL models, the introduction of structural breaks improves the performance of the model in terms of the volatility of exchange-rate returns without changing the order of model performance in the one-regime case.

Second, in the one-regime case, constant-gain learning and self-referential learning perform better in explaining why exchange rates and deviations from the fundamentals display substantial persistence than decreasing-gain learning and present-value learning, respectively. Tables 7 and 8 show that, under decreasing-gain learning models, the returns and the deviations are negatively serially correlated, even over short horizons, with some exceptions for self-referential learning. Although constant-gain learning performs slightly better than decreasing gain learning, present-value learning with constant gain does not generate variance ratios that are greater than unity over short return horizons and is not even superior to self-referential learning with decreasing gain. I found that only self-referential learning with constant gain, model (iv), can produce positive autocorrelations over short horizons and negative autocorrelations over long horizons, with a sole exception for the Deutsche mark. This may be because self-referential learning combined with a constant-gain algorithm generates escape-type dynamics. These escape dynamics could endogenously generate what look like regime

[33] Timmermann (1996) also found that, in the context of present-value asset pricing models, present-value learning has a better ability in matching volatility and predictability than self-referential learning.

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**Table 13**
Long-run implications of monetary models: Swiss franc.

| | RE ($n = 130$) | RE | AE | Adaptive learning | | | |
|---|---|---|---|---|---|---|---|
| | | | | (i) | (ii) | (iii) | (iv) |
| *A. Volatility*: *standard deviation* | | | | | | | |
| (1) Exchange-rate returns | | | | | | | |
| | 2.20 | 2.20 | 2.37 | 2.83 | 3.17 | 2.29 | 2.89 |
| (2) Deviations from fundamentals | | | | | | | |
| | 0.03 | 0.03 | 0.01 | 0.03 | 0.05 | 0.17 | 0.41 |
| *B. Persistence*: *variance ratio statistic* | | | | | | | |
| (1) Exchange-rate returns | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 0.80 | 0.84 | 1.31 | 0.85 | 0.85 | 0.86 | 0.97 |
| 16 | 0.60 | 0.69 | 1.12 | 0.71 | 0.71 | 0.72 | 0.95 |
| 32 | 0.35 | 0.48 | 0.81 | 0.52 | 0.51 | 0.54 | 0.92 |
| (2) Deviations from fundamentals | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 0.79 | 0.84 | 0.18 | 0.93 | 1.14 | 0.87 | 0.93 |
| 16 | 0.60 | 0.69 | 0.09 | 0.86 | 0.94 | 0.77 | 0.87 |
| 32 | 0.34 | 0.48 | 0.04 | 0.71 | 0.57 | 0.60 | 0.77 |
| *C. Predictability*: $\hat{\beta}_k$ | | | | | | | |
| 1 | −0.18 | −0.14 | 0.64 | 0.02 | 0.04 | 0.00 | 0.00 |
| 4 | −0.67 | −0.50 | 0.71 | 0.09 | 0.18 | 0.01 | 0.00 |
| 8 | −1.19 | −0.92 | 0.45 | 0.26 | 0.36 | 0.02 | 0.01 |
| 16 | −1.97 | −1.53 | 0.06 | 0.22 | 0.56 | 0.05 | 0.02 |

*Note*: Table entries are median values of statistics of 5,000 simulations in a large sample of $n = 1,000$. The statistics implied by the RE model in the small sample ($n = 130$) are in the second column.

shifts, even though the underlying process for the fundamentals does not undergo structural changes. Next and more importantly, I investigate whether these persistent and large deviations of exchange rates are also possible in other specifications of AL models with a small number of discrete structural shifts. In the multiple-regime case, the variance ratios of both returns and deviations generally are greater than one over short horizons and less than one and decreasing with return horizon over long horizons for all AL specifications. Therefore, I conclude that changes in the structure of the model, combined with AL, is an important source of substantial persistence of exchange rates.

Third, the AL models now are capable of accounting for the difference between short-run and long-run correlations between exchange rates and monetary fundamentals, as presented in Tables 9–12. For all learning specifications, the slope coefficients and $t$ statistics are consistently positive and increase with return horizons, and $R^2$'s start low but then rise. This suggests that incomplete information about the structure of economy causes exchange rates to deviate from their long-run equilibrium values over short horizons, but exchange rates tend to move to the values that are determined by economic fundamentals over long horizons. This result is quite robust to the presence of structural changes, although the pattern of estimates and statistics in the return regression becomes much weaker as the economy moves to the RE environment in large samples. Finally, neither the specification of AL nor the structural shift in fundamentals has much of an effect on out-of-sample forecast accuracy of the monetary model. For most cases, the $U$ statistics and the $DM$ statistics suggest that the monetary model cannot dominate the random walk model, even over long horizons.

