Investment, Production + the Current Account

- Remember, $CA = S - I$. In an endowment economy, $I = 0$. Hence, so far we've only discussed half the story! We now take a major step toward realism by assuming that output must be produced. This gives an economy a little more flexibility, since it can reallocate output over time through investment.

- We make the following 3 assumptions:

1.) Capital is the only input to production. Labor inputs are ignored. Capital + output are the same good (one-sector production technology).

2.) Now what is given is the initial stock of capital.

3.) The output + factor markets are competitive. Profit-maximizing firms produce output. Households own the firms.

- Let K_1 : capital stock at beginning of period 1 (given)

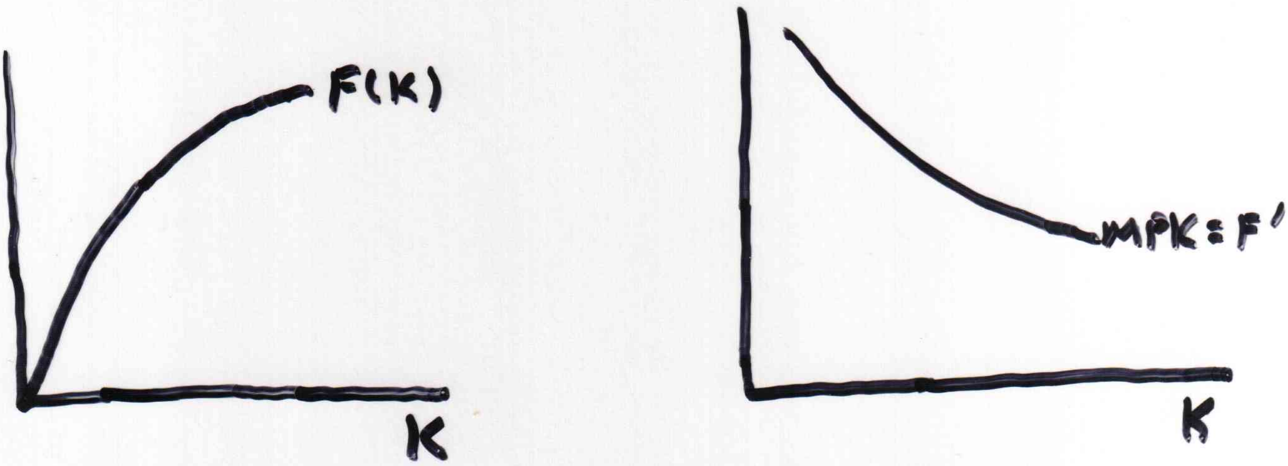
  K_2 : capital stock at beginning of period 2

- Now output is an increasing function of capital

  $Q_1 = F(K_1)$ > since K_1 is fixed, so is Q_1

  $Q_2 = F(K_2)$

- We assume $F(\cdot)$ is increasing and concave, so that MPK' < 0 (i.e., $F' > 0$, $F'' < 0$).



- Capital depreciates at rate δ, so that

  $K_2 = (1-\delta) K_1 + I_1$

  where I_1 = period 1 gross investment

  (net investment = $K_2 - K_1$)

Firms

- Firms sell output at a price of 1 (output is the "numeraire"). Their costs of production consist of the rental rate on capital (sometimes called the "user cost of capital"). [Even if firms own their own capital, the rental rate represents the opportunity cost of capital].

- The rental rate consists of 2 parts:

  1.) The interest rate, r

  2.) The depreciation rate, δ

- Therefore, period 1 and 2 profits are:

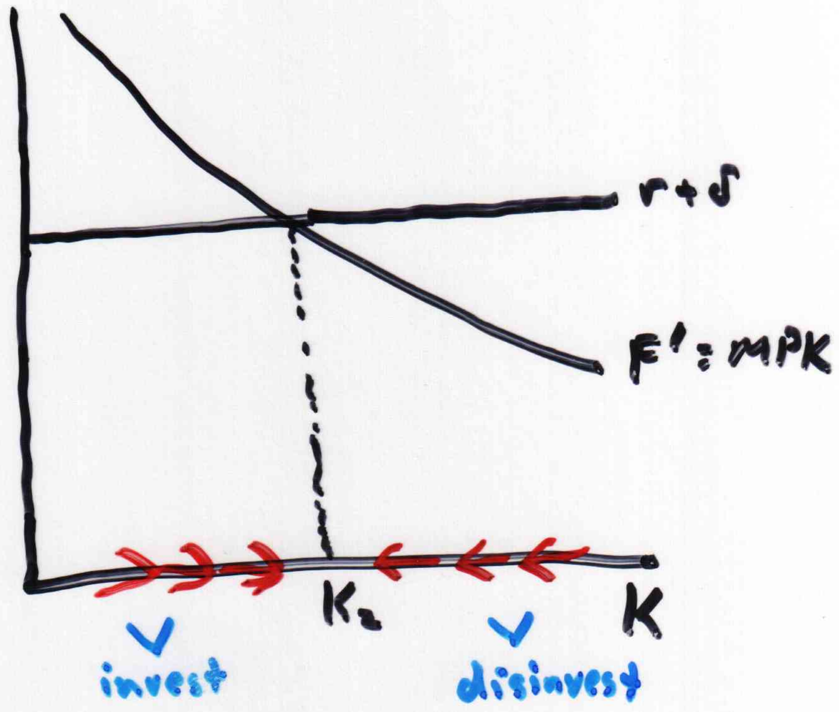
$$\pi_1 = F(K_1) - (r_0 + \delta)K_1 \quad \text{(given)}$$

