Labor Hoarding and the Business Cycle

Craig Burnside

University of Pittsburgh and Queen's University

Martin Eichenbaum

Northwestern University, National Bureau of Economic Research, and Federal Reserve Bank of Chicago

Sergio Rebelo

Portuguese Catholic University, Bank of Portugal, University of Rochester, and National Bureau of Economic Research

This paper investigates the sensitivity of Solow residual based measures of technology shocks to labor hoarding behavior. Using a structural model of labor hoarding and the identifying restriction that innovations to technology shocks are orthogonal to innovations in government consumption, we estimate the fraction of the variability of the Solow residual that is due to technology shocks. Our results support the view that a significant proportion of movements in the Solow residual are artifacts of labor hoarding behavior. Specifically, we estimate that the variance of innovations to technology is roughly 50 percent less than that implied by standard real business cycle models. In addition, our results suggest that existing real business cycle studies substantially overstate the extent to which technology shocks account for the variability of postwar aggregate U.S. output.

I. Introduction

Hall (1988) has challenged the assumption of most real business cycle (RBC) models that movements in the Solow residual represent exoge-

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nous technology shocks (see, e.g., Prescott 1986). He argues that "under competition and constant returns to scale, the Solow residual is uncorrelated with all variables known to be neither causes of productivity shifts nor to be caused by productivity shifts" (p. 924). In fact, the Solow residual is significantly correlated with military expenditures (Hall 1988), various monetary aggregates (Evans 1992), and government consumption (see Sec. IV). Hall (1988, 1989) suggests that imperfect competition and increasing returns to scale are essential ingredients of an empirically plausible explanation of these types of correlations.

Not only is the Solow residual correlated with government consumption, but innovations to these two variables are also positively correlated (see Sec. IV). Suppose that analogously to Hall (1988, 1989) we take as given the identifying restriction that innovations to technology ought to be uncorrelated with innovations to government consumption. Then, two questions arise. First, how sensitive to this restriction are conventional measures of technology shocks and the performance of standard RBC models? Second, what are quantitatively convincing mechanisms to explain the observed correlation between the Solow residual and government consumption? This paper deals with these two questions.

To address the first question, we examine the effects of imposing orthogonality between innovations to technology (measured by innovations to the Solow residual) and government consumption within the Hansen (1985) and Rogerson (1988) indivisible labor model. We find that imposing this restriction reduces the variance of the innovation to technology by roughly 60 percent and leads to a significant overall deterioration in the empirical performance of the model.

To address the second question, we construct and empirically implement a general equilibrium model that allows for labor hoarding behavior. By imposing the identifying restriction that innovations to true technology shocks are orthogonal to innovations to government consumption, we can estimate the fraction of the variability in the innovation to the Solow residual that is due to true technology shocks. Our model is able to account for the observed correlation between the Solow residual and government consumption as well as the observed correlation between the innovations to those variables. It also does as well as the standard model in accounting for the relative volatility of those economic aggregates typically stressed in RBC studies. This is true even though our identifying restrictions imply that the variance of innovations to technology is roughly 50 percent less than the variance implied by standard RBC models and that roughly 15 percent of the standard deviation of total labor input into market production is attributable to variations in labor effort. Depending on exactly which procedure we use to estimate the labor hoarding model, the fraction of output volatility accounted for by technology shocks drops between 30 and 60 percent relative to the standard model.[1]

The remainder of this paper is organized as follows. In Section II we describe our basic model. Section III describes our econometric methodology. In Section IV we present our empirical results. Section V discusses some shortcomings of our analysis. Finally, Section VI contains some concluding remarks.

II. A Model of Time-varying Effort and the Business Cycle

In this section we present a variation of Hansen's (1985) indivisible labor model modified to allow for labor hoarding. Our model economy is populated by a large number of infinitely lived individuals. To go to work each individual must incur a fixed cost, ξ, denominated in terms of hours of forgone leisure. Once at work, an individual stays for a fixed shift length of f hours. The momentary utility at time t of such a person is given by

$$\ln(C_t) + \theta \ln(T - \xi - W_t f). \tag{1}$$

Here, T is a scalar denoting the individual's time endowment, θ is a positive scalar, C_t denotes time t privately purchased consumption, and W_t denotes the level of time t effort. According to this specification, what individuals care about is total *effective* work, $W_t f$. The time t utility of a person who does not go to work is given by

$$\ln(C_t) + \theta \ln(T). \tag{2}$$

Output, Y_t, is produced via the Cobb-Douglas production function

$$Y_t = A_t K_t^{1-\alpha}[f N_t W_t(\gamma^t)]^{\alpha}, \tag{3}$$

where $0 < \alpha < 1$, γ represents the growth rate of exogenous labor-augmenting technological progress, N_t denotes the total number of individuals going to work at time t, and K_t denotes the beginning of period t capital stock. The variable A_t represents the stochastic shock to technology, which evolves according to

$$\ln(A_t) = (1 - \rho_a)\ln(A) + \rho_a \ln(A_{t-1}) + \epsilon_{at}. \tag{4}$$

The unconditional mean of $\ln(A_t)$ equals $\ln(A)$, $|\rho_a| < 1$, and ϵ_{at} is the innovation to $\ln(A_t)$ with a standard deviation of $\sigma_{\epsilon a}$.

[1] The models in this paper allow for stochastic shocks in government consumption. While these shocks have a substantial impact on some properties of the model, we find that they have virtually no impact on the volatility of model output. This is consistent with results in Christiano and Eichenbaum (1992).

The aggregate resource constraint is given by

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq Y_t. \tag{5}$$

The parameter δ represents the depreciation rate on capital ($0 < \delta < 1$). The random variable G_t denotes time t government consumption, which evolves according to

$$G_t = \gamma^t g_t, \tag{6}$$

where g_t has the law of motion

$$\ln(g_t) = (1 - \rho_g)\ln(g) + \rho_g \ln(g_{t-1}) + \epsilon_{gt}. \tag{7}$$

Here $\ln(g)$ is the mean of $\ln(g_t)$, $|\rho_g| < 1$, and ϵ_{gt} is the innovation to $\ln(g_t)$ with standard deviation σ_{ϵ_g}.

In the presence of complete markets the decentralized competitive equilibrium corresponds to the solution of a social planning problem. Proceeding as in Rogerson (1988), one can easily show that, since agents' criteria functions are separable across consumption and leisure, the social planner will equate the consumption of employed and unemployed individuals. Under these circumstances, the Pareto-optimal competitive equilibrium corresponds to the solution of the following planning problem: maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t[\ln(C_t) + \theta N_t \ln(T - \xi - W_t f) + \theta(1 - N_t)\ln(T)], \tag{8}$$

subject to (3)–(7) and K_0, by choice of contingency plans for $\{C_t, K_{t+1}, N_t, W_t\colon t \geq 0\}$. In (8), we have normalized the number of agents in the economy to one. Also, E_0 is the time 0 conditional expectations operator, and β is the subjective discount rate ($0 < \beta < 1$).

To complete the specification of the model we must specify the planner's time t information set, Ω_t^*. If A_t and G_t are seen *before* N_t and W_t are chosen, then the model is observationally equivalent to the standard indivisible labor model, modified to incorporate government consumption into the analysis (see Christiano and Eichenbaum 1992). Here we suppose that N_t must be chosen *before* A_t and G_t are known. Let Ω_t denote agents' common information set at the beginning of time t, which includes the lagged values of all variables in the model. Let Ω_t^* consist of Ω_t plus (A_t, G_t). Then the planner's contingency plans for N_t will be a function of the elements of Ω_t, and the contingency plans for W_t, K_{t+1}, and C_t will be functions of the elements of Ω_t^*.[2]

[2] If $\xi = 0$, it is efficient for all individuals to go to work in every period ($N_t = 1$), given that they can adjust their labor effort, W_t, in response to shocks. For this reason we assume that $\xi > 0$.