### 4.2. Assessment of the monetary models

Up to this point, I have compared model implications under AL to those generated under alternative specifications of expectations using median values of statistics. I now employ a better means to assess the models: joint empirical distribution of statistics to form Wald statistics. This offers the advantage of reducing the performance of models to a single dimension, which makes it easy to compare and rank the models. Since there is no claim that an artificial economy or a calibrated model explains all the characteristics of the actual data (Watson, 1993), I follow Cecchetti et al. (1993) who developed a measure of fit for calibrated models and a generalized standard calibration methodology to incorporate statistical inference. Let $\zeta$ be a vector of population values, for example, $\zeta' = (\beta_1\ \beta_4\ \beta_8\ \beta_{16})$ of the slope coefficients of the $k$-period exchange-rate return regressions for $k = 1, 4, 8$, and 16 that are found in the historical data, and let $\varpi(\varphi; \lambda, \eta, \gamma)$ be a vector of statistics implied by a model when market participants use $\varphi$ to make forecasts of the variable of interest, given a set of the model's parameter values.[34] The statistic constructed by Cecchetti et al. (1993) to test the hypothesis that

[34] $\varphi$ differs across models. For example, $\varphi$ is $\bar{\phi}' = (\bar{a}\ \bar{b}\ \bar{c})$ under the RE model and $\phi' = (a\ b\ c)$ under self-referential learning, respectively.

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**Table 14**
Joint tests: Swiss franc.

| | Historical data | Simulated data | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One-regime case | | | | | | Multiple-regime case | | | | | |
| | | RE | AE | Adaptive learning | | | | RE | AE | Adaptive learning | | | |
| | | | | (i) | (ii) | (iii) | (iv) | | | (i) | (ii) | (iii) | (iv) |
| *A. Persistence ($\Delta s$): variance ratio statistics* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.01 | 0.80 | 1.11 | 0.68 | 0.91 | 0.95 | 1.12 | 1.03 | 1.25 | 0.64 | 0.92 | 1.19 | 1.32 |
| 16 | 0.81 | 0.60 | 0.88 | 0.48 | 0.63 | 0.80 | 1.09 | 0.83 | 1.03 | 0.43 | 0.83 | 1.10 | 1.23 |
| 32 | 0.25 | 0.35 | 0.51 | 0.24 | 0.33 | 0.54 | 0.91 | 0.42 | 0.56 | 0.22 | 0.55 | 0.68 | 0.81 |
| $\hbar$ | | 4.10 | 1.65 | 5.42 | 6.43 | 1.66 | 1.82 | 1.98 | 2.02 | 1.75 | 1.47 | 1.78 | 4.74 |
| *B. Persistence ($\xi$): variance ratio statistics* | | | | | | | | | | | | | |
| 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 8 | 1.13 | 0.79 | 0.18 | 0.48 | 0.49 | 0.96 | 1.07 | 0.46 | 0.21 | 0.90 | 1.03 | 1.02 | 1.10 |
| 16 | 0.89 | 0.60 | 0.08 | 0.29 | 0.29 | 0.83 | 1.05 | 0.30 | 0.11 | 0.71 | 0.96 | 0.99 | 1.13 |
| 32 | 0.27 | 0.34 | 0.04 | 0.14 | 0.13 | 0.57 | 0.83 | 0.13 | 0.03 | 0.41 | 0.62 | 0.68 | 0.86 |
| $\hbar$ | | 6.21 | 4,311.23 | 6.43 | 3.86 | 3.04 | 2.62 | 97.93 | 11,610.38 | 13.25 | 5.43 | 3.69 | 2.67 |
| *C. long-horizon U.S. dollar return regression: $\beta_k$* | | | | | | | | | | | | | |
| 1 | 0.06 | −0.18 | 0.69 | 0.20 | 0.18 | 0.01 | 0.02 | −0.12 | 0.97 | 0.10 | 0.03 | 0.01 | 0.01 |
| 4 | 0.24 | −0.67 | 0.95 | 0.64 | 0.60 | 0.06 | 0.08 | −0.41 | 1.74 | 0.39 | 0.12 | 0.07 | 0.08 |
| 8 | 0.47 | −1.19 | 0.94 | 0.92 | 0.90 | 0.11 | 0.16 | −0.64 | 1.79 | 0.68 | 0.22 | 0.16 | 0.18 |
| 16 | 0.72 | −1.97 | 0.98 | 1.00 | 1.00 | 0.22 | 0.30 | −0.74 | 1.37 | 1.05 | 0.61 | 0.37 | 0.42 |
| $\hbar$ | | 18.57 | 7.69 | 3.91 | 5.41 | 1.23 | 5.88 | 15.34 | 119.22 | 5.12 | 0.01 | 4.38 | 7.68 |