$$\pi_2 = F(K_2) - (r_1 + \delta) K_2$$

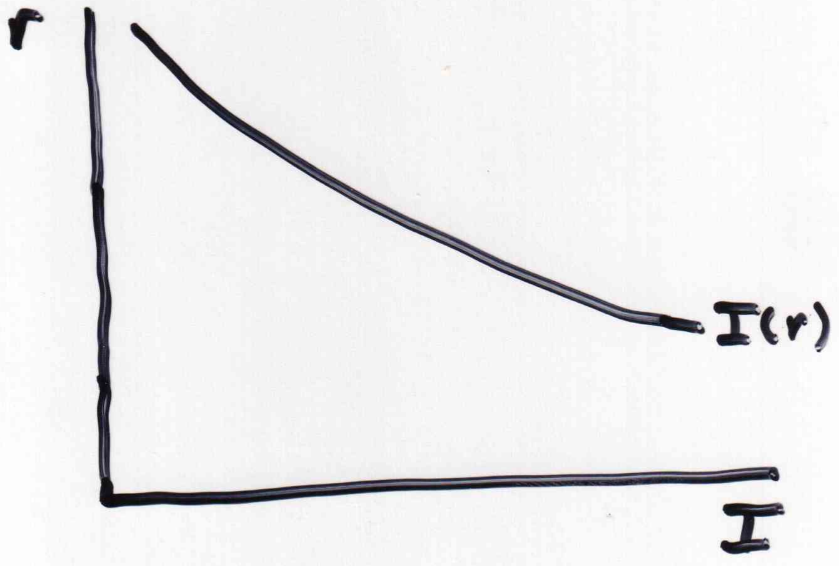
- Clearly, profits are maximized when

$$\boxed{F'(K_2) = r_1 + \delta}$$

- We can visualize this as follows:



- From the above graph, it is clear that $r\uparrow \Longrightarrow K_2 \downarrow$. Since $I_1 = K_2 - (1-\delta)K_1$, we then know there is an inverse relationship between r and I



Households

- Households have 2 sources of income:
  1.) Firm profits (dividends)
  2.) Rental payments / asset income
- Households have 2 uses for income:
  1.) Consumption
  2.) Saving / Asset Accumulation
- Letting W_i = wealth at end of period i

$$\left.\begin{aligned} C_1 + (W_1 - W_0) &= r_0 W_0 + \pi_1 \\ C_2 + (W_2 - W_1) &= r_1 W_1 + \pi_2 \end{aligned}\right\} \text{budget constraints}$$

- Now the TVC becomes $W_2 = 0$. Therefore, the period 2 budget constraint becomes:

$$C_2 = (1+r_1)W_1 + \pi_2$$

- Substituting this into the period 1 budget constraint, we get the following intertemporal budget constraint

$$\boxed{C_1 + \frac{C_2}{1+r_1} = (1+r_0)W_0 + \pi_1 + \frac{\pi_2}{1+r_1}}$$

Equilibrium in a Closed Economy

- In a closed economy, the only form of wealth is the domestic capital stock:

$$W_0 = K_1$$
$$W_1 = K_2$$

- Substituting for π_1 & π_2 in the budget constraints, we recover the National Income Accounting identities:

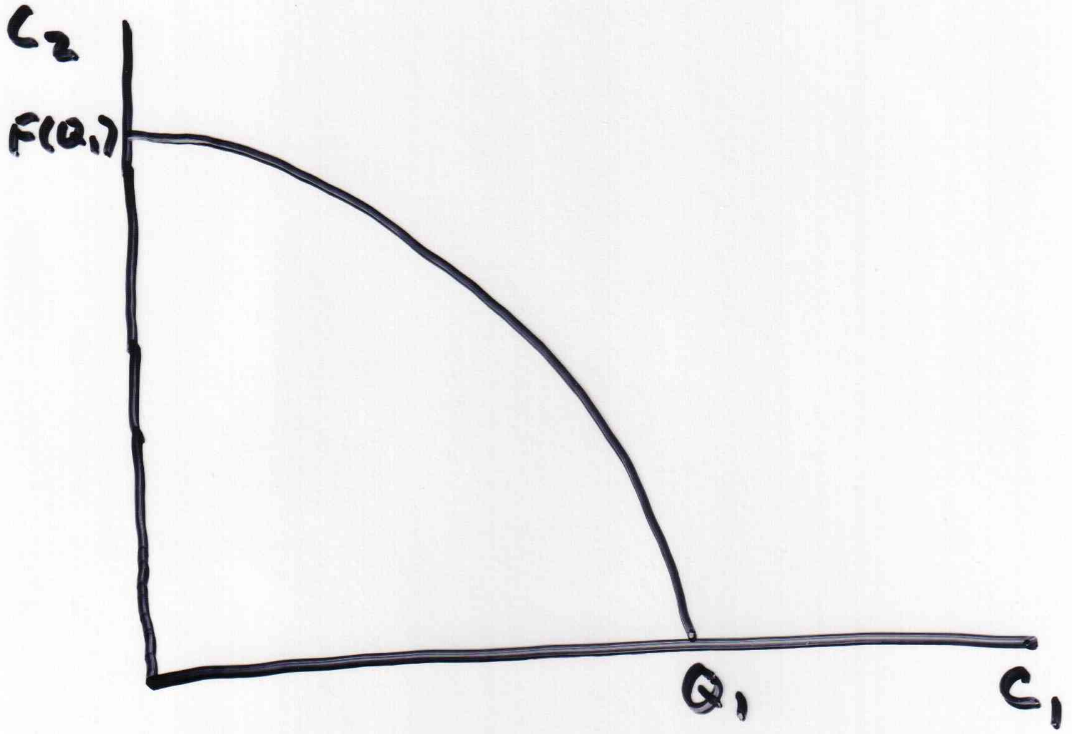
$$Q_1 = C_1 + K_2 - (1-\delta)K_1 = C_1 + I_1$$
$$Q_2 = C_2 - (1-\delta)K_2 = C_2 + I_2$$