The previous formalization of the planning problem incorporates the notion that firms must make employment decisions conditional on their views about the future state of demand and technology. Once employment decisions are made, firms adjust to observed shocks along other dimensions. In our model this adjustment occurs through variations in the labor effort that workers are asked to supply. Workers' compensation will depend on the effort supplied. However, to compute the laws of motion for the quantity variables, we do not have to be precise about the exact compensation scheme adopted by firms.

In the nonstochastic steady state of this economy, Y_t, K_t, C_t, and G_t all grow at rate γ; W_t and N_t are constants. Throughout this paper we use lowercase letters to denote detrended variables (e.g., $y_t = Y_t/\gamma^t$). In general, it is not possible to solve this model analytically. Here we use King, Plosser, and Rebelo's (1988) log-linear modification of the procedure used by Kydland and Prescott (1982) to obtain an approximate solution to the planning problem. The resulting decision rules express $\{\ln(W_t), \ln(N_t), \ln(k_{t+1}), \ln(c_t)\}$ as linear functions of $\ln(k_t)$, $\ln(N_t)$, $\ln(A_t)$, and $\ln(g_t)$.

We conclude this section by considering the implications of our model for the standard RBC practice of interpreting Solow residuals as exogenous technology shocks. Most RBC studies (see, e.g., Prescott 1986) assume that output is produced via the Cobb-Douglas production function:

$$Y_t = S_t K_t^{1-\alpha}(H_t\gamma^t)^\alpha. \tag{9}$$

Here H_t denotes total hours worked. Under the maintained assumptions of the standard indivisible labor model, H_t equals total individuals at work times the fixed shift length, f:

$$H_t = N_t f. \tag{10}$$

Implicit in this formulation is the assumption that effort is constant over time.

According to our model, the logarithm of the conventionally measured Solow residual, S_t, is related to the logarithms of the true technology shock, A_t, and effort, W_t, via the relationship

$$\ln(S_t) = \ln(A_t) + \alpha\ln(W_t). \tag{11}$$

It follows that objects that are correlated with $\ln(W_t)$ will also be correlated with $\ln(S_t)$, even though they are not correlated with $\ln(A_t)$. Since our model predicts that $\ln(W_t)$ depends on $\ln(g_t)$, our model is consistent with the fact that the Solow residual is correlated with government consumption.

Given our estimates of the model's structural parameters, it is optimal for agents to work harder in response to a positive innovation to government consumption. It follows from (11) that the Solow residual and average labor productivity will rise in response to such a shock. Naive Solow residual accounting falsely attributes the increase in average productivity to a shift in technology. It is also optimal for agents to work harder in response to a technology shock. Consequently, the Solow residual and average labor productivity will rise by more than a technology shock; that is, the innovation to $\ln(S_t)$ will be larger than the corresponding innovation to $\ln(A_t)$. We conclude that, according to our model, (i) effort is procyclical and (ii) naive Solow residual accounting systematically overestimates the level of technology in booms, systematically underestimates the level of technology in recessions, and systematically overestimates the variance of the true technology shock. In the next section we discuss our econometric method for studying the quantitative importance of this bias.

III. Econometric Method

The key problem in empirically implementing the model of Section II is that we do not have data on effort. In this paper, we adopt two strategies for dealing with this identification problem. Our first strategy, referred to as labor hoarding I, exploits functional form assumptions on the representative agent's utility function, as well as the assumption of perfect competition, to deduce a time series on effort that is a function of observable variables and a subset of the model parameters.

In the competitive equilibrium of our economy, effort is allocated so that the marginal product of an extra unit of effort times the marginal utility of an extra unit of consumption is equal to the marginal disutility of effort of those engaged in work, that is,

$$\theta(T - \xi - W_t f)^{-1} = \frac{\alpha Y_t}{C_t W_t H_t}, \tag{12}$$

where $H_t = N_t f$ denotes total hours worked at time t. Relation (12) allows us to deduce a time series for W_t, given observations on $\{Y_t, C_t, H_t\}$ and values for θ, T, ξ, f, and α. Given this information, we use (9) and (11) to deduce a time series on technology shocks.

To assess the robustness of our results, we pursue a second identification strategy, referred to as labor hoarding II. This strategy exploits the full solution to the social planner's problem, rather than just one of the planner's Euler equations. The linearized equilibrium law of motion for effort can be expressed as

$$\ln(W_t) = \pi_0 + \pi_1 \ln(k_t) + \pi_2 \ln(H_t) + \pi_3 \ln(A_t) + \pi_4 \ln(g_t). \tag{13}$$

The parameters π_i are functions of the model's underlying structural parameters. Relation (13) expresses the equilibrium value of $\ln(W_t)$ as a function of the unknown structural parameters of the model, two observable variables (H_t and g_t), and the unobserved technology shock, $\ln(A_t)$. We also know that the true technology shocks are related to the Solow residual and W_t via

$$\ln(A_t) = \ln(y_t) - (1 - \alpha)\ln(k_t) - \alpha \ln(H_t) - \alpha \ln(W_t). \quad (14)$$

Solving (13) and (14) for each time period, we can obtain time series on W_t and A_t. Given estimates of the structural parameters of the model, (13) and (14) constitute two equations in two unknowns, $\ln(W_t)$ and $\ln(A_t)$, at each point in time. Consequently, we can obtain a time series on both $\ln(W_t)$ and $\ln(A_t)$ by solving these equations for each point in our sample.

A. Estimation and Diagnostic Procedures

This paper uses a variant of the generalized method of moments (GMM) procedure discussed in Christiano and Eichenbaum (1992) to estimate and assess the empirical performance of the model. Our estimation criterion is designed to allow the model to equate model and sample first moments of the data.[3] We use the estimated model to calculate selected second moments of the data. These same second moments can also be estimated directly in a way that does not involve the model. When one abstracts from sampling uncertainty, the two sets of second-moment estimates ought to coincide if the model has been specified correctly. To test this hypothesis, we employ a Wald-type statistic developed in Christiano and Eichenbaum (1992). This statistic, which we denote by $\tilde{J}$, is discussed in the Appendix. Unlike standard calibration exercises, this diagnostic procedure takes into account (i) the sampling error associated with our non-model-based estimates of the second moments and (ii) the sampling error in the model-based estimates of the second moments.[4]

Let Ψ_1 denote the 11×1 vector of structural parameters to be estimated:

$$\Psi_1 = \{\delta, \theta, \alpha, \rho_a, \sigma_{\epsilon a}, \rho_g, \sigma_{\epsilon g}, \gamma_g, g, \gamma, Y\}. \quad (15)$$

[3] We recognize that there is no a priori reason to use a small subset of the model's implications when estimating the structural parameters. Ignoring some implications of our model at the estimation stage of the analysis affects the asymptotic efficiency of our estimator but not its consistency.

[4] This error arises solely from sampling uncertainty in our point estimates of the structural parameters.

Here $\ln(Y)$ and $\ln(g)$ denote the unconditional means of linearly detrended $\ln(Y_t)$ and $\ln(G_t)$.[5] The parameters γ_g and γ denote the unconditional growth rates of government purchases and output, respectively. In describing our model, we assumed $\gamma = \gamma_g$.

The parameters T, β, f, and ξ were not estimated. Instead we fixed T at 1,369 hours per quarter. We set β equal to $(1.03)^{-.25}$. The parameter f was chosen so that the steady state of effort equals one. We experimented with a variety of values of ξ and found that our results were very insensitive to choices of ξ between 20 and 120. The results reported in Section IV correspond to a value of ξ equal to 60.

