Strategic Trading / Market Microstructure

- Grossman-Stiglitz study trading when investors are competive price-takers. The focus is on the decision to acquire information and signal extraction.

- In a 1985 paper entitled "Continuous Auctions & Insider Trading", Kyle studies optimal trading when an informed investor's trades influence the price, and so trading must be strategic. This paper is the foundation of the market microstructure literature.

- The basic problem the informed trader faces is that if he trades too aggressively in response to his inside info., the price will move against him (increase if he's buying, decrease if he's selling). This is the same basic problem a monopolist producer confronts when deciding what price to charge (i.e, high price / low sales vs. low price / high sales).

- Kyle studies 2 versions of his model: (1) A static model with just 1 trading period (here the focus is on the optimal trade size), and (2) A more complex model featuring a sequence of trading periods (here the focus is on the optimal sequence of trades).

- Today I will mainly focus on the static model, since it conveys most of the key results.

Assumptions

1.) There are 2 dates, 0 and 1. The asset is traded at date 0, and its random value, $\tilde{v}$, is realized at date 1. $\tilde{v} \sim N(\bar{v}, \sigma_v^2)$.

2.) Before trading, a single risk-neutral investor (costlessly) observes $\tilde{v}$.

3.) There are also random uninformed noise/liquidity trades, $\tilde{z}$, where $\tilde{z} \sim N(0, \sigma_z^2)$.

4.) The market price is set by a (competitive) risk-neutral "market maker". The market maker cannot distinguish between informed + uninformed trades. He just observes the total (net) order flow. In equilibrium, he will lose money (on average) to the informed trader, but will make money from the noise traders. In equilibrium, his (expected) profits are zero. In contrast, the informed trader makes positive expected profits, while the noise ~~5.) The informed~~ traders lose on average.

Strategies

- Let $\tilde{x}$ = Informed trader's optimal order. Can be positive or negative. (Short sales permitted);

  $\tilde{z}$ = Noise traders exogenous orders

  $\tilde{y} = \tilde{x} + \tilde{z}$ = Total net orders submitted to market maker, (If $\tilde{x}$ & $\tilde{z}$ are opposite signs, the informed & noise traders first trade between themselves).

- Given the above assumptions we know,

$$p(\tilde{y}) = E[\tilde{v} \mid \tilde{y}]$$

$$x(\tilde{v}) = \underset{x}{\operatorname{argmax}} \left(\tilde{v} - E[p(x + \tilde{z})] \right) \cdot x$$

- Note, the informed trader takes account of his own influence on the realization of price, but he cannot condition his order on the noise trades. He must submit his order before $\tilde{z}$ is realized.

Equilibrium

- There is a unique <u>linear</u> equilibrium (nonlinear equilibria are still the subject of research).

$$\boxed{\begin{array}{lll} p(y) = \delta + \lambda y & \delta = \bar{v} & \lambda = \frac{1}{2}\frac{\sigma_v}{\sigma_z} \\ x(\hat{v}) = \alpha + \beta \tilde{v} & \alpha = -\delta\beta & \beta = \frac{1}{2\lambda} \end{array}}$$

- The sensitivity of price to order flow, λ, has come to be known as "<u>Kyle's Lambda</u>" in the mkt. microstructure literature. It captures the endogenous degree of market <u>liquidity</u>. If λ is small, the market is liquid (price doesn't move much in response to trades).

Equilibrium Verification

- We need to verify the strategies are best responses to each other.
- Given the informed trader's strategy, the mkt. maker's optimal strategy is,

$$E[\tilde{v} \mid \tilde{x}+\tilde{z}] = E[\tilde{v} \mid \beta\tilde{v}+\tilde{z}] = \bar{v} + \frac{\operatorname{cov}(\tilde{v}, \beta\tilde{v}+\tilde{z})}{\operatorname{var}(\beta\tilde{v}+\tilde{z})}(\beta\tilde{v}+\tilde{z}-\beta\bar{v})$$

$$= \bar{v} + \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_z^2}(\beta\tilde{v}+\tilde{z}-\beta\bar{v})$$

$$= \bar{v} + \lambda(\tilde{x}+\tilde{z}) \quad \checkmark$$

with $\lambda \equiv \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_z^2}$ $\quad \delta = \bar{v}$

and $\tilde{x} = \beta(\tilde{v}-\bar{v}) = \alpha + \beta\tilde{v}$ (with $\alpha = -\beta\delta$ and $\delta = \bar{v}$)