- Since $Q_2 = F(K_2)$ and Q_1 is given, we can combine these two to get the economy's "Production Possibility Frontier" (PPF):

$$\boxed{C_2 = F(Q_1 + (1-\delta)K_1 - C_1) + (1-\delta)[Q_1 + (1-\delta)K_1 - C_1]}$$

- When $\delta = 1$ this simply becomes

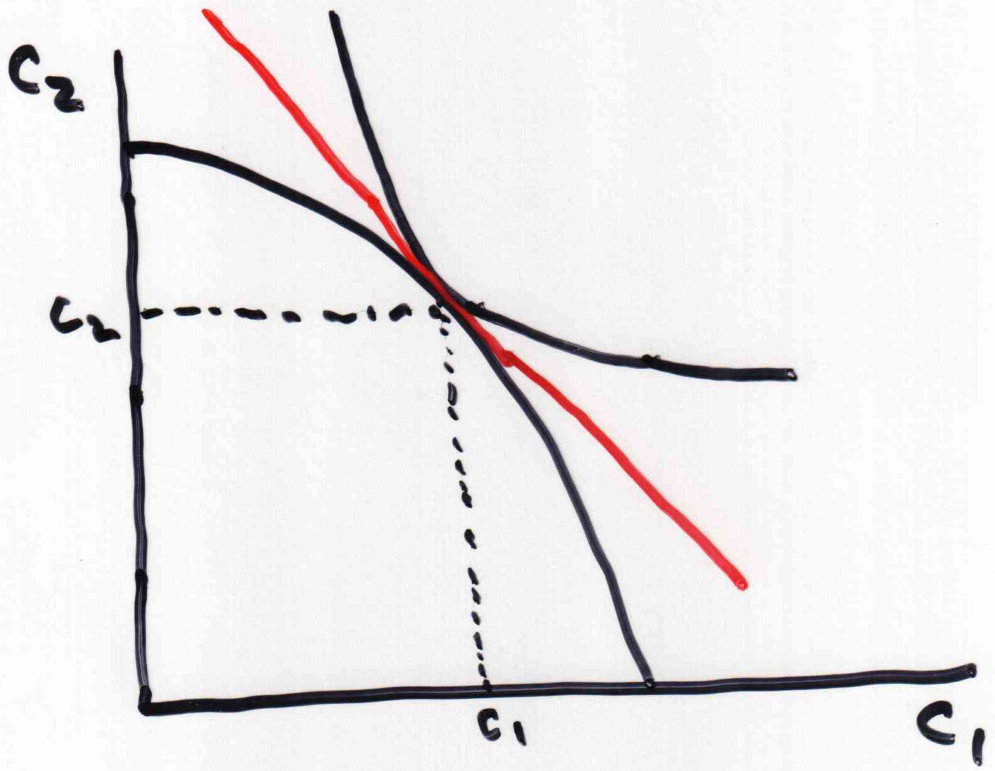
$$C_2 = F(Q_1 - C_1)$$



- When $\delta = 1$, the slope of the PPF is just $-F'(Q_1 - C_1)$

When $\delta < 1$, the slope of the PPF is $-F'(\cdot) - (1-\delta)$

- In a closed economy, households must stay within the PPF



- In equilibrium, the slope of the Indifference Curve equals the slope of the PPF.

  slope of IC $= -\dfrac{U_1}{U_2}$

  slope of PPF $= -F'(\cdot) - (1-\delta)$

- Since $F'(\cdot) = r+\delta$, we have

$$\boxed{\frac{U_1}{U_2} = 1+r}$$

- In a closed economy r is <u>endogenous</u>

Equilibrium in Small Open Economies

- Small open economies differ in two important ways:

1.) The interest rate is exogenous (or given)

2.) Households now confront a portfolio allocation problem, i.e, they can invest in either domestic capital or foreign bonds.

- That is, household wealth now consists of claims on foreigners, B^*, and claims to domestic capital, K.

$$W_0 = K_1 + B_0^*$$
$$W_1 = K_2 + B_1^*$$

- However, with no uncertainty or default risk, the two assets become perfect substitutes. No arbitrage implies they must therefore offer the same return. Hence, domestic investment occurs up until the point

$$\boxed{F'(K_2^*) = r^* + \delta}$$

Beyond this point all domestic saving gets channelled into foreign bonds.

Comments

1.) Later we discuss a more interesting and realistic portfolio allocation problem. [Ventura (2002)].

