1. Use the Solow model to analyze the macroeconomic consequences of a sudden decline in population, as occurred for example during the so-called ‘Black Death’ in the 1340s. Suppose the economy is in a steady state when the plague hits, and that as a result of the plague a certain fraction of the population suddenly dies, but the underlying growth rate of the population stays the same. What are your predictions concerning the time paths of per capita output, wages, and interest rates?

2. Now consider how unemployment might be incorporated in the Solow model. Suppose that output is produced according to the production function

\[ Y = K^{\theta}[(1 - u)N]^{1-\theta} \]

where \( u \) is ‘natural rate’ of unemployment (we will talk about this more after the midterm). As usual, assume the national saving rate is \( s \), the population growth rate is \( n \), and the depreciation rate is \( \delta \).

(a) Express per capita output \( (y = Y/N) \) as a function of capital per worker \( (k = K/N) \) and the natural rate of unemployment, \( u \). Describe the steady state of the economy.

(b) Suppose that some change in government policy reduces the natural rate of unemployment. Use a graph to describe how this policy change affects output, both immediately and over time. Is the steady-state effect larger or smaller than the short-run effect? Explain.

3. In Canada, capital’s share of GDP is about 30 percent; the average growth in output is about 3.0 percent per year; the depreciation rate is about 4.0 percent per year; and the capital/output ratio is about 2.5. Suppose that the Canadian production function is Cobb-Douglas, so that capital’s share in output is constant, and assume that Canada has reached its steady state.

(a) Use the above data to infer what the Canadian saving rate is. [Hint: Use the steady-state relationship, \( sy = (\delta + n + g)k \)].

(b) What is the marginal product of capital?

(c) Now suppose that public policy raises the saving rate so that the economy reaches the “Golden Rule” level of capital. What will the new marginal product of capital be? Compare this to the original marginal product of capital computed in part b. Explain the difference.

(d) What must the new saving rate be in order to reach the Golden Rule steady state?
4. Suppose Canada’s production function is \( Y = A \cdot K^{1/3}L^{2/3} \), and that Canada is currently in a steady with a savings rate of 20% (i.e., \( s = .20 \)), a population growth rate of 1.7% (i.e., \( n = .017 \)), and a depreciation rate of 5% (i.e., \( \delta = .05 \)). For simplicity, suppose productivity, \( A \), is constant.

(a) If per capita income in Canada is $30,000, what must be the value of \( A \)? (Hint: Derive a formula relating steady state per capita output to \( A, s, n \) and \( \delta \) and then solve for \( A \)).

(b) Demographers predict that Canada’s population growth rate could drop to zero this century. Use the Solow model to forecast the consequences of this demographic shift on total output, per capita output, the growth rate of total and per capita output, and the wage rate. Consider the effects in both the new steady state and in the transition to the new steady state.