The Appendix formally describes our estimator of $\mathbf{\Psi}_1$. When we restrict ourselves to an exactly identified GMM procedure, our estimator results in parameter estimates that have three appealing features. First, they are very similar to those used in most RBC studies. This allows us to isolate the effects of labor hoarding per se in those models. Second, the model succeeds in reproducing the first moments of the data. Third, our estimator can be given a very simple interpretation. In particular, our estimator of δ corresponds to the average rate of depreciation in the empirical capital stock and investment series. The estimators of α and θ are designed to make the model reproduce the sample average value of the capital/output ratio and hours worked. The point estimates of ρ_a and σ_{ϵ_a} are obtained by running ordinary least squares on an AR(1) specification of the natural log of our measure of technology shocks. Our point estimates of $\ln(\gamma)$ and $\ln(Y)$ are obtained by regressing the natural log of our measure of output on a constant and time. We estimate $\ln(g)$ and $\ln(\gamma_g)$ by regressing the natural log of our measure of government consumption on a constant and time. Finally, our point estimates of ρ_g and σ_{ϵ_g} are obtained by applying ordinary least squares to $\ln(G_t/\gamma_g^t)$.

This estimator of $\mathbf{\Psi}_1$ does *not* guarantee that innovations to technology shocks will be orthogonal to innovations to government purchases; that is, our exactly identified estimator does not impose the condition

$$E\epsilon_{gt}\epsilon_{at} = 0. \tag{16}$$

While we can test whether this condition is satisfied using the Wald-type statistic ($\tilde{J}$) discussed above, condition (16) can also be imposed during the estimation procedure. When this is done, the GMM system has one overidentifying restriction, which can be tested using Hansen's (1982) J statistic.[6]

[5] We need not estimate $\ln(A)$ because it can be deduced from the other parameters of our model.

[6] This J statistic is asymptotically distributed as a χ^2 statistic with one degree of freedom.

In order to compare our model to a standard RBC model, we also estimate the structural parameters of the Hansen (1985)–Rogerson (1988) indivisible labor model. In this model the planner's criterion function takes the form $E_0 \, \Sigma_{t=0}^{\infty} \, [\ln(C_t) \; - \; \tilde{\theta} N_t]$, where $\tilde{\theta}$ is some positive scalar.[7] With $\tilde{\theta}$ replacing θ, $\boldsymbol{\Psi}_1$ continues to be given by (15). The Appendix formally describes our GMM estimator of $\boldsymbol{\Psi}_1$ for the Hansen-Rogerson model. Since W_t is, by assumption, constant in this model, $\ln(A_t) \equiv \ln(S_t)$; that is, technology shocks are assumed to coincide with the Solow residual.

To implement our diagnostic procedures, we must estimate various moments of the data-generating process. The vector $\boldsymbol{\Psi}_2$ denotes the set of second moments to be estimated. For some of the tests we conduct, $\boldsymbol{\Psi}_2$ is specified as

$$\boldsymbol{\Psi}_2 = \{\sigma_c/\sigma_y, \sigma_i/\sigma_y, \sigma_g/\sigma_y, \sigma_h, \sigma_h/\sigma_{\mathrm{APL}}\}. \tag{17}$$

Here APL denotes the average productivity of labor, i denotes gross investment, and σ_x denotes the standard deviation of the variable x, $x = \{c, y, \mathrm{APL}, i, h\}$. This specification of $\boldsymbol{\Psi}_2$ is useful for considering the implications of our model for the moments of the data typically stressed in existing RBC studies. Since the data display marked time trends, some stationary inducing transformation must be adopted to ensure that the moments in (17) exist. To this end, we detrend both the time series emerging from the model and actual data with the Hodrick and Prescott (1980) filter. Under these circumstances the population moments in $\boldsymbol{\Psi}_2$ pertain to Hodrick-Prescott filtered versions of the data.

For other specifications of $\boldsymbol{\Psi}_2$, we do not work with Hodrick-Prescott filtered data, since the object of interest corresponds to data that have been rendered stationary via some other transformation. Specifically, in some of our exercises, we specify $\boldsymbol{\Psi}_2$ to equal

$$\boldsymbol{\Psi}_2 = \{b_g, b(\epsilon_s, \epsilon_g)\}. \tag{18}$$

Here b_g refers to the regression coefficient of the growth rate of the Solow residual on the growth rate of government consumption, and $b(\epsilon_s, \epsilon_g)$ denotes the regression coefficient of the innovation to the Solow residual on the innovation to government consumption.

Irrespective of the precise specification of $\boldsymbol{\Psi}_2$, any estimator of $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$ will be based on the same data set. Moreover, the Solow residual involves the unknown parameter α (which is an element of $\boldsymbol{\Psi}_1$). These facts imply the existence of a nonzero covariance between estimators of $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$. Consequently, we estimate $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$ simultaneously in order to obtain the correct sampling distribution for our estimator (see the Appendix).

[7] The parameters θ and $\tilde{\theta}$ are related according to $\tilde{\theta} = \theta \ln[T/(T - \xi - f)]$.

B. Data

Private consumption, C_t, was measured as the sum of private-sector expenditures on nondurable goods plus services plus the imputed service flow from the stock of durable goods. Government consumption, G_t, was measured by real government purchases of goods and services minus real government (federal, state, and local) investment. The capital stock, K_t, was measured as the sum of consumer durables, producer structures and equipment, and government and private residential capital plus government nonresidential capital. Data on gross investment, I_t, are the flow data that match the capital stock concept. Gross output, Y_t, was measured as C_t plus G_t plus I_t plus time t inventory investment. Our basic measure of hours worked is the one constructed by Hansen (1984), which we refer to as household hours. The data are quarterly, cover the period 1959:1–1984:1, and were converted to per capita terms using an efficiency-weighted measure of the population.[8] Finally, we incorporate Prescott's (1986) model of measurement error into our analysis. In particular, we assume that the log of reported hours worked differs from the log of actual hours worked by an independently and identically distributed random variable that has mean zero and standard errors σ_v. To estimate σ_v we exploit two different measures of hours worked. The first is Hansen's (1984) measure, which is based on the household survey conducted by the Bureau of the Census. The second is based on the establishment survey conducted by the Bureau of Labor Statistics (see the Appendix for details regarding the estimation procedure).

IV. Empirical Results

In this section we report our empirical results. We begin by discussing the Hansen-Rogerson model. Here we focus on three key results. First, this model is inconsistent with the observed correlation between the growth rate of government purchases and the Solow residual. Second, innovations to the measure of technology shocks used in RBC models (the Solow residual) display a positive, statistically significant correlation with innovations to government purchases. Third, when we impose the condition that these innovations ought to be orthogonal, the point estimates of the model's parameters move to inadmissible values.

After discussing the Hansen-Rogerson model, we consider the empirical implications of our labor hoarding model. Our main results here can be summarized as follows. First, the model is consistent with

[8] For further details on our data set, see Christiano (1987, appendix).

the observed correlation between the growth rates of government consumption and the Solow residual. Second, we cannot reject the null hypothesis that innovations to the measure of technology shocks that emerge from the model are orthogonal to innovations to government consumption. Third, these technology shocks are much less volatile than innovations to the Solow residual. This translates into a marked reduction in the percentage of output volatility that can be attributed to technology shocks. Fourth, the labor hoarding model does at least as well as the Hansen-Rogerson model at accounting for the volatility of hours worked and the relative volatility of consumption, investment, average productivity, and government consumption.

Table 1 presents our parameter estimates for the Hansen-Rogerson model. Column 1 reports parameter estimates for the unconstrained version of the model; that is, condition (16) is not imposed. These parameter estimates are very similar to the values typically used in RBC studies (see, e.g., Hansen 1985). Notice in particular that the estimated value of α (.655) is very close to the value used by Hansen (1985) and Prescott (1986). Consequently, our time series on the Solow residual is virtually identical to the one used in those studies.[9]

The first two columns of table 2 report the implications of the Hansen-Rogerson model for (i) $b(\epsilon_s, \epsilon_g)$, the regression coefficient of the innovation to the Solow residual on the innovation to government consumption; (ii) $b(\epsilon_a, \epsilon_g)$, the regression coefficient of the innovation to the technology shock on the innovation to government consumption; and (iii) b_g, the regression coefficient of the growth rate of the Solow residual on the growth rate of government consumption.