2.) The fact that domestic capital + foreign bonds are perfect substitutes means there is no loss in generality in ruling out FDI, and assuming domestic residents own the entire domestic capital stock.

Resource Constraint

- To characterize the equilibrium, we need to combine the PPF with the aggregate resource constraint.
- For notational simplicity assume $\delta = 1$ and $B_0^* = 0$.
- As before, the household budget constraint is,

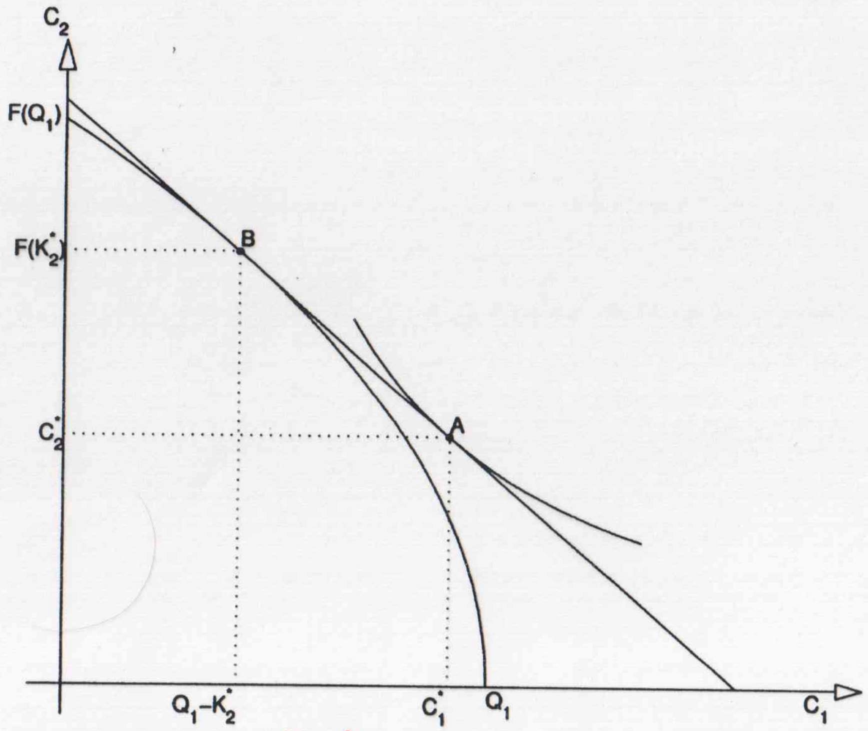
$$c_1 + \frac{c_2}{1+r^*} = (1+r^*)W_0 + \Pi_1 + \frac{\Pi_2}{1+r^*}$$

Substituting for Π_1 + Π_2 gives

$$\boxed{c_2 = (1+r^*)(Q_1 - c_1 - K_2^*) + F(K_2^*)}$$

- This says that households consumption in period 2 is equal to output plus the interest and principal on net foreign assets.
- Combining the PPF + Resource Constraint gives the following picture –

Figure 3.8: Equilibrium in the production economy: the small open economy case



CA
deficit

The "Separation Theorem"

- Note that finding the equilibrium in small open economies consists of 2 distinct steps:

1.) Pick the production point (pt. B in previous graph) that maximizes the economy's wealth (evaluated at world prices). That is, find the point on the PPF that is tangent to the resource constraint.

2.) Pick the best point on the Resource Constraint (pt. A in previous graph). That is, find the point tangent to an Indifference Curve.

- Hence, in contrast to closed economies, in open economies production decisions are completely <u>separated</u> from consumption decisions. Optimal production + investment is independent of preferences. All that matters is the world interest rate + the domestic production function.

Comments

1.) **Warning** - the Separation Theorem will break down if labor enters the production function and consumption & leisure enter non-separably in the utility function.

2.) The converse does not apply - consumption decisions will of course depend on investment decisions, since investment affects the total resources available to be consumed.

3.) The Separation Theorem is the key intuition behind the Feldstein-Horioka Puzzle. According to the Separation Theorem, there is no *necessary* relationship between S & I. Regressing I on S should produce a low R^2 and insignificant t-stats. Feldstein & Horioka found the opposite.

Comparative Statics

- Suppose $Q_1 = A_1 F(K_1)$ and $Q_2 = A_2 F(K_2)$
- Let's consider 3 types of shocks:

1.) Temporary Productivity Shock $[A_1 \downarrow]$

2.) Permanent Productivity Shock $[A_1 \downarrow$ and $A_2 \downarrow]$

3.) World Interest Rate Shock $[r^* \downarrow]$.

Temporary Productivity Shock (see fig. 3.9 in following graph).

1.) Leftward parallel shift of PPF

2.) r^* doesn't change

$\Longrightarrow$ investment and Q_2 don't change

3.) Consumption-smoothing implies $C_1 \downarrow$ less than $Q_1 \downarrow$

$\Longrightarrow CA \downarrow$ (since $S \downarrow$ and I is constant)

(How would a closed-economy respond?)

