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Lawrence J. Christiano; Martin Eichenbaum

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Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations

By LAWRENCE J. CHRISTIANO AND MARTIN EICHENBAUM*

Hours worked and the return to working are weakly correlated. Traditionally, the ability to account for this fact has been a litmus test for macroeconomic models. Existing real-business-cycle models fail this test dramatically. We modify prototypical real-business-cycle models by allowing government consumption shocks to influence labor-market dynamics. This modification can, in principle, bring the models into closer conformity with the data. Our empirical results indicate that it does. (*JEL* E32, C12, C52, C13, C51)

In this paper, we assess the quantitative implications of existing real-business-cycle (RBC) models for the time-series properties of average productivity and hours worked. We find that the single most salient shortcoming of existing RBC models lies in their predictions for the correlation between these variables. Existing RBC models predict that this correlation is well in excess of 0.9, whereas the actual correlation is much closer to zero.[1] This shortcoming leads us to add to the RBC framework aggregate demand shocks that arise from stochastic movements in government consumption. According to our empirical results, this change substantially improves the models' empirical performance.

The ability to account for the observed correlation between the return to working and the number of hours worked has traditionally been a litmus test for aggregate economic models. Thomas J. Sargent (1987 p. 468), for example, states that one of the primary empirical patterns casting doubt on the classical and Keynesian models has been the observation by John T. Dunlop (1938) and Lorie Tarshis (1939) "alleging the failure of real wages to move countercyclically." The classical and Keynesian models share the assumption that real wages and hours worked lie on a stable, downward-sloped marginal productivity-of-labor curve.[2] Consequently, they both counterfactually predict a strong negative correlation between real wages and hours worked. Modern versions of what Sargent (1987 p. 468) calls the "Dunlop–Tarshis observation" continue to play a central role in assessing the empirical plausibility of different business-cycle models.[3] In discussing Stanley Fischer's (1977)

*Christiano: Federal Reserve Bank of Minneapolis, Minneapolis, MN 55480; Eichenbaum: Northwestern University, Evanston, IL 60208, National Bureau of Economic Research, and Federal Reserve Bank of Chicago. This paper is a substantially revised version of NBER Working Paper No. 2700, "Is Theory Really Ahead of Measurement? Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations." We thank Rao Aiyagari, Paul Gomme, Finn Kydland, Ed Prescott, and Mark Watson for helpful conversations. Any views expressed here are ours and not necessarily those of any part of the Federal Reserve System.

[1]This finding is closely related to Bennett McCallum's (1989) observation that existing RBC models generate grossly counterfactual predictions for the correlation between average productivity and output.

[2]As John Maynard Keynes (1935 p. 17) says, "...I am not disputing this vital fact which the classical economists have (rightly) asserted as indefeasible. In a given state of organisation, equipment and technique, the real wage earned by a unit of labour has a unique (inverse) correlation with the volume of employment."

[3]For example, Robert J. Barro and Herschel I. Grossman (1971) cite the Dunlop–Tarshis observation to motivate their work on disequilibrium theories. Also, Edmund S. Phelps and Sidney G. Winter, Jr. (1970 p. 310) and Franco Modigliani (1977 p. 7) use this observation to motivate their work on noncompetitive approaches to macroeconomics. Finally, Robert E. Lucas, Jr. (1981 p. 13) cites the Dunlop–Tarshis observation in motivating his work on capacity and overtime.

sticky-wage business-cycle model, for example, Bennett McCallum (1989 p. 191) states that

> ...the main trouble with the Fischer model concerns its real wage behavior. In particular, to the extent that the model itself explains fluctuations in output and employment, these should be inversely related to real wage movements: output should be high, according to the model, when real wages are low. But in the actual U.S. economy there is no strong empirical relation of that type.