In the Hansen-Rogerson model, technology shocks are, by assumption, equal to the Solow residual. Consequently, $b(\epsilon_s, \epsilon_g)$ ought to equal $b(\epsilon_a, \epsilon_g)$. Under the maintained assumptions of existing RBC models, both ought to equal zero. Hall (1988) has argued that b_g ought to equal zero. These properties are summarized by the zeros in column 2 of table 2.

To assess the empirical plausibility of these model properties, we estimated the values of b_g, $b(\epsilon_s, \epsilon_g)$, and $b(\epsilon_a, \epsilon_g)$ in the data.[10] Our results are contained in column 1 of table 2. Numbers in parentheses denote standard errors. To evaluate the null hypothesis that sample and model population moments are drawn from the same data-

[9] The only other parameters that are relevant for calculating the Solow residual are σ_v, the standard error of the measurement error in household hours worked, and the unconditional growth rate of technology. Our estimates of these parameters are virtually identical to those used by Prescott (1986).

[10] The only "model" parameter involved in this estimation problem is α, which is required to construct the Solow residual.

TABLE 1

PARAMETER ESTIMATES

	Hansen Rogerson		Labor Hoarding I		Labor Hoarding II	
	Exactly Identified (1)	Overidentified (2)	Exactly Identified (3)	Overidentified (4)	Exactly Identified (5)	Overidentified (6)
θ	...	...	3.61	3.61	3.68	3.68
			(.025)	(.013)	(.034)	(.027)
$\tilde{\theta}$	.0037	.0038				
	(.00004)	(.00003)				
ρ_a	.986	1.013	.983	.982	.977	.970
	(.026)	(.029)	(.024)	(.018)	(.029)	(.027)
σ_{ϵ_a}	.0089	.0055	.0059	.0053	.0072	.0063
	(.0013)	(.0018)	(.0006)	(.0006)	(.0012)	(.0009)
$\ln Y$	8.57	8.57	8.57	8.57	8.57	8.57
	(.018)	(.009)	(.018)	(.009)	(.018)	(.009)
$\ln \gamma$	.004	.004	.004	.004	.004	.004
	(.0003)	(.0002)	(.0003)	(.0002)	(.0003)	(.0002)
$\ln \gamma_g$	.002	.002	.002	.002	.002	.002
	(.0004)	(.0002)	(.0004)	(.0002)	(.0004)	(.0002)
$\ln G$	6.95	6.94	6.95	6.95	6.95	6.95
	(.028)	(.014)	(.028)	(.015)	(.028)	(.015)
ρ_g	.982	.989	.982	.987	.982	.997
	(.021)	(.016)	(.021)	(.016)	(.021)	(.013)
σ_{ϵ_g}	.015	.014	.015	.014	.015	.014
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
δ	.021	.021	.021	.021	.021	.021
	(.0003)	(.0001)	(.0003)	(.0001)	(.0003)	(.0001)
σ_ν	.009	.012	.009	.012	.009	.012
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
α	.655	.657	.655	.654	.655	.655
	(.006)	(.005)	(.006)	(.005)	(.006)	(.007)
J	...	8.53	...	4.86	...	2.56
		(.004)		(.03)		(.11)

TABLE 2

COVARIANCE OF TECHNOLOGY AND GOVERNMENT SHOCKS

	Hansen-Rogerson		Labor Hoarding I		Labor Hoarding II	
	Sample (1)	Model Population (2)	Sample (3)	Model Population (4)	Sample (5)	Model Population (6)
b_g	.176 (.067)	0 ... [.009]	.176 (.067)	.107 (.025) [.38]	.176 (.067)	.107 (.025) [.38]
$b(\epsilon_a, \epsilon_g)$	.183 (.069)	0 ... [.008]	.077 (.036)	0 ... [.03]	.072 (.058)	0 ... [.22]
$b(\epsilon_s, \epsilon_g)$	.183 (.069)	0 ... [.008]	.183 (.069)	.106 (.026) [.35]	.185 (.068)	.106 (.026) [.34]

generating process, we employed the $\tilde{J}$ statistic discussed above. Numbers in brackets denote the corresponding probability values.

Two key results emerge here. First, innovations to the Solow residual are positively correlated (.183) with innovations to government purchases. The null hypothesis that $b(\epsilon_s, \epsilon_g)$ equals zero can be rejected at less than the 1 percent significance level. The only way to reconcile this finding with the notion that the Solow residual measures exogenous technology shocks is to suppose that government consumption responds to such shocks within the quarter. This would violate one of the basic identifying assumptions of this paper. Second, the regression coefficient of the growth rate of government consumption on the growth rate of the Solow residual is positive (.176) and statistically significant. The null hypothesis that b_g actually equals zero can be rejected at less than the 1 percent significance level. This finding mirrors Hall's (1988) result that the growth rate in his measure of government spending is significantly correlated with the growth rate of the Solow residual.

Column 2 of table 1 reports parameter estimates for the Hansen-Rogerson model when overidentifying restriction (16) is imposed during the estimation procedure. The last entry in this column is Hansen's J statistic, which we use to test the overidentified system. The number in parentheses denotes the corresponding probability value. Notice that the overidentified system is rejected at less than the 1 percent significance level. This provides confirming evidence that the measure of technology shocks used in standard RBC models is inconsistent with restriction (16). Also notice that when (16) is imposed, the estimated value of ρ_a exceeds one. Evidently, the model cannot accommodate this restriction with an admissible value of ρ_a. This provides strong additional evidence regarding the incompatibility of the model with restriction (16).[11]

Columns 3–6 of table 1 present our parameter estimates for the unconstrained and constrained labor hoarding I and labor hoarding II cases. By constrained we mean that condition (16) is imposed during estimation. In principle the estimated labor hoarding model allows us to disentangle actual technology shocks from movements in the Solow residual. The key issue is why one should take this model more seriously than standard versions of the RBC model. We have already established that innovations to the Solow residual are not orthogonal to innovations to government consumption. Moreover, we showed that the growth rates of government consumption and the Solow residual are positively correlated. Neither finding can be accounted for by the standard model. We now show that both findings can be accounted for by the labor hoarding model. Moreover, one cannot reject the hypothesis that innovations to the time series on technology shocks generated by the estimated labor hoarding model are orthogonal to innovations to government purchases.

According to the labor hoarding model, the correlation between innovations to technology shocks and those to government consumption, $b(\epsilon_a, \epsilon_g)$, ought to equal zero (see col. 4 or 6 of table 2). Our exactly identified GMM estimation procedure does *not* impose this restriction, for either of the two identification strategies pursued. However, given our point estimates of the structural parameters, we can compute the implied value for $b(\epsilon_a, \epsilon_g)$. This value is reported in columns 3 and 5 of table 2. Numbers in parentheses denote standard errors.[12] Numbers in brackets denote the probability values of our statistic for testing whether $b(\epsilon_a, \epsilon_g)$ equals zero in population. Notice that we cannot reject the null hypothesis that $b(\epsilon_a, \epsilon_g)$ is equal to zero at the 3 percent and 22 percent significance levels, for the labor hoarding I and labor hoarding II cases, respectively. These results

[11] Burnside, Eichenbaum, and Rebelo (1991) redo the empirical analysis in this paper allowing for a break in the sample at 1969:4. The resulting point estimates of ρ_a are always less than one. This finding aside, the main conclusions of this paper are very robust to splitting the sample. They find that imposing (16) (i) reduces the estimated variance of innovations to technology shocks by approximately 33 percent and 61 percent in the first and second sample periods, respectively; (ii) reduces the unconditional volatility of technology shocks by 31 percent and 25 percent in the first and second sample periods, respectively; and (iii) reduces the percentage of the variance of output that the model can account for by 33 percent and 68 percent in the first and second sample periods, respectively.