Figure 3.9: The effect of a temporary output decline in the small-open economy with production

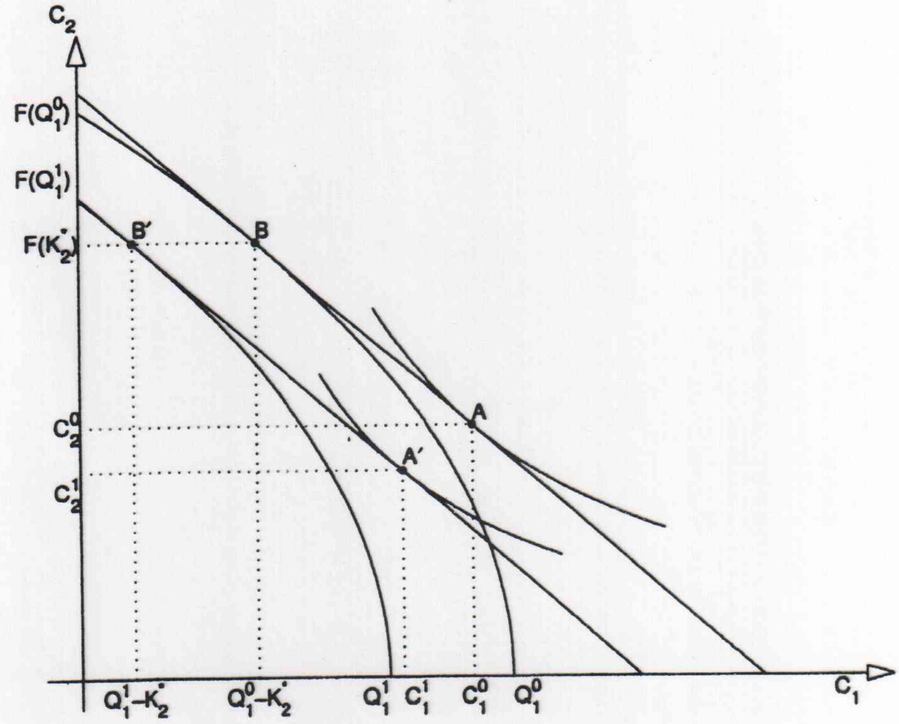
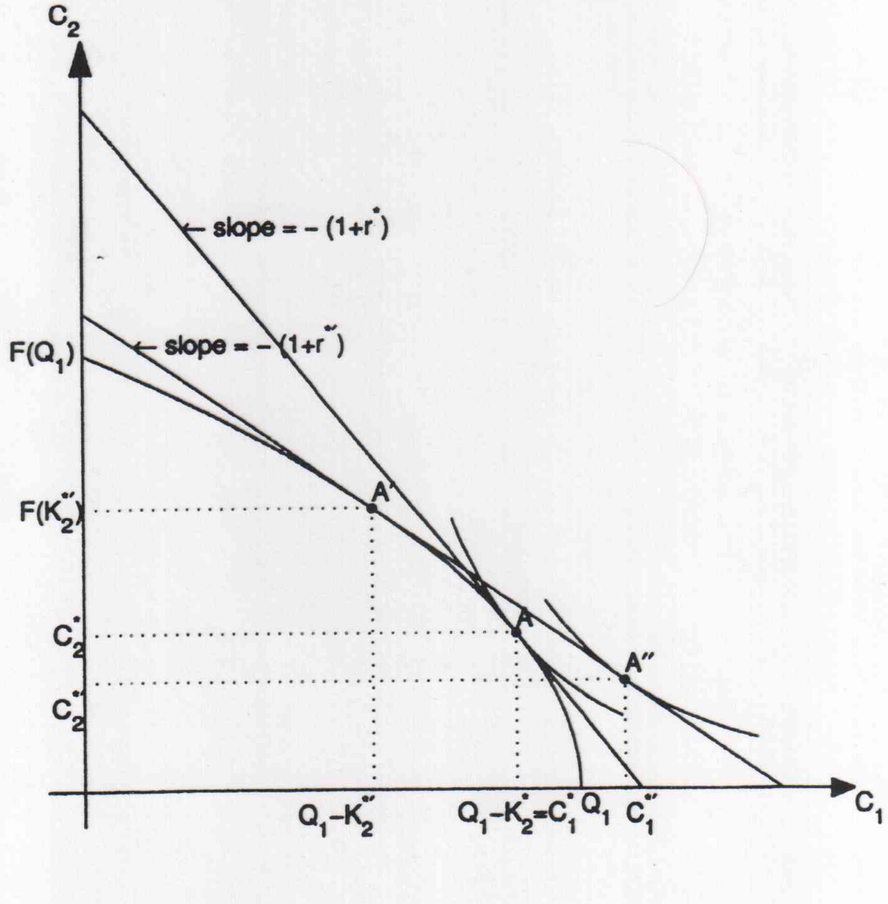


Figure 3.10: A decline in the world interest rate from r^* to $r^{*\prime}$



Permanent Productivity Shock

1.) Now $A_2 \downarrow$ too $\Rightarrow K_2 \downarrow$ $\quad (A_2 F'(K_2) = 1 + r^*)$

$\Rightarrow I_1 \downarrow$

2.) $Q_2 \downarrow$ for two reasons : 1.) $A_2 \downarrow$

2.) $K_2 \downarrow$

3.) Therefore, $Q_2 \downarrow$ more than Q_1

$\Rightarrow S \uparrow$

$\Rightarrow CA \uparrow$ ($S \uparrow$ and $I \downarrow$).

World Interest Rate Shock (see fig. 3.10 in previous graph).