In remarks particularly relevant to RBC models, Robert E. Lucas (1981 p. 226) says that "observed real wages are not constant over the cycle, but neither do they exhibit consistent pro- or countercyclical tendencies. This suggests that any attempt to assign systematic real wage movements a central role in an explanation of business cycles is doomed to failure." Existing RBC models fall prey to this (less well-known) Lucas critique. Unlike the classical and Keynesian models, which understate the correlation between hours worked and the return to working, existing RBC models grossly overstate that correlation. According to existing RBC models, the only impulses generating fluctuations in aggregate employment are stochastic shifts in the marginal product of labor. Loosely speaking, the time series on hours worked and the return to working are modeled as the intersection of a stochastic labor-demand curve with a fixed labor-supply curve. Not surprisingly, therefore, these theories predict a strong positive correlation between hours worked and the return to working.[4]

Several strategies exist for modeling the observed weak correlation between measures of these variables. One is to consider models in which the return to working is unaffected by shocks to agents' environments, regardless of whether the shocks are to aggregate demand or to aggregate supply. Pursuing this strategy, Olivier Jean Blanchard and Stanley Fischer (1989 p. 372) argue that the key assumption of Keynesian macro models—nominal wage and price stickiness—is motivated by the view that aggregate demand shocks affect employment but not real wages. Another strategy is simply to abandon one-shock models of aggregate fluctuations and suppose that the business cycle is generated by a variety of impulses. Under these conditions, the Dunlop–Tarshis observation imposes no restrictions per se on the response of real wages to any particular type of shock. Given a specific structural model, however, it does impose restrictions on the relative frequency of different types of shocks. This suggests that one strategy for reconciling existing RBC models with the Dunlop–Tarshis observation is to find measurable economic impulses that shift the labor-supply function.[5] With different impulses shifting the labor-supply and labor-demand functions,

[4]Although Finn E. Kydland and Edward C. Prescott (1982) and Prescott (1986) never explicitly examine the hours/real-wage correlation implication of existing RBC models, Prescott (1986 p. 21) does implicitly acknowledge that the failure to account for the Dunlop–Tarshis observation is the key remaining deviation between economic theory and observations: "The key deviation is that the empirical labor elasticity of output is less than predicted by theory." To see the connections, denote the empirical labor elasticity by η. By definition, $\eta \equiv \text{corr}(y,n)\sigma_y/\sigma_n$, where $\text{corr}(i,j)$ is the correlation between i and j, σ_i is the standard deviation of i, y is the logarithm of detrended output, and n is the logarithm of hours. Simple arithmetic yields $\text{corr}(y-n,n) = [\eta - 1](\sigma_n/\sigma_{y-n})$. If, as Prescott claims, RBC models do well at reproducing the empirical estimates of σ_n/σ_{y-n}, then saying that the models overstate η is equivalent to saying that they overstate $\text{corr}(y-n,n)$. In Prescott's model and with his assumed market structure, $\text{corr}(y-n,n)$ is exactly the same as the correlation between real wages and hours worked. (Also, under log detrending, $y-n$ is log detrended productivity.)

[5]An alternative strategy is pursued by Valerie R. Bencivenga (1992), who allows for shocks to labor suppliers' preferences. Matthew D. Shapiro and Mark W. Watson (1988) also allow for unobservable shocks to the labor-supply function. Jess Benhabib et al. (1991) and Jeremy Greenwood and Zvi Hercowitz (1991) explore the role of shocks to the home production technology.

there is no a priori reason for hours worked to be correlated in any particular way with the return to working.

Candidates for such shocks include tax rate changes, innovations to the money supply, demographic changes in the labor force, and shocks to government spending. We focus on the last of these. By ruling out any role for government-consumption shocks in labor-market dynamics, existing RBC models implicitly assume that public and private consumption have the same impact on the marginal utility of private spending. Robert J. Barro (1981, 1987) and David Alan Aschauer (1985) argue that if $1 of additional public consumption drives down the marginal utility of private consumption by less than does $1 of additional private consumption, then positive shocks to government consumption in effect shift the labor-supply curve outward. With diminishing labor productivity, but without technology shocks, such impulses will generate a negative correlation between hours worked and the return to working in RBC models.

In our empirical work, we measure the return to working by the average productivity of labor rather than real wages. We do this for both empirical and theoretical reasons. From an empirical point of view, our results are not very sensitive to whether the return to working is measured by real wages or average productivity: Neither displays a strong positive correlation with hours worked, so it seems appropriate to refer to the low correlation between the return to working and hours worked as the *Dunlop–Tarshis observation*, regardless of whether the return to working is measured by the real wage or average productivity. From a theoretical point of view, a variety of ways exist to support the quantity allocations emerging from RBC models. By using average productivity as our measure of the return to working, we avoid imposing the assumption that the market structure is one in which real wages are equated to the marginal product of labor on a period-by-period basis. Also, existing parameterizations of RBC models imply that marginal and average productivity of labor are proportional to each other. For the calculations we perform, the two are interchangeable.