[12] These standard errors reflect the sampling uncertainty in our point estimates of the model's parameters.

are consistent with the tests of the overidentified GMM systems in which (16) is imposed (see table 1).[13]

This result would not be very interesting if the labor hoarding model accommodated (16) at the cost of generating counterfactual implications for $b(\epsilon_s, \epsilon_g)$ or b_g. This is not the case. Table 2 indicates that (i) irrespective of which identification strategy is used, we cannot reject the null hypothesis that the values of $b(\epsilon_s, \epsilon_g)$ implied by our parameter estimates and the non-model-based estimates are equal in population; and (ii) the labor hoarding model can account for the observed correlation between the growth rates of the Solow residual and government consumption. The value of b_g implied by both labor hoarding I and II equals .107 with a standard error of .025. The estimated value of b_g in the data equals .176 with a standard error of .07. Testing the hypothesis that the two regression coefficients are the same in population, we obtain a value for the χ^2 statistic that has a probability value of .38. Consequently, one cannot reject, at conventional significance levels, the view that our model fully succeeds in accounting for the regression coefficient in question.

Hall (1988) interprets his positive estimate of b_g as evidence in favor of the notion that imperfect competition and increasing returns to scale are important determinants of the time-series properties of average productivity. While he does not construct and test a model incorporating these features, he does review and reject alternative explanations of his regression results. To argue that unobserved variation in labor effort is not a plausible explanation, he calculates the growth rate of effective labor input required to explain *all* the observed movements in total factor productivity. From this measure he subtracts the growth rate of actual hours of work to generate a time series on the growth rate in work effort. On the basis of these calculations, Hall argues that the implied movements in work effort are implausibly large. However, this calculation is not germane to our analysis because it presumes that there are *no* technology shocks whatsoever. In our context the relevant issue is what the time-series properties of effort must be to explain the regression coefficient in question, *not* whether time-varying effort can explain *all* movements in total factor productivity.

One way to quantify the importance of time-varying effort in the estimated labor hoarding model is to calculate the ratio, Λ, of the standard deviation of the log of effort to the standard deviation of

[13] The J statistics in cols. 4 and 6 of table 1 indicate that the overidentified labor hoarding I and II models cannot be rejected at the 3 and 11 percent significance levels, respectively.

the log of total effective labor input. Below we report the values of Λ implied by the two labor hoarding cases.

Labor Hoarding I		Labor Hoarding II	
Exactly Identified	Overidentified	Exactly Identified	Overidentified
.14	.13	.16	.09

Notice that the values of Λ obtained under the two identification schemes are quite similar and decline when (16) is imposed. In all cases the log of effort accounts for less than a fifth of the standard deviation of the log of total effective labor input. Evidently one can account for Hall's results and the positive correlation between innovations to the Solow residual and government consumption without assuming that most of the movement in total effective labor input is due to variations in effort.

Consider now the performance of the two models with respect to the volatility of hours worked, σ_h, the relative volatility of consumption, investment, and government purchases to output—σ_c/σ_y, σ_i/σ_y, and σ_g/σ_y, respectively—and the volatility of hours worked relative to average productivity, $\sigma_h/\sigma_{\text{APL}}$. Table 3 reports the models' predictions for these moments as well as our estimates of the corresponding data moments. Each element in this table contains three numbers. The top number equals the value of the moment implied by the relevant model. The middle number (in parentheses) is the estimated standard error of the first number. For each moment we tested the null hypothesis that the model moment equals the data population moment. The bottom number (in brackets) equals the probability value of the $\tilde{J}$ statistic.

Table 3 reveals that it is very difficult to distinguish between the models on the basis of their implications for the moments in question. Indeed, there is very little evidence against the individual hypotheses that the value of σ_h, σ_c/σ_y, σ_i/σ_y, σ_g/σ_y, or $\sigma_h/\sigma_{\text{APL}}$ that emerges from any of the models is different from the corresponding data population moment. We conclude that allowing for time-varying effort does not cause the model's performance to deteriorate with respect to these moments. Burnside et al. (1991) argue that the labor hoarding model is better able to account for the joint behavior of average productivity and hours worked than the standard model.[14]

[14] In particular, the labor hoarding model is consistent with three key facts. First, average productivity and hours worked do not display any marked contemporaneous correlation. Second, average productivity leads the cycle in the sense that it is positively correlated with future hours worked. Third, average productivity is negatively correlated with lagged hours. Gordon (1979) presents evidence on this last phenomenon, which he refers to as the "End-of-Expansion-Productivity-Slowdown." McCallum (1989) documents a similar pattern for the dynamic correlation between average productivity and output.

TABLE 3

SELECTED SECOND MOMENTS

		Hansen-Rogerson		Labor Hoarding I		Labor Hoarding II	
Moment	U.S. Data	Exactly Identified	Overidentified	Exactly Identified	Overidentified	Exactly Identified	Overidentified
σ_c/σ_y	.44	.53	. . .	.53	.52	.48	.45
	(.03)	(.24)		(.19)	(.18)	(.19)	(.15)
		[.69]		[.62]	[.63]	[.81]	[.94]
σ_i/σ_y	2.22	2.65	. . .	2.64	2.66	2.77	2.87
	(.07)	(.59)		(.48)	(.44)	(.45)	(.37)
		[.46]		[.38]	[.31]	[.22]	[.08]
σ_g/σ_y	1.15	1.11	. . .	1.62	1.70	1.29	1.35
	(.20)	(.35)		(.35)	(.36)	(.15)	(.17)
		[.88]		[.19]	[.14]	[.50]	[.39]
σ_h/σ_{APL}	1.22	1.056	. . .	1.01	1.02	1.017	1.08
	(.12)	(.46)		(.15)	(.08)	(.23)	(.12)
		[.73]		[.22]	[.15]	[.40]	[.38]
σ_h	.017	.013	. . .	.012	.014	.013	.015
	(.002)	(.005)		(.002)	(.002)	(.003)	(.002)
		[.47]		[.02]	[.13]	[.15]	[.56]

We now turn to the question of how inference about the volatility of technology shocks is affected by allowing for labor hoarding. Table 4 reproduces our point estimates of ρ_a and σ_{ϵ_a} and reports σ_A, the unconditional standard deviation of technology shocks, corresponding to the different models. The variance of the innovation to technology shocks drops by 56 percent and 35 percent when we move from the exactly identified Hansen-Rogerson model to the exactly identified labor hoarding I and labor hoarding II cases, respectively. Comparing the exactly identified Hansen-Rogerson model to the overidentified labor hoarding I and II cases reveals that the estimated value of $\sigma^2_{\epsilon a}$ drops by 65 percent and 50 percent, respectively. A similar picture emerges if we compare estimates of σ^2_A, the unconditional variance of technology shocks. For example, moving from the unconstrained Hansen-Rogerson model to the labor hoarding I and II cases generates a drop in the estimated value of σ^2_A equal to 62 percent and 57 percent, respectively. Evidently, incorporating time-varying effort in the standard model substantially reduces point estimates of both the unconditional and conditional variance of technology shocks. We interpret these results as providing substantial support for the view that a large percentage of the movements in the observed Solow residual are artifacts of labor hoarding type behavior.

An important question is how the findings discussed above translate into the percentage of the variability of output that the different models can account for. To this end we computed the statistic $\lambda = \sigma^2_{ym}/\sigma^2_y$ for the different models. Here the numerator denotes the variance of Hodrick-Prescott filtered output implied by the estimated model and the denominator denotes the variance of Hodrick-Prescott filtered U.S. output. Kydland and Prescott (1989) have emphasized the importance of this statistic. Their claim that technology shocks account for most of the fluctuations in postwar U.S. output corresponds to the claim that λ is a large number, with the current estimate being between .75 and 1.0, depending on exactly which RBC model is used. From table 4 we see that the estimated value of λ for the exactly identified Hansen-Rogerson model is equal to .81.

Comparing the exactly identified Hansen-Rogerson model to the exactly identified labor hoarding I and II cases, we see that λ drops by 54 percent and 28 percent, respectively. Moving from the exactly identified Hansen-Rogerson model to the overidentified labor hoarding I and II models induces a decline in λ of 62 percent and 37 percent, respectively (the corresponding point estimate of λ falls from .81 to .31 in the labor hoarding I case and .51 in the labor hoarding II case). We conclude that the percentage of output variability that technology shocks can account for is substantially reduced once time-varying effort is allowed for.