1.) $r^* \downarrow \Rightarrow K_2$ and $I_1 \uparrow$ $\quad (F'(K_2) = 1 + r^*)$

2.) Substitution effect $\Rightarrow C_1 \uparrow$

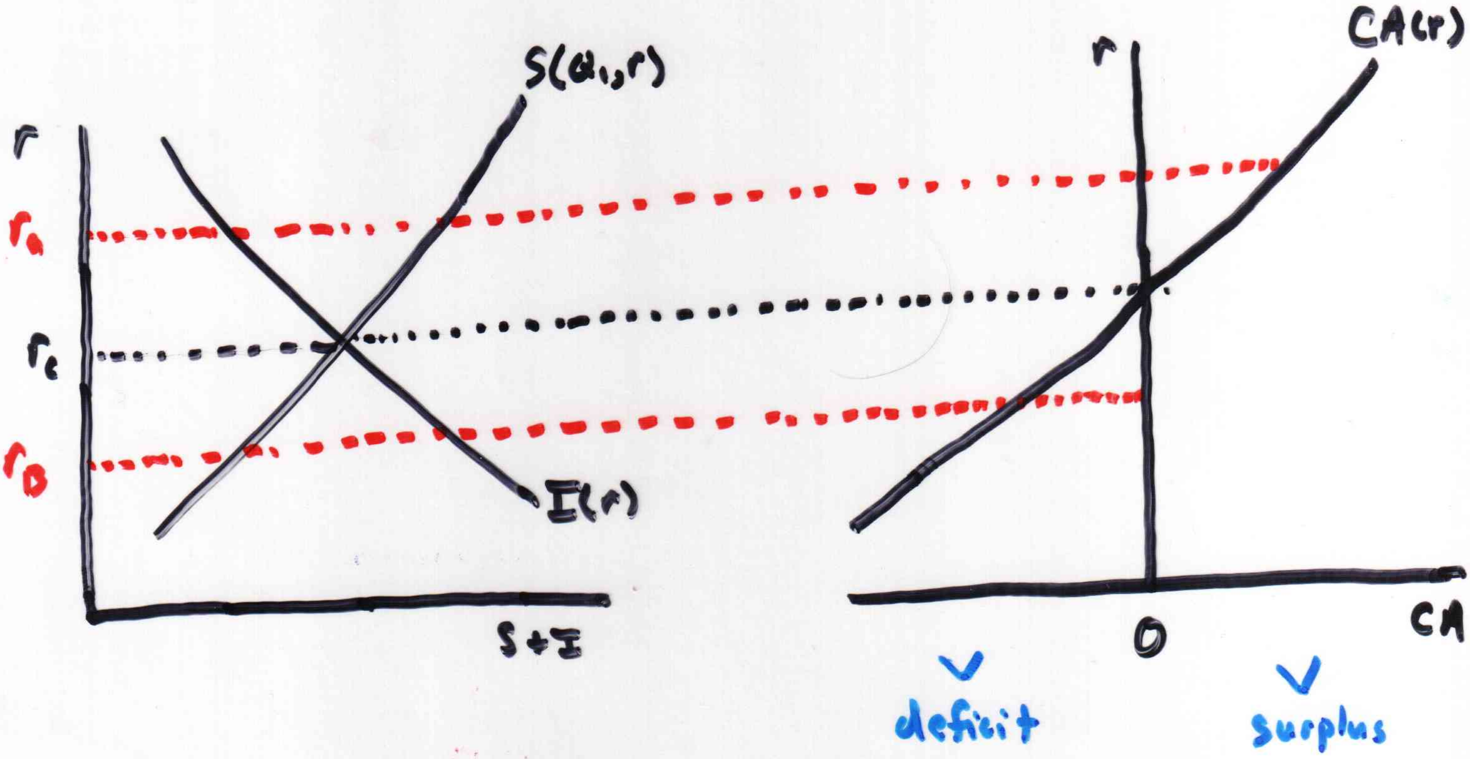
$\Rightarrow S \downarrow$ (Q_1 is given)

3.) $CA \downarrow$ $(S \downarrow, I \uparrow)$

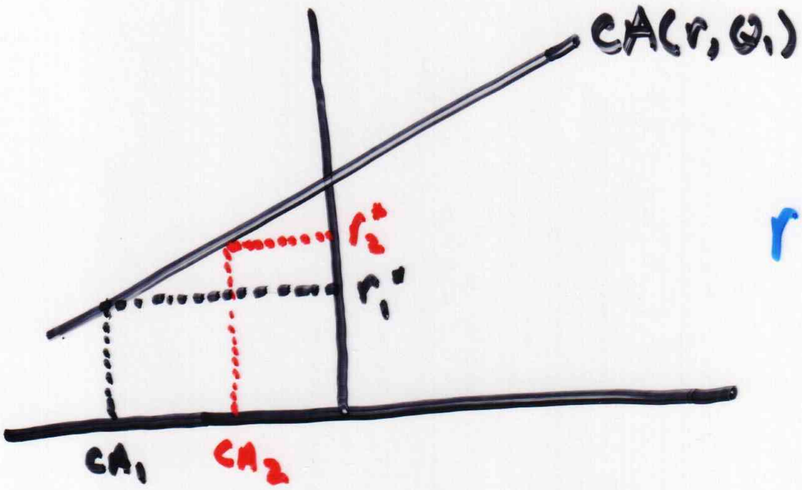
(How does this compare to response of endowment economy?