Our empirical results show that incorporating government into the analysis substantially improves the RBC models' performance. Interestingly, the impact of this perturbation is about as large as allowing for nonconvexities in labor supply of the type stressed by Gary D. Hansen (1985) and Richard Rogerson (1988). Once government is incorporated into the analysis, we cannot reject the hypothesis that a version of the Hansen-Rogerson indivisible-labor model is consistent with both the observed correlation between hours worked and average productivity and the observed volatility of hours worked relative to average productivity. This is not true if government is excluded from the analysis.

The paper is organized as follows. In Section I, we describe a general equilibrium model that nests as special cases a variety of existing RBC models. In Section II, we present our econometric methodology for estimating and evaluating the empirical performance of the model. In Section III, we present our empirical results. In Section IV, we offer some concluding remarks.

I. Two Prototypical Real-Business-Cycle Models

In this section, we present two prototypical real-business-cycle models. One is a stochastic version of the one-sector growth model considered by Kydland and Prescott (1980 p. 174). The other is a version of the model economy considered by Hansen (1985) in which labor supply is indivisible. In both of our models, we relax the assumption implicit in existing RBC models that public and private spending have identical effects on the marginal utility of private consumption.

We make the standard RBC assumption that the time series on the beginning-of-period-t per capita stock of capital (k_t), private time-t consumption (c_t^p) and hours worked at time t (n_t) correspond to the solution of a social-planning problem which can be decentralized as a Pareto-optimal competitive equilibrium. The following

problem nests both our models as special cases. Let N be a positive scalar that denotes the time-t endowment of the representative consumer, and let γ be a positive scalar. The social planner ranks streams of consumption services (c_t), leisure $(N-n_t)$, and publicly provided goods and services (g_t) according to the criterion function

$$(1) \quad E_0 \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \gamma V(N-n_t)\}.$$

Following Barro (1981, 1987), Roger C. Kormendi (1983), and Aschauer (1985), we suppose that consumption services are related to private and public consumption as follows:

$$(2) \quad c_t = c_t^p + \alpha g_t$$

where α is a parameter that governs the sign and magnitude of the derivative of the marginal utility of c_t^p with respect to g_t.[6] Throughout, we assume that agents view g_t as an uncontrollable stochastic process. In addition, we suppose that g_t does not depend on the current or past values of the endogenous variables in the model.[7]

We consider two specifications for the function $V(\cdot)$. In the *divisible-labor model*, $V(\cdot)$ is given by

$$(3) \quad V(N-n_t) = \ln(N-n_t) \qquad \text{for all } t.$$

In the *indivisible-labor model*, $V(\cdot)$ is given by

$$(4) \quad V(N-n_t) = N-n_t \qquad \text{for all } t.$$

This specification can be interpreted in at least two ways. One is that the specification simply reflects the assumption that individual utility functions are linear in leisure. The other interpretation builds on the assumption that labor supply is indivisible. Under this second interpretation, individuals can either work some positive number of hours or not work at all. Assuming that agents' utility functions are separable across consumption and leisure, Rogerson (1988) shows that a market structure in which individuals choose lotteries rather than hours worked will support a Pareto-optimal allocation of consumption and leisure. The lottery determines whether individuals work or not. Under this interpretation, (4) represents a reduced-form preference-ordering that can be used to derive the Pareto-optimal allocation by solving a fictitious social-planning problem. This is the specification used by Hansen (1985).

Per capita output y_t is produced using the Cobb-Douglas production function

$$(5) \quad y_t = (z_t n_t)^{(1-\theta)} k_t^{\theta}$$

where $0 < \theta < 1$ and z_t is an aggregate shock to technology that has the time-series representation

$$(6) \quad z_t = z_{t-1} \exp(\lambda_t).$$

Here λ_t is a serially uncorrelated independent and identically distributed process with mean λ and standard error σ_λ. The aggregate resource constraint is given by

$$(7) \quad c_t^p + g_t + k_{t+1} - (1-\delta)k_t \leq y_t.$$

That is, per capita consumption and investment cannot exceed per capita output.