TABLE 4

VARIABILITY OF OUTPUT ACCOUNTED FOR BY MODELS

	Hansen-Rogerson		Labor Hoarding I		Labor Hoarding II	
	Exactly Identified (1)	Overidentified (2)	Exactly Identified (3)	Overidentified (4)	Exactly Identified (5)	Overidentified (6)
ρ_a	.986	1.013	.983	.982	.977	.970
	(.026)	(.029)	(.024)	(.018)	(.029)	(.027)
σ_{ϵ_a}	.0089	.0055	.0059	.0053	.0072	.0063
	(.0013)	(.0018)	(.0005)	(.0006)	(.0012)	(.0009)
σ_A	.052	. . .	.032	.028	.034	.026
	(.046)		(.023)	(.017)	(.026)	(.017)
λ	.81	. . .	.37	.31	.58	.51
	(.56)		(.17)	(.14)	(.14)	(.12)

V. Shortcomings of the Analysis

According to our labor hoarding model, firms can contemporaneously adjust to unanticipated changes in demand and productivity only by using labor more intensively. In reality there are a variety of margins along which firms can adjust. In this section we briefly discuss three such margins and indicate the nature of the biases that they are likely to impart on our results.

A. *Endogenous Capital Utilization Rates*

Our model does not allow for endogenous capital utilization rates. Allowing for them would strengthen our main conclusions. While poorly measured, capital utilization rates are clearly procyclical (Shapiro 1989). Consequently, the measurement error involved in using the stock of capital to calculate the Solow residual would also be procyclical. The same sorts of impulses that cause labor effort to increase would presumably also induce increases in capital utilization rates. To the extent that this is true, our results understate the sensitivity of RBC models to more general types of "hoarding" behavior.[15]

B. *Time-varying Shift Lengths and Overtime Labor*

A key maintained assumption of empirical work that documents the procyclical nature of average productivity, as well as the properties of the Solow residual, is that straight-time and overtime hours are perfect substitutes (Bodkin and Klein 1967; Prescott 1986). Lucas (1970), Sargent (1987, chap. 16), and Hansen and Sargent (1988) have argued that the procyclical nature of average productivity is, in part, an artifact of cyclical changes in the straight time/overtime labor mix. The same considerations imply that the Solow residual overstates the cyclical importance of technology shocks. To see this consider a special case of the model in Sargent (1987) in which total output, Y_t, is given by

$$Y_t = A_t K_t^{1-\alpha} n_{1t}^{\alpha} + A_t K_t^{1-\alpha} n_{2t}^{\alpha}.$$

Here n_{1t} and n_{2t} denote total hours of straight-time and overtime labor. Conventional Solow residual accounting proceeds as though n_{1t} and n_{2t} were perfect substitutes, that is, as though the production function were $Y_t = A_t K_t^{1-\alpha}(n_{1t} + n_{2t})^{\alpha}$. The logarithm of the resulting Solow residual is composed of two parts: a true technology shock,

[15] For examples of models in which endogenous capital utilization rates have this sort of effect, see Rotemberg and Woodford (1991).

$\ln(A_t)$, and a component that reflects the fact that straight-time and overtime labor are imperfect substitutes ($\ln[X_t]$): $\ln(S_t) = \ln(A_t) + \ln(X_t)$, where $\ln(X_t) = \ln(n_{1t}^{\alpha} + n_{2t}^{\alpha}) - \alpha \ln(n_{1t} + n_{2t})$.

Consider a shock that generates an increase in the equilibrium levels of n_{1t} and n_{2t}. It is straightforward to show that $\ln(X_t)$ will increase provided that two plausible conditions are satisfied: (i) $n_{1t} > n_{2t}$, that is, total straight-time hours exceed total overtime hours; and (ii) the elasticity of n_{2t} with respect to this shock exceeds that of n_{1t}. Sargent (1987) produces an example in which condition ii is satisfied because of differential costs of adjustment in n_{1t} and n_{2t}. This has three important implications.

First, Solow residual accounting overstates the volatility of the technology shocks to the extent that n_{1t} and n_{2t} are not perfect substitutes.[16] Second, average productivity can in principle be procyclical even in the absence of any technology shocks and in the presence of constant returns to scale production functions. Third, the previous example suggests a way of seeing whether the straight time/overtime distinction can explain the fact that the growth rate of the Solow residual is correlated with the growth rate of government consumption. According to the previous example, the "correct" Solow residual is given by $\ln(Y_t) - (1 - \alpha)\ln(K_t) - \ln(n_{1t}^{\alpha} + n_{2t}^{\alpha})$. We calculated a time series on this residual using the measures of straight-time and overtime hours adopted by Hansen and Sargent (1988, p. 291), using values of α ranging from .55 to .75. In every instance, the regression coefficient of the growth rate in the "correct" Solow residual on the growth rate in government consumption exceeded .13 and was statistically significant. In contrast, the value of this regression coefficient using conventionally measured Solow residuals is approximately .18 (see table 2). So, while the regression coefficient becomes somewhat smaller when we correct for cyclical changes in the straight time/overtime labor mix, this route does not appear, in and of itself, capable of resolving the basic problem.

One of the main points of this paper was that the volatility of output that is attributable to technology shocks has been overstated by existing RBC studies. Since we proceeded under the standard assumption that straight time and overtime are perfect substitutes when calculating the Solow residual, the importance of technology shocks in explaining output volatility is likely to be even smaller than our analysis indicates.

[16] Since shocks to government consumption increase equilibrium hours worked, the previous example shows how ignoring the distinction between straight-time and overtime labor could cause an analyst to infer that a technology shock has occurred when in fact none has.

A second major purpose of our paper was to explore the role of labor hoarding in the business cycle. By ignoring the distinction between straight time and overtime in our model, we are likely to overstate the role of labor hoarding. In fact there is a trivial reinterpretation of our model in which time t effort, W_t, represents overtime labor. Under this reinterpretation, the firm cannot change the number of people hired at time t following an innovation to A_t or G_t, but it could increase overtime labor, that is, shift length. There is nothing in our model that prevents such a reinterpretation. However, assuming that straight-time and overtime labor are perfect substitutes would lead us back to where we started from. Labor input would be correctly measured and innovations to the Solow residual would accurately measure innovations to technology. But then one could not explain the observed correlation between innovations to the Solow residual and innovations to government purchases without violating our basic identifying restriction, namely, that innovations to technology ought to be orthogonal to innovations to government consumption.

C. Contemporaneous Adjustments in Employment

Our model assumes that firms cannot adjust the total number of people hired in response to unanticipated shocks in technology or government purchases. To the extent that this assumption is incorrect, our results will overstate the importance of labor hoarding and understate the importance of technology shocks.

To explore some of the ramifications of allowing firms some flexibility in changing employment after seeing the time t realizations of A_t and G_t, we nest the Hansen-Rogerson model and our labor hoarding model within a more general setup. Suppose that the representative consumer's preferences and technology are the same as described in Section II with the following modification: firms must make an initial time t employment plan on the basis of the information set Ω_t, which does not include the time t innovations to A_t and G_t. Denote the value of planned time t employment by N_t^*. In contrast to the model of Section II, suppose that after the innovations to A_t and G_t are realized, the firm can revise and set actual employment to N_t. However, there is an adjustment cost, $(\mu/2)(N_t - N_t^*)^2$, associated with deviations of actual employment from planned employment. The model of Section II corresponds to the case of $\mu = \infty$, in which case N_t will always equal planned employment N_t^*. The Hansen-Rogerson model corresponds to the case of $\mu = 0$ so that N_t^* is irrelevant.