A Useful Shortcut

- Although examining changes in the PPF, Resource Constraint, and Indifference Curves is quite informative, for many questions we don't need that much detail. Often it is sufficient to work directly with S & I graphs. These can easily be combined to illustrate the CA.

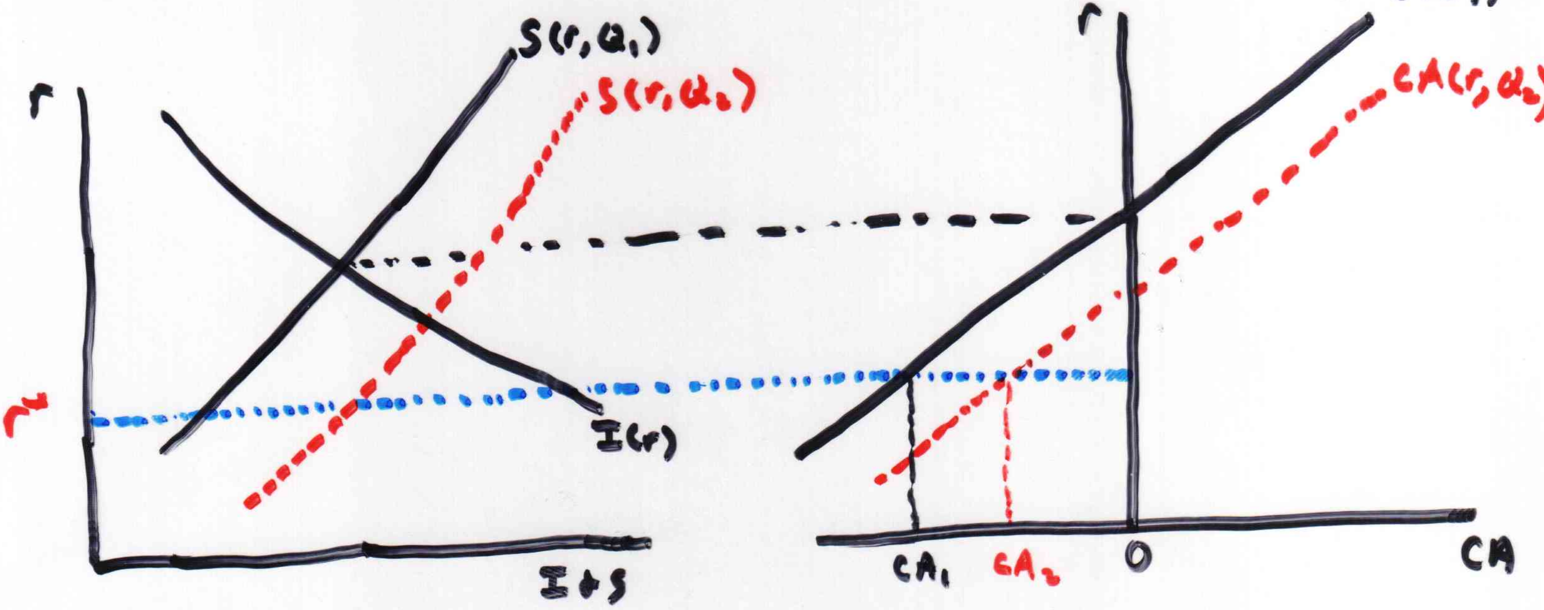


World Interest Rate Shock ($r^* \uparrow$)



$r \uparrow \Rightarrow CA \uparrow$
(deficits shrinks)

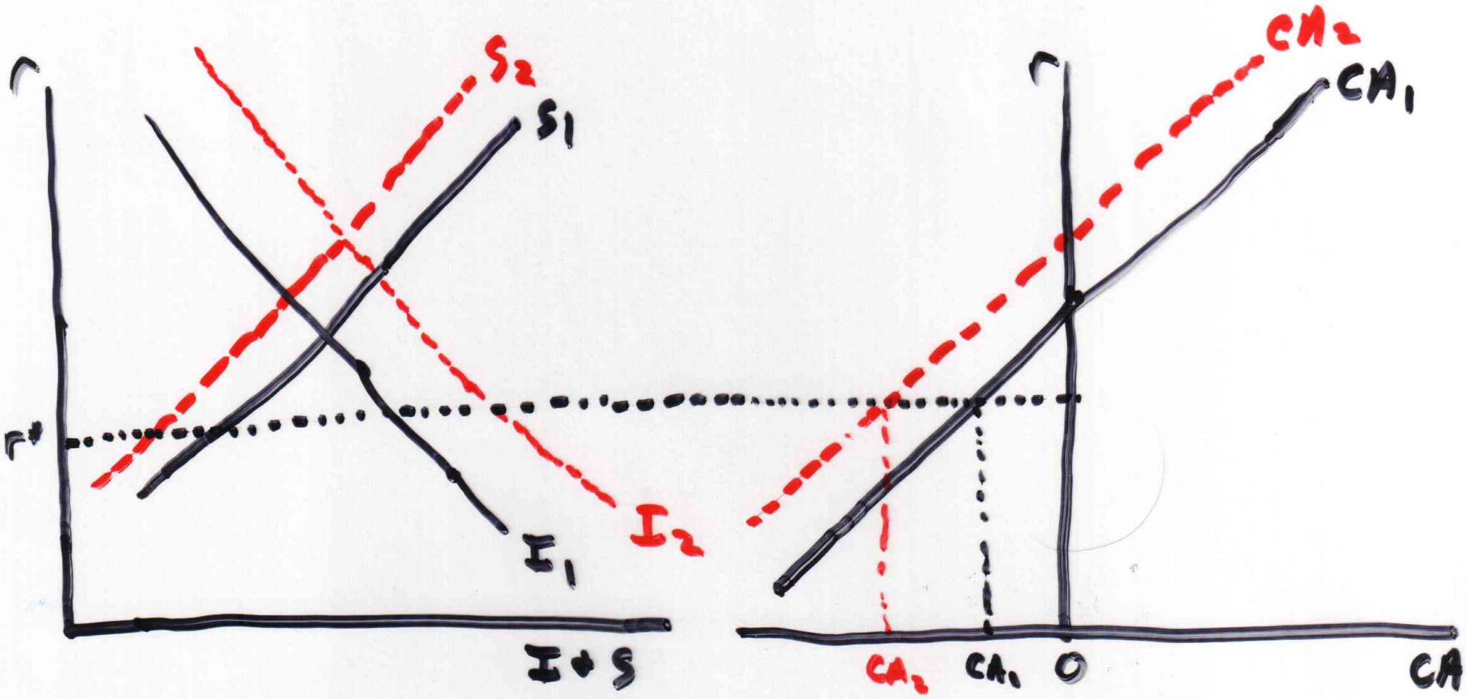
Temporary Productivity Increase ($A_1 \uparrow$)