At time 0, the social planner chooses contingency plans for $\{c_t^p, k_{t+1}, n_t: t \geq 0\}$ to maximize (1) subject to (3) or (4), (5)–(7), k_0, and a law of motion for g_t. Because of the nonsatiation assumption implicit in (1),

[6] We can generalize the criterion function (1) by writing it as $\ln(c_t) + \gamma V(N-n_t) + \phi(g_t)$, where $\phi(\cdot)$ is some positive concave function. As long as g_t is modeled as an exogenous stochastic process, the presence of such a term has no impact on the competitive equilibrium. However, the presence of $\phi(g_t) > 0$ means that agents do not necessarily feel worse off when g_t is increased. The fact that we have set $\phi(\cdot) \equiv 0$ reflects our desire to minimize notation, not the view that the optimal level of g_t is zero.

[7] Under this assumption, g_t is isomorphic to an exogenous shock to preferences and endowments. Consequently, existing theorems which establish that the competitive equilibrium and the social-planning problem coincide are applicable.

we can, without loss of generality, impose strict equality in (7). Substituting (2), (5), and this version of (7) into (1), we obtain the following *social-planning problem*: maximize

$$(8)\quad E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln\Big[(z_t n_t)^{(1-\theta)} k_t^{\theta} + (1-\delta)k_t - k_{t+1} + (\alpha - 1) g_t \Big] + \gamma V(N - n_t) \Big\}$$

subject to k_0, a law of motion for g_t, and $V(\cdot)$ given by either (3) or (4), by choice of contingency plans for $\{k_{t+1}, n_t\colon t \geq 0\}$.

It is convenient to represent the social-planning problem (8) in a way such that all of the planner's decision variables converge in nonstochastic steady state. To that end, we define the following detrended variables:

$$(9)\quad \bar{k}_{t+1} = k_{t+1}/z_t \qquad \bar{y}_t = y_t/z_t \qquad \bar{c}_t = c_t/z_t \qquad \bar{g}_t = g_t/z_t.$$

To complete our specification of agents' environment, we assume that $\bar{g}_t$ evolves according to

$$(10)\quad \ln(\bar{g}_t) = (1-\rho)\ln(\bar{g}) + \rho \ln(\bar{g}_{t-1}) + \mu_t$$

where $\ln(\bar{g})$ is the mean of $\ln(\bar{g}_t)$, $|\rho| < 1$, and μ_t is the innovation in $\ln(\bar{g}_t)$ with standard deviation σ_μ. Notice that g_t has two components, z_t and $\bar{g}_t$. Movements in z_t produce permanent changes in the level of government consumption, whereas movements in $\bar{g}_t$ produce temporary changes in g_t. With this specification, the factors giving rise to permanent shifts in government consumption are the same as those that permanently enhance the economy's productive ability.

Substituting (9) into (8), we obtain the criterion function:

$$(11)\quad \kappa + E_0 \sum_{t=0}^{\infty} \beta^t r\left(n_t, \bar{k}_t, \bar{k}_{t+1}, \bar{g}_t, \lambda_t\right)$$

where

$$(12)\quad r\left(n_t, \bar{k}_t, \bar{k}_{t+1}, \bar{g}_t, \lambda_t\right) = \Big\{ \ln\Big[n_t^{(1-\theta)} \bar{k}_t^{\theta} \exp(-\theta\lambda_t) + \exp(-\lambda_t)(1-\delta)\bar{k}_t - \bar{k}_{t+1} + (\alpha - 1)\bar{g}_t \Big] + \gamma V(N - n_t) \Big\}$$

and where $\kappa = E_0 \sum_{t=0}^{\infty} \beta^t \ln(z_t)$ and $V(\cdot)$ is given by either (3) or (4). Consequently, the original planning problem is equivalent to the problem of maximizing (11), subject to $\bar{k}_0$, (10), and (12), and $V(\cdot)$ is given by either (3) or (4). Since κ is beyond the planner's control, it can be disregarded in solving the planner's problem.