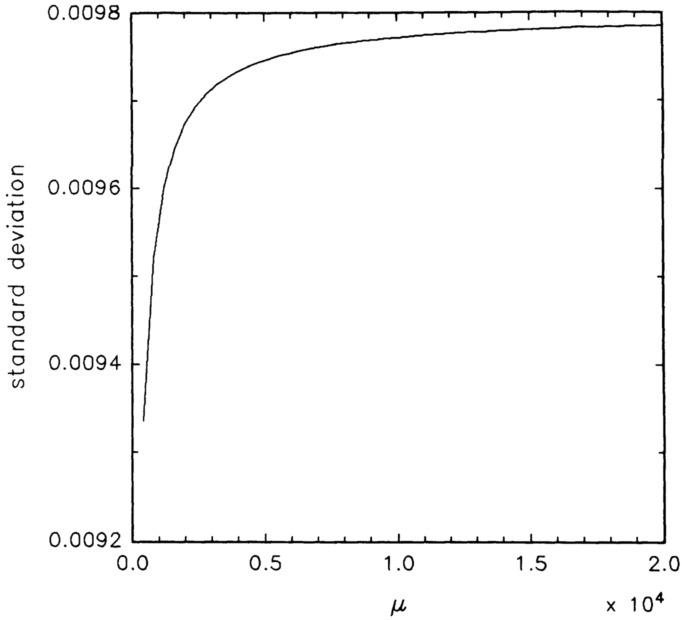

FIG. 1.—Variability of $\log(Y_t/C_tH_t)$ as a function of μ

To explore the implications of the adjustment cost μ, recall the Euler equation for effort, which we exploited to obtain a measure of W_t:

$$\frac{W_t\theta/\alpha}{T - \xi - W_t f} = \frac{Y}{C_t H_t}.$$

The standard deviation of $\ln(Y_t/C_tH_t)$ is .031. The variability of the left-hand side is monotonically increasing in μ. Figure 1 shows the effect of varying μ on the variability of the value of $\ln(Y_t/C_tH_t)$ implied by the model.[17] Notice that the standard deviation of $\ln(Y_t/C_tH_t)$ (as well as W_t itself) is monotonically increasing in the adjustment cost parameter, μ. The Hansen-Rogerson model counterfactually implies that the variance of $\ln(Y_t/C_tH_t)$ should be zero. As figure 1 reveals, the version of the nested model that comes closest to reproducing the variability of $\ln(Y_t/C_tH_t)$ is $\mu = \infty$, that is, our labor hoarding model. Indeed, even that version of the model does not generate sufficient volatility in effort to render it consistent with this dimension of the data. Allowing firms more flexibility to adjust employment at

[17] This figure was generated using the parameter values contained in col. 3 of table 1.

time t (i.e., moving closer to the Hansen-Rogerson model) would only exacerbate this shortcoming.[18]

VI. Conclusion

This paper investigated the sensitivity of Solow residual based measures of technology shocks to labor hoarding type behavior. In addition, we analyzed claims in the literature that technology shocks account for most of the volatility in postwar aggregate U.S. output. Our results are supportive of the view that a significant proportion of movements in the Solow residual are artifacts of labor hoarding type behavior. In addition, they strongly suggest that existing RBC studies substantially overstate the extent to which technology shocks account for the volatility of postwar aggregate U.S. output.

Appendix

In this Appendix we accomplish three tasks. First, we describe the GMM procedures used to estimate the vectors $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$. Second, we discuss the test statistics J and $\tilde{J}$. Third, we discuss how measurement error was incorporated into the analysis.

For all models the following moment conditions were used to estimate $\boldsymbol{\Psi}_1$:

$$E[\ln(Y_t) - \ln(Y) - \ln(\gamma)t] = 0; \tag{A1}$$

$$\frac{E[\ln(Y_t) - \ln(Y) - \ln(\gamma)t]t}{T} = 0; \tag{A2}$$

$$E[\ln(G_t) - \ln(g) - \ln(\gamma_g)t] = 0; \tag{A3}$$

$$\frac{E[\ln(G_t) - \ln(g) - \ln(\gamma_g)t]t}{T} = 0; \tag{A4}$$

$$E[[\ln(g_t) - (1 - \rho_g)\ln(g) - \rho_g \ln(g_{t-1})]\ln(g_{t-1})] = 0; \tag{A5}$$

$$E[[\ln(g_t) - (1 - \rho_g)\ln(g) - \rho_g \ln(g_{t-1})]^2 - \sigma^2_{\epsilon_g}] = 0; \tag{A6}$$

$$E\left[\delta - \left(1 + \frac{I_t}{K_t} - \frac{K_{t+1}}{K_t}\right)\right] = 0; \tag{A7}$$

$$E\left[\beta^{-1} - \frac{\{[(1-\alpha)Y_{t+1}/K_{t+1}] + (1-\delta)\}C_t}{C_{t+1}}\right] = 0; \tag{A8}$$

$$E[[\ln(A_t) - (1 - \rho_a)\ln(A) - \rho_a \ln(A_{t-1})]\ln(A_{t-1})] = 0; \tag{A9}$$

$$E[[\ln(A_t) - (1 - \rho_a)\ln(A) - \rho_a \ln(A_{t-1})]^2 - \sigma^2_{\epsilon_a}] = 0. \tag{A10}$$

[18] The time series of Y_t/C_tH_t actually exhibits a negative trend. This trend behavior has the effect of magnifying estimates of the series' standard deviation. To correct for trend behavior, we also estimated the standard deviation of the Hodrick-Prescott filtered version of Y_t/C_tH_t and obtained a value equal to .0127. Evidently, the failure of the labor hoarding model on this dimension of the data is substantially overstated by using the nondetrended time series on $\ln(Y_t/C_tH_t)$.

Equations (A1)–(A7) and (A9)–(A10) follow directly from assumptions in the text. Equation (A8) follows from applying the law of iterated expectations to the planner's Euler equation for capital.

The previous equations were augmented by an additional unconditional moment condition to estimate θ. For the Hansen-Rogerson model, we used the unconditional expected version of the planner's Euler equation for employment:

$$E\left[\tilde{\theta} - \frac{\alpha Y_t}{C_t H_t}\right] = 0. \tag{A11}$$

For the labor hoarding II case, we used the unconditional expected version of the planner's Euler equation for effort,

$$E\left[\theta(T - \xi - W_t f)^{-1} - \frac{\alpha Y_t}{C_t W_t H_t}\right] = 0, \tag{A12}$$

to estimate θ. Relation (A12) cannot be used for this purpose in the labor hoarding I case since it is used to construct the time series on effort. Instead, we estimated θ by imposing the condition

$$E[H_t - EH] = 0, \tag{A13}$$

where EH is the unconditional mean of hours worked implied by the model.

In overidentified estimators we impose (16), which can be written as

$$\begin{aligned} E[[\ln(A_t) - (1 - \rho_a)\ln(A) - \rho_a \ln(A_{t-1})] \\ \times [\ln(g_t) - (1 - \rho_g)\ln(g) - \rho_g \ln(g_{t-1})]] = 0. \end{aligned} \tag{A14}$$

The unconditional moment conditions underlying our estimator for the version of Ψ_2 defined in (17) are given by

$$E\left[y_t^2 \frac{\sigma_x^2}{\sigma_y^2} - x_t^2\right] = 0 \quad \text{for } x = c, i, g; \tag{A15}$$

$$E[h_t^2 - \sigma_h^2] = 0; \tag{A16}$$

$$E\left[\mathrm{APL}_t^2 \frac{\sigma_h^2}{\sigma_{\mathrm{APL}}^2} - h_t^2\right] = 0; \tag{A17}$$

$$E[y_t^2 - \sigma_y^2] = 0. \tag{A18}$$

Here we have used the fact that Hodrick-Prescott filtered data have zero mean by construction. The unconditional moment conditions underlying our estimator of the version of Ψ_2 defined in (18) are given by

$$E[\Delta \ln(\hat{S}_t)\Delta \ln(\hat{g}_t) - [\Delta \ln(\hat{g}_t)]^2 b_g] = 0, \tag{A19}$$

$$E[\epsilon_{st} \ln(\hat{S}_{t-1})] = 0, \tag{A20}$$

and

$$E[\epsilon_{st}\epsilon_{gt} - b(\epsilon_s, \epsilon_g)\epsilon_{gt}^2] = 0, \tag{A21}$$

where $\epsilon_{st} = \ln(\hat{S}_t) - \rho_s \ln(\hat{S}_{t-1})$. Here the log of a hatted variable denotes the log of that variable minus the mean of the log of that variable.