- Given the mkt. maker's strategy, the informed trader's opt. strategy is,

$$\max_x E\{x[\tilde{v} - \delta - \lambda(x+\tilde{z})]\} \Rightarrow x = \frac{\tilde{v}-\delta}{2\lambda} \quad \checkmark$$

$$= \alpha + \beta\tilde{v}$$

with $\beta \equiv \frac{1}{2\lambda}$ $\quad \alpha = -\beta\delta$

- Let's now compute λ in terms of the exogenous variables

First, $\beta = \frac{1}{2\lambda} = \frac{1}{2}\left(\frac{\beta^2\sigma_v^2+\sigma_z^2}{\beta\sigma_v^2}\right) \Rightarrow \beta^2\sigma_v^2 = \frac{1}{2}(\beta^2\sigma_v^2+\sigma_z^2)$

$$\Rightarrow \beta = \frac{\sigma_z}{\sigma_v}$$

sub back into λ,

$$\boxed{\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_z^2} = \frac{\sigma_z/\sigma_v \cdot \sigma_v^2}{\sigma_z^2/\sigma_v^2 \cdot \sigma_v^2 + \sigma_v^2} = \frac{1}{2}\frac{\sigma_v}{\sigma_z}}$$

Comments

1.) Note that mkt. liquidity, $\frac{1}{\lambda} = 2\frac{\sigma_z}{\sigma_v}$, increases with the variance of noise/liquidity trades (obviously), but decreases with the variance of the asset value (more interesting). The info. advantage of the informed trader is increasing with σ_v^2. Hence, the mkt. maker's adverse selection problem gets worse when $\sigma_v^2 \uparrow$.

2.) The informed trader's expected profits are

$$E[\tilde{x}(\tilde{v} - p(\tilde{x}+\tilde{z}))] = \beta E[(\tilde{v} - \bar{v})[\tilde{v} - \delta - \lambda\beta(\tilde{v} - \bar{v}) - \lambda\tilde{z}]]$$

$$= \frac{1}{2}\sigma_v \sigma_z$$

Hence, his expected profits are higher when he has more private info. and when there is more liquidity trading. The expected profits of liq. traders are $E[\tilde{z}(\tilde{v} - p(\tilde{x}+\tilde{z}))] = -\frac{1}{2}\sigma_v\sigma_z$

Hence, the informed trader takes money from the liquidity traders, while the market maker breaks even (on average).

3.) In this 1-period model, half of the informed trader's info. gets leaked into the price. That is,

$$var(\tilde{v} \mid y) = E[\tilde{v} - E[\tilde{v} \mid y]]^2$$

$$= E[\tilde{v} - \bar{v} - \lambda y]^2 = \frac{1}{2}\sigma_v^2$$

In other words, the market maker's uncertainty about $\tilde{v}$ gets cut in half after witnessing the order flow.

Continuous-Time

- The informed trader can do better if he can trade gradually, just like a monopolist can do better if he can price discriminate. For example, suppose $P = P_0 + \lambda x$ and you buy a quantity, $x = Q$, in a single large batch. Then your total cost is,

$$Q(P_0 + \lambda Q) = P_0 Q + \lambda Q^2$$

However, if you can buy the same total amount using a sequence of infinitesimal purchases, your total cost is,

$$\int_0^Q (P_0 + \lambda x)\, dx = P_0 Q + \tfrac{1}{2} \lambda Q^2$$

which is lower. This is the basic idea in the continuous-time version.

- If the informed trader can trade continuously over a finite interval (normalized to $[0, 1]$) he can double his expected profits to $\sigma_v \sigma_z$. With a Brownian Motion noise trader process $dz = \sigma_z dB$, the optimal strategies are:

$$dP = \lambda\, dY \qquad \text{with } \lambda = \frac{\sigma_v}{\sigma_z}$$

$$dX = \theta_t\, dt \qquad \text{with } \theta_t = \frac{\tilde{v} - P_t}{\lambda(1-t)}$$

- Note that $P_t \to \tilde{v}$ as $t \to 1$. Hence, the informed trader "uses up" all his info. over time.

- The informed trader trades "gradually" in the sense that his orders are $\sim dt$ (whereas price & noise trades $\sim dB$).