The only case in which an analytical solution for this problem is possible occurs when $\alpha = \delta = 1$ and the function $V(\cdot)$ is given by (3). John B. Long, Jr., and Charles I. Plosser (1983) provide one analysis of this case. Analytical solutions are not available for general values of α and δ. We use Christiano's (1988) log-linear modification of the procedure used by Kydland and Prescott (1982) to obtain an approximate solution to our social-planning problem. In particular, we approximate the optimal decision rules with the solution to the linear-quadratic problem obtained when the function r in (12) is replaced by a function R, which is quadratic in $\ln(n_t)$, $\ln(\bar{k}_t)$, $\ln(\bar{k}_{t+1})$, $\ln(\bar{g}_t)$, and λ_t. The function R is the second-order Taylor expansion of $r[\exp(A_1), \exp(A_2), \exp(A_3), \exp(A_4), A_5]$ about the point

$$[A_1, A_2, A_3, A_4, A_5] = \left[\ln(n), \ln(\bar{k}), \ln(\bar{k}), \ln(\bar{g}), \lambda\right].$$

Here n and $\bar{k}$ denote the steady-state values of n_t and $\bar{k}_t$ in the nonstochastic version of (11) obtained by setting $\sigma_\lambda = \sigma_\mu = 0$.

Results in Christiano (1988) establish that the decision rules which solve this problem are of the form

$$(13)\quad \bar{k}_{t+1} = \bar{k}\left(\bar{k}_t/\bar{k}\right)^{r_k}\left(\bar{g}_t/\bar{g}\right)^{d_k} \times \exp\left[e_k(\lambda_t - \lambda)\right]$$

and

$$(14)\quad n_t = n\left(\bar{k}_t/\bar{k}\right)^{r_n}\left(\bar{g}_t/\bar{g}\right)^{d_n}\exp\left[e_n(\lambda_t - \lambda)\right].$$

In (13) and (14), r_k, d_k, e_k, r_n, d_n, and e_n are scalar functions of the model's underlying structural parameters.[8]

To gain intuition for the role of $\bar{g}_t$ in aggregate labor-market fluctuations, it is useful to discuss the impact of three key parameters (α, ρ, and γ) on the equilibrium response of n_t to $\bar{g}_t$. This response is governed by the coefficient d_n.

First, notice that when $\alpha = 1$ the only way c_t^p and g_t enter into the social planner's preferences and constraints is through their sum, $c_t^p + g_t$. Thus, exogenous shocks to g_t induce one-for-one offsetting shocks in c_t^p, leaving other variables like y_t, k_{t+1}, and n_t unaffected. This implies that the coefficients d_n and d_k in the planner's decision rules for k_{t+1} and n_t both equal zero. Consequently, the absence of a role for g_t in existing RBC models can be interpreted as reflecting the assumption that $\alpha = 1$.

Second, consider what happens when $\alpha < 1$. The limiting case of $\alpha = 0$ is particularly useful for gaining intuition. Government consumption now is formally equivalent to a pure resource drain on the economy; agents respond to an increase in government consumption as if they had suffered a reduction in their wealth. (As footnote 6 indicates, this does not imply that they have suffered a reduction in utility.) The coefficient d_n is positive, since we assume that leisure is a normal good. That is, increases in $\bar{g}_t$ are associated with increases in n_t and decreases in y_t/n_t. Continuity suggests that d_n is decreasing in α. The same logic suggests that d_n is an increasing function of ρ, since the wealth effect of a given shock to $\bar{g}_t$ is increasing in ρ. For a formal analysis of the effects of government consumption in a more general environment than the one considered here, see S. Rao Aiyagari et al. (1990).

Finally, consider the impact of γ on aggregate labor-market fluctuations. In several experiments, we found that e_n and d_n were increasing in γ (for details, see Christiano and Eichenbaum [1990a]). To gain intuition into this result, think of a version of the divisible-labor model in which the gross investment decision rule is fixed exogenously. In this simpler model economy, labor-market equilibrium is the result of the intersection of static labor-supply and labor-demand curves. Given our assumptions regarding the utility function, the response of labor supply to a change in the return to working is an increasing function of γ; that is, the labor-supply curve becomes flatter as γ increases. By itself, this makes the equilibrium response of n_t to λ_t (which shifts the labor-demand curve) an increasing function of γ. This relationship is consistent with the finding that e_n is increasing in γ in our model. With respect to d_n, it is straightforward to show that, in the static framework, the extent of the shift in the labor-supply curve induced by a change in $\bar{g}_t$ is also an increasing function of γ. This is also consistent with the finding that d_n is an increasing function of γ in our model.