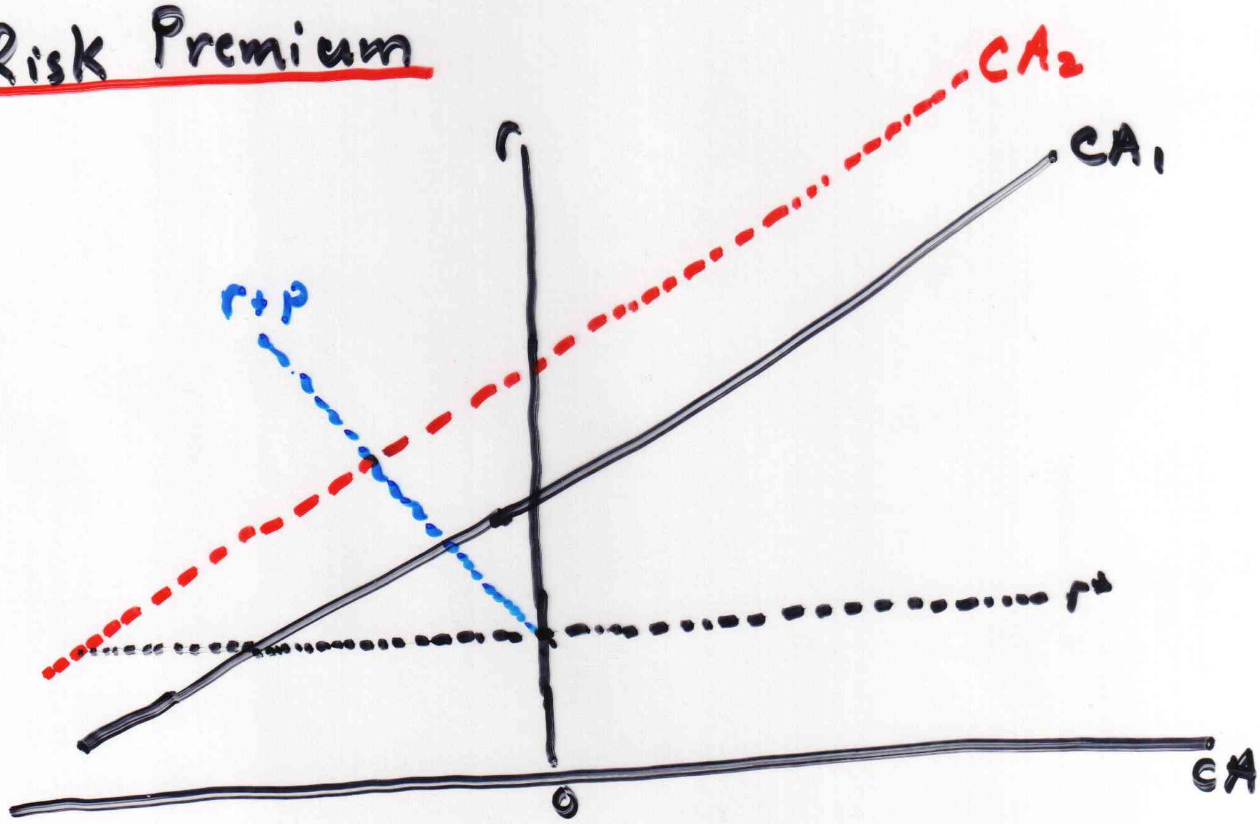
$A_1 \uparrow \Rightarrow S \uparrow$, I constant

$\Rightarrow CA \uparrow$ (deficit shrinks)

Future Productivity Increase ($A_2 \uparrow$)



Risk Premium



Now CA is less reponsive, since domestic interest rate rises.