*Note*: The 5 percent critical value for the $\chi^2_{(4)}$ is 9.49.

model-implied moments match the population values, $H_0 : \boldsymbol{\zeta} = \boldsymbol{\varpi}(\boldsymbol{\varphi}; \lambda, \eta, \gamma)$,[35] is

$$\hbar = T[\zeta_T - \boldsymbol{\varpi}(\boldsymbol{\varphi}; \lambda, \eta, \gamma)]' \Omega^{-1}(\boldsymbol{\varphi}; \lambda, \eta, \gamma)[\zeta_T - \boldsymbol{\varpi}(\boldsymbol{\varphi}; \lambda, \eta, \gamma)], \tag{24}$$

where $\Omega(\boldsymbol{\varphi}; \lambda, \eta, \gamma)$ is calculated from model-generated data and $T$ is the sample size. Under the null, $\hbar \sim \chi^2_{(k)}$, where $k$ is the dimensionality of $\boldsymbol{\varpi}$.

Table 14 presents some selected $\hbar$-statistics that measure the ability of the model for the Swiss franc in simultaneously matching the statistics of interest found in the historical data.[36] Panels A and B present the $\hbar$ statistics that measure the ability of a model to jointly match variance ratios at horizons 1, 8, 16, and 32. In general, the RE model does not perform as poorly as it is evaluated in terms of each individual statistic. However, the AL models, especially the self-referential learning model with constant gain, perform considerably better than the RE model in matching persistence of deviations from monetary fundamentals. In panel C, I report the $\hbar$ statistics that test the ability to match the joint slope coefficients of the return regressions at horizon 1, 4, 8, and 16. For any specification of the AL models, we cannot reject the null hypothesis at the 5 percent significance level, and this suggests that AL models clearly dominate alternative specifications of expectations with regard to predictability.

## 5. Conclusion

I consider a monetary model of exchange rates where market participants, who have incomplete knowledge about the true structure of the economy, learn about the economic environment by employing AL rules and compare the predictions of the models with AL to those generated under standard RE and under AE. In addition, I model unanticipated structural shifts in the monetary fundamentals that market participants must contend with. Although the AL monetary models do not produce completely realistic descriptions about how exchange rates behave, simulation results suggest that, given trend-stationary economic fundamentals, any of the AL models studied in this paper dominates the alternative specifications of expectations in the following ways: its ability to account for why the fundamentals predict exchange-rate returns over long

[35] Note that $\zeta_T$ consistently estimates its population values $\zeta$ both under the null hypothesis, $H_0 : \zeta = \boldsymbol{\varpi}(\boldsymbol{\varphi}; \lambda, \eta, \gamma)$, as well as under the alternative hypothesis, $H_1 : \zeta \neq \boldsymbol{\varpi}(\boldsymbol{\varphi}; \lambda, \eta, \gamma)$. On the other hand, the implied moment vector $\boldsymbol{\varpi}(\boldsymbol{\varphi}; \lambda, \eta, \gamma)$ consistently estimates its population value only if $H_0$ is true.

[36] Results for other statistics and currencies are similar and are suppressed.

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horizons; for generating exchange-rate return volatility in excess of fundamentals volatility; and, in generating persistent deviations of the exchange rate from the fundamentals. In regard to the relative performance between learning rules, I found that the model under constant-gain learning, especially combined with structural changes, performs better in the sense that it comes close to matching the volatility and persistence of the exchange rates. Therefore, I conclude that the underlying uncertainty of the structure of the economy goes far in helping to resolve some longstanding puzzles in the foreign exchange market.

The results of this paper suggest fruitful extensions in a number of directions. First, the relaxation of the partial-equilibrium assumptions of the models, for example the Lucas (1982) two-country model, should prove useful. Under RE, the Lucas model requires an unreasonably small risk premium to explain the data, and the implied volatility of depreciation is much too small to be consistent with the data. Introducing AL may be a promising alternative without increasing model complexity.[37] Second, deviations from RE in accordance with AL may play a key role in explaining the delayed overshooting behavior of the exchange rate as documented by Eichenbaum and Evans (1995) and Clarida and Galí (1994). The maximal change in the exchange rate that occurs some period after the initial monetary shock may be rationalized when market participants adjust their expectations about future inflation in an adaptive fashion.[38] Finally, it should prove useful to study how model uncertainty, not parameter uncertainty, helps to explain the data in the foreign exchange market.

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[37] In their recent study, Chakraborty and Evans (2009) theoretically show that perpetual learning can induce a large downward asymptotic bias in the estimated forward premium regression coefficient as the fundamentals process approaches a random walk in the context of a monetary model.

[38] Kim and Mark (2006) show that a stochastic dynamic model of exchange rates under adaptive expectations generates the delayed overshooting dynamics of exchange rates.

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