To define our joint estimator of Ψ_1 and Ψ_2, consider the following generic representation of our moment conditions:

$$E[\mathbf{M}_t(\boldsymbol{\Psi}^0)] = 0, \quad \forall t \geq 0,$$

where $\boldsymbol{\Psi}^0$ is the true value of $(\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2)$. Here $\mathbf{M}_t(\cdot)$ denotes the vector of moment conditions underlying our estimator of $\boldsymbol{\Psi}^0$ before expectations are applied. Let g_T denote the vector valued function

$$g_T(\boldsymbol{\Psi}) = \frac{1}{T}\sum_{t=0}^{T} \mathbf{M}_t(\boldsymbol{\Psi}).$$

Under the conditions set forth in Hansen (1982), $\boldsymbol{\Psi}^0$ can be consistently estimated by choosing the value of $\boldsymbol{\Psi}$, say $\boldsymbol{\Psi}_T$, that minimizes the quadratic form $J_T = Tg_T(\boldsymbol{\Psi})S_T^{-1}g_T(\boldsymbol{\Psi})$. Here S_T is a consistent estimate of the spectral density matrix of $\mathbf{M}_t(\boldsymbol{\Psi}^0)$ at frequency zero.[19]

A consistent estimator of the variance-covariance matrix of $\boldsymbol{\Psi}_T$ is given by

$$\text{var}(\boldsymbol{\Psi}_T) = \frac{1}{T}[D_T' S_T^{-1} D_T]^{-1},$$

where $D_T = \partial g_T(\boldsymbol{\Psi}_T)/\partial \boldsymbol{\Psi}'$. For exactly identified systems, the minimized value of J_T is zero. When (A14) is imposed, we can test the overidentified system by using the fact, established by Hansen (1982), that the minimized value of J_T is asymptotically distributed as a χ^2 random variable with one degree of freedom.

Let $\boldsymbol{\omega}$ denote the $q \times 1$ vector of moments to be studied, $q > 0$. Let the value of $\boldsymbol{\omega}$ implied by the model by given by $\Pi(\boldsymbol{\Psi}_1^0)$, where Π: $R^{11} \rightarrow R^q$. The value of $\boldsymbol{\omega}$ in the data-generating process is given by $\mathbf{B}\boldsymbol{\Psi}^0$, where $\mathbf{B}$ is a conformable matrix of zeros and ones. We are interested in studying hypotheses of the form H_0: $F(\boldsymbol{\Psi}^0) = \Pi(\boldsymbol{\Psi}_1^0) - \mathbf{B}\boldsymbol{\Psi}^0 = 0$. Christiano and Eichenbaum (1992) show that

$$\text{var}[F(\boldsymbol{\Psi}_T)] = T[F'(\boldsymbol{\Psi}_T)][\text{var}(\boldsymbol{\Psi}_T)][F'(\boldsymbol{\Psi}_T)]'$$

and the test statistic

$$\tilde{J} = F(\boldsymbol{\Psi}_T)'\,\text{var}[F(\boldsymbol{\Psi}_T)]^{-1}F(\boldsymbol{\Psi}_T)$$

is asymptotically distributed as a χ^2 random variable with q degrees of freedom.

We now show how the unconditional moment restrictions implied by the labor hoarding model must be modified to take account of measurement error in hours worked. Proceeding as in Prescott (1986), we assume that the two measures of hours worked at our disposal are related to true hours in the following way:

$$\begin{aligned} \ln(H_t^e) &= \ln(H_t^*) + v_t^e, \\ \ln(H_t^h) &= \ln(H_t^*) + v_t^h, \end{aligned} \tag{A22}$$

where H_t^* denotes true hours worked, H_t^e represents establishment hours worked, H_t^h represents Hansen's (1984) household hours, and v_t^e and v_t^h are independently and identically distributed, orthogonal to each other and to

[19] To construct our estimator S_T, we use the damped autocovariance estimator discussed in Eichenbaum and Hansen (1990). The reported results were calculated by truncating after four lags.

$\ln(H_t^*)$. Since we use household hours in our empirical work, we estimate $\sigma_{v^h}^2$ by adding the following moment condition to the GMM estimators described above:

$$E\{\sigma_{v^h}^2 - .5[\Delta \ln(H_t^h)]^2 + .5\Delta \ln(H_t^e)\Delta \ln(H_t^h)\} = 0. \quad \text{(A23)}$$

The moment conditions depending on our measures of hours must be modified to account for this measurement error. Asterisks are used to denote true values.

In labor hoarding I, effort equals

$$W_t = \frac{(T - \xi)\alpha Y_t}{\alpha f Y_t + \theta C_t H_t} = \frac{(T - \xi)\alpha Y_t}{\alpha f Y_t + \theta C_t H_t^* \exp(v_t^h)}.$$

Taking a first-order Taylor expansion of the logarithm of W_t around $v_t^h = 0$ yields

$$\ln(W_t) \approx \ln(W_t^*) - \frac{\theta C_t H_t^*}{\alpha f Y_t + \theta C_t H_t^*} v_t^h.$$

We make a further approximation that $\ln(W_t) = \ln(W_t^*) - \phi v_t^h$, where ϕ is the steady-state value of $\theta C_t H_t^*/(\alpha f Y_t + \theta C_t H_t^*)$ implied by our model. Equation (3) implies

$$\begin{aligned}\ln(A_t) &= \ln(Y_t) - \alpha(\ln\gamma)t - (1 - \alpha)\ln(K_t) - \alpha\ln(H_t) - \alpha\ln(W_t)\\ &= \ln(A_t^*) - \alpha(1 - \phi)v_t^h.\end{aligned}$$

Therefore, (A9) must be modified as

$$E[[\ln(A_t) - (1 - \rho_a)\ln(A) - \rho_a\ln(A_{t-1})]\ln(A_{t-1})] = -\rho_a\alpha^2(1 - \phi)^2\sigma_{v^h}^2.$$

Similarly, (A10) must be modified since

$$\begin{aligned}&E[[\ln(A_t) - (1 - \rho_a)\ln(A) - \rho_a\ln(A_{t-1})]^2\\ &\quad - \sigma_{\epsilon_a}^2 - (1 + \rho_a^2)\alpha^2(1 - \phi)^2\sigma_{v^h}^2] = 0.\end{aligned}$$

In labor hoarding II, A_t is related to A_t^* according to

$$\begin{aligned}\ln(A_t) &= \ln(Y_t) - \alpha(\ln\gamma)t - (1 - \alpha)\ln(K_t) - \alpha\ln(H_t) - \alpha\ln(W_t)\\ &= \ln(A_t^*) - \phi v_t^h,\end{aligned}$$

where $\phi = \alpha(1 + \pi_2)/(1 + \alpha\pi_3)$. Furthermore, $\ln(W_t) = \ln(W_t^*) + (\pi_2 - \phi\pi_3)v_t^h$. In this case, equations (A9) and (A10) must be modified as

$$E[[\ln(A_t) - (1 - \rho_a)\ln(A) - \rho_a\ln(A_{t-1})]\ln(A_{t-1}) + \rho_a\phi^2\sigma_{v^h}^2] = 0,$$

$$E[[\ln(A_t) - (1 - \rho_a)\ln(A) - \rho_a\ln(A_{t-1})]^2 - \sigma_{\epsilon_a}^2 - (1 + \rho_a^2)\phi^2\sigma_{v^h}^2] = 0.$$

The equations used to identify θ need not be modified to take account of measurement error, since the terms involving measurement error in the first-order Taylor series expansions have mean zero. For the Hansen-Rogerson model, the same modifications to (A9) and (A10) used in labor hoarding II are used again, except that α is substituted for ϕ wherever it appears.

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