SIMON FRASER UNIVERSITY
Department of Economics

Econ 305
Intermediate Macroeconomic Theory

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PROBLEM SET 1
(Solutions)

1. (25 points). In class we discussed one reason why Americans might work more than Europeans. This question explores one reason why Americans might work *less* than Chinese people. Suppose the preferences of Americans and Chinese are the *same*, and are given by

$$U(C,N) = \alpha C - \frac{C^2}{N} - \frac{1}{2}\beta N^2$$

where C stands for consumption of market goods, N stands for (weekly) hours worked, and (α, β) are fixed parameters, which are the same in the USA and China. Assume both countries are competitive, and output is produced by firms using the linear production function

$$Y = zN$$

where N denotes labor input and z denotes productivity. For simplicity, assume there is no government, or non-labor income.

(a) Assuming the market (real) wage is w, derive expressions for the household's consumption and labor supply decisions as a function of w. (Hint: Note that in terms of N, the household's budget constraint is $C = wN$ and its optimality condition is $-U_N/U_C = w$). Sketch a plot of the labor supply curve. Does the income effect ever dominate the substitution effect? If so, at what wage is labor supply maximized?

The household's optimality condition is

$$\frac{-U_N}{U_C} = \frac{\beta N - (C/N)^2}{\alpha - 2(C/N)} = w$$

From the budget constraint, we know $C = wN$. Using this to sub out for C/N above gives

$$\frac{\beta N - w^2}{\alpha - 2w} = w$$

Solving for N give us the following labor supply curve

$$N^s = \frac{1}{\beta} w(\alpha - w)$$

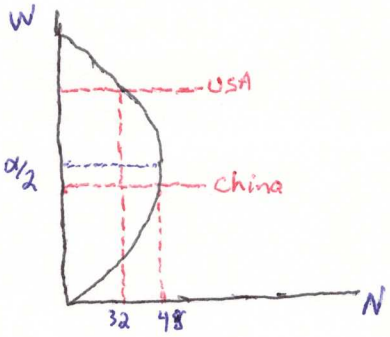
This is upward sloping for small values of w, reaches a maximum at $w = \alpha/2$, and then falls back to 0 at $w = \alpha$. That is, for $w < \alpha/2$ the substitution effect dominates the income effect, while for $w > \alpha/2$ the income effect dominates the substitution effect.

(b) Derive an expression for the firm's labor demand curve, and illustrate it in a graph, along with the labor supply curve. What is the market equilibrium wage rate?

Since the production function features constant returns to labor, the labor demand curve is simple. It is just a straight line at $w = z = MPL$. (It is zero for $w > z$ and infinite for $w < z$). Hence, the market equilibrium wage is clearly $w = z$.

(c) Assume that $\alpha = 20$ and $\beta = 2$ in both countries. However, assume that the USA is more productive. Specifically, suppose $z = 16$ in the USA and $z = 8$ in China. Compute the equilibrium (weekly) hours of work in each country. Finally, suppose z is growing at 7% in China, but only at 1% in the USA. How long will it be before Chinese labor supply matches the USA?

Notice that given the above data, the USA is on the backward bending portion of the labor supply curve, whereas China is on the upward sloping portion. We can depict the situation as follows



Weekly hours worked in the USA will be

$$N^u = \frac{1}{2}16(20-16) = 32$$

Weekly hours worked in China will then be

$$N^c = \frac{1}{2}8(20-8) = 48$$

Americans work less than Chinese because, given their already relatively high living standard, they value leisure relatively more than do Chinese people. However, China is catching up. If China's productivity continues to grow 7% while US productivity only grows at 1%, the following calculation shows that labor supply in the two countries will converge in about 12 years.

$$z_0^u(1+g^u)^t = z_0^c(1+g^c)^t \quad \Rightarrow \quad \frac{16}{8} = \left(\frac{1.07}{1.01}\right)^t \quad \Rightarrow \quad t = \frac{\ln(2)}{\ln(1.07/1.01)} \approx 12$$

Warning: This question is not meant to be quantitatively serious. I just made the numbers up, in order to make the math simple. However, I do think it captures at least some of what is going on. Obviously, there are many other factors involved.

2. (25 points). Consider a subsistence farmer who can produce food, Y, according to the production function, $Y = z\sqrt{N}$, where N denotes the hours of work he does. Suppose this farmer also values leisure in addition to food, and has h total hours of time available per period.

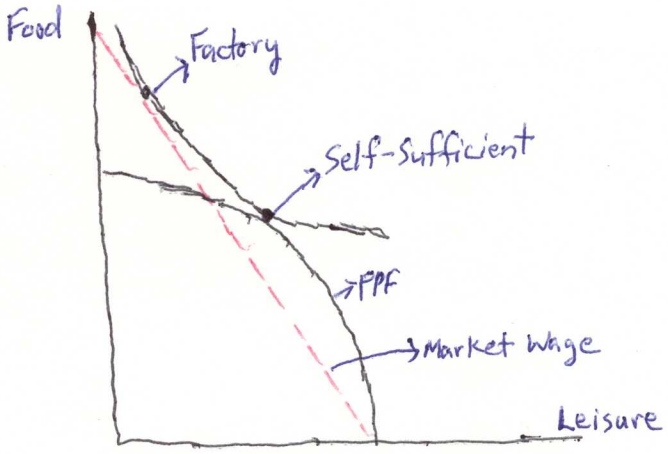
(a) Draw the farmer's production possibilities frontier, and use it to illustrate his optimal choice of food and leisure. (For simplicity, ignore the fact that there might be some miinimum amount of food required to live!)

To derive the PPF, just substitute the time constraint, $N = h - \ell$, into the production function. This produces a concave, negatively sloped relationship between Food and Leisure. It is concave because production is subject to diminishing returns. (See the graph in part (b))

(b) Now suppose a factory opens up nearby, giving him the opportunity to work for a fixed wage rate, which he can then use to buy his food in the town market. Draw a picture illustrating the minimum wage required to induce him to leave farming and work in the factory. Assuming

a farmer decides to take a job in a factory, does he work more or less than before? Explain intuitively.

The followiing graph describes the situation



Evidently, people who leave farming to work in factories will work <u>more</u> than before.

(c) Do you think this picture has any relevance for developing countries like China?

It has often been observed that during the early phases of industrialization workers work long hours. True, subsistence farmers work long hours too, but moving to a factory allows workers to work for a <u>fixed</u> wage, which allows them to escape the work-discouraging effects of diminishing returns. This might partly explain why workers in China put in so many hours.