SIMON FRASER UNIVERSITY
Department of Economics

Econ 305                                                                                 Prof. Kasa
Intermediate Macroeconomic Theory                                   Summer 2019

## PROBLEM SET 1
### (Solutions)

1. (25 points). Suppose household preferences are described by the utility function

$$U(C, \ell) = \alpha C - \frac{1}{2}(\beta - \ell)^2$$

where $C$ stands for consumption of market goods and $\ell$ stands for leisure. For simplicity, assume there is no government in this economy.

(a) Assuming the market (real) wage is $w$ and the total amount of time available is $h$, derive expressions for the household's consumption and labor supply decisions as a function of $w$ and $h$. (For simplicity, assume the household has no nonmarket income). Does the income effect ever dominate the substitution effect? How does labor supply depend on income and consumption? Explain intuitively.

*The household's optimality condition is*

$$\frac{U_\ell}{U_c} = \frac{\beta - \ell}{\alpha} = w$$

*Solving for $\ell$ we get*

$$\ell = \beta - \alpha w$$

*Substituting this into the time constraint, $h = \ell + N^s$, we get*

$$N^s = h - \beta + \alpha w$$

*Note this is always increasing in $w$, so the substitution effect always dominates the income effect. Substituting the solution for $\ell$ into budget constraint, $C + w\ell = hw$, we can solve for consumption as a function of the market wage*

$$C = (h - \beta)w + \alpha w^2$$

(b) Now suppose output, $Y$, is produced by competitive firms with technology $Y = zN$ where $N$ denotes labor inputs, and $z$ is an index of productivity. Derive an expression of the firm's labor demand, and illustrate it with a graph.

*The firm's optimality condition is to set the marginal product of labor equal to the wage rate. Here the marginal product of labor is <u>constant</u>, and equal to $z$. As a result, the labor demand curve is discontinuous - it is equal to zero for all $w > z$, it is a flat line (i.e., indeterminate) for $w = z$, and then jumps to infinity for $w < z$ (since each worker yields a profit, why not hire an infinite number of them!)*

**(c)** Using your answers to parts (a) and (b), derive an expression for the market-clearing wage rate. How does the equilibrium wage change when $z$ increases? How does the equilibrium wage change when $\alpha$ increases? Use a Labor Supply/Labor Demand graph to illustrate these changes.

*The answer to part (b) implies that the market-clearing wage must be $w = z$. So to determine the equilibrium employment level, we just need to replace $w$ with $z$ in the household's labor supply curve. This gives*

$$N^e = h - \beta + \alpha z$$

*If $z$ increases, the horizontal labor demand curve shifts up, wages rise, and the equilibrium employment level increases. If $\alpha$ increases, household's value consumption increases. As a result, they work more. The labor supply curve shifts out, and equilibrium employment increases. Note, however, that wages remain fixed at $z$.*
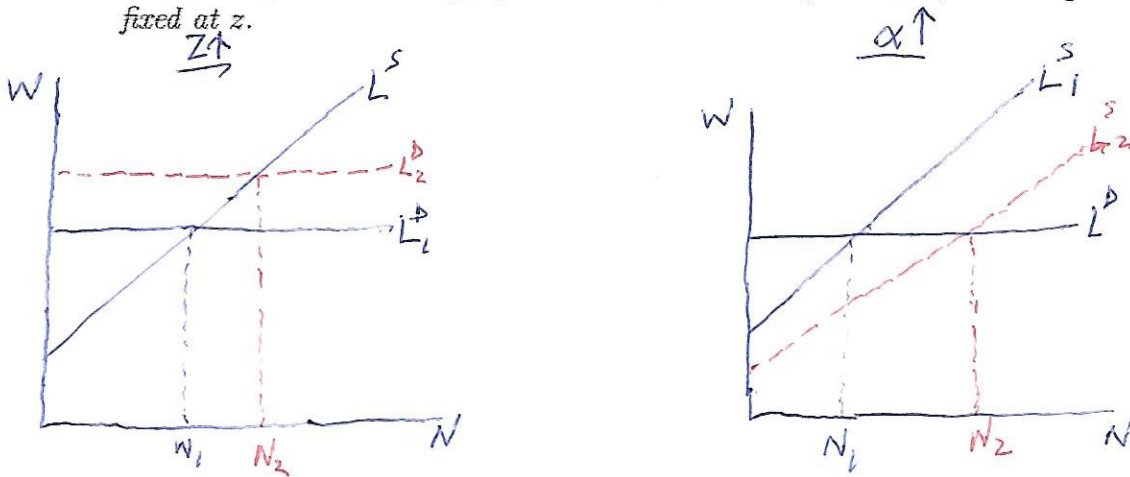


2. (25 points). Suppose that government spending makes private firms more productive, e.g., spending on roads and bridges might reduce transport costs.

**(a)** Using the production possibilities graph described in chpt. 5 of the text and in Lecture 5, show how an increase productive government spending could increase household welfare.

*In general, an increase in productive government spending has ambiguous effects on private welfare. The increased taxes needed to pay for it lowers welfare, by shifting down the net-of-taxes PPF. However, when the spending increases productivity, it causes the PPF to become steeper and higher at each $\ell < h$.*



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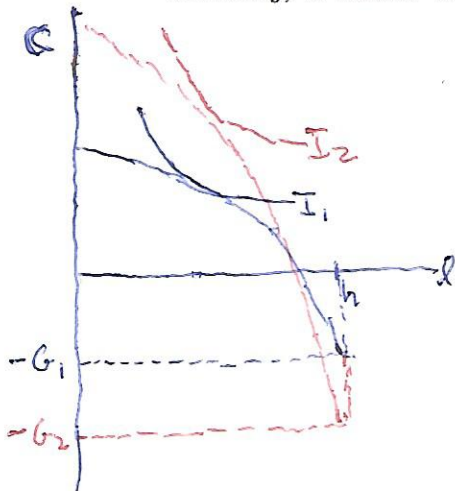
**(b)** Show that the equilibrium effects on consumption and hours worked are ambiguous. but but that output definitely increases. Hint: Consider the induced income and substitution effects.

*Since the PPF becomes steeper, the substitution effect reduces leisure but increases consumption. There are two separate income effects. An unambiguously negative one due to the higher taxes, which causes both $C$ and $\ell$ to fall, and a possibly positive one caused by the increased productivity. The magnitude of the second income effect depends on the initial equilibrium. It is larger, and more likely to be positive, the higher the initial labor supply, since productivity improvements are more important when you are producing more to begin with. Unless the initial equilibrium features a very high $\ell$, it is likely that $C$ increases. The effects on leisure are more ambiguous, since the net income effect and substitution effect are offsetting. As drawn above, consumption and household welfare increase, while there is little change in $\ell$.*