That e_n and d_n are increasing in γ leads us to expect that the volatility of hours worked will also be an increasing function of γ. However, we cannot say a priori what impact larger values of γ will have on the Dunlop–Tarshis correlation, because larger values of e_n drive that correlation up, but larger values of d_n drive it down.

[8] Christiano (1987a, 1988 footnotes 9, 18) discusses the different properties of the log-linear approximation that we use here and linear approximations of the sort used by Kydland and Prescott (1982).

II. Econometric Methodology

In this section, we describe three things: our strategy for estimating the structural parameters of the model and various second moments of the data, our method for evaluating the model's implications for aggregate labor-market fluctuations, and the data used in our empirical analysis. While similar in spirit, our empirical methodology is quite different from the methods typically used to evaluate RBC models. Much of the existing RBC literature makes little use of formal econometric methods, either when model parameter values are selected or when the fully parameterized model is compared with the data. Instead, the RBC literature tends to use a variety of informal techniques, often referred to as *calibration*. In contrast, we use a version of Lars Peter Hansen's (1982) generalized method-of-moments (GMM) procedure at both stages of the analysis. Our estimation criterion is set up so that, in effect, estimated parameter values equate model and sample first moments of the data. It turns out that these values are very similar to the values used in existing RBC studies. An important advantage of our GMM procedures, however, is that they let us quantify the degree of uncertainty in our estimates of the model's parameters. This turns out to be an important ingredient of our model-evaluation techniques.

A. *Estimation*

Now we will describe our estimation strategy. The parameters of interest can be divided into two groups. Let Ψ_1 denote the model's eight structural parameters:

$$(15) \qquad \Psi_1 = \{\delta, \theta, \gamma, \rho, \bar{g}, \sigma_\mu, \lambda, \sigma_\lambda\}.$$

The parameters N, β, and α were not estimated. Instead, we fixed N at 1,369 hours per quarter and set the parameter β so as to imply a 3-percent annual subjective discount rate; that is, $\beta = (1.03)^{-0.25}$. Two alternative values of α were considered: $\alpha = 0$ and $\alpha = 1$.

Given estimated values of $\Psi_1, \hat{\Psi}_{1,T}$, and distribution assumptions on μ_t and λ_t, our model provides a complete description of the data-generating process. (Here T denotes the number of observations in our sample.) This can be used to compute the second moments of all the variables of the model. Suppose, for the time being, that we can abstract from sampling uncertainty in $\hat{\Psi}_{1,T}$, say, because we have a large data sample. Then the second moments implied by $\hat{\Psi}_{1,T}$ will coincide with the second moments of the stochastic process generating the data only if the model has been specified correctly.

This observation motivates our strategy for assessing the empirical plausibility of the model. First we calculate selected second moments of the data using our model evaluated at $\hat{\Psi}_{1,T}$. Then we estimate the same second moments directly, without using the model. Our test then compares these two sets of second moments and determines whether the differences between them can be accounted for by sampling variation under the null hypothesis that the model is correctly specified.

To this end, it is useful to define Ψ_2 to be various second moments of the data. Our measures of c_t^p, dk_t, k_t, y_t, $(y/n)_t$, and g_t all display marked trends, so some stationarity-inducing transformation of the data must be adopted for second moments to be well defined. (Here dk_t denotes gross investment.) The transformation we used corresponds to the Hodrick and Prescott (HP) detrending procedure discussed by Robert J. Hodrick and Prescott (1980) and Prescott (1986). We used the HP transformation because many researchers, especially Kydland and Prescott (1982, 1988), G. Hansen (1985), and Prescott (1986), have used it to investigate RBC models. Also, according to our model, the logarithms of c_t^p, dk_t, k_t, y_t, $(y/n)_t$, and g_t are all difference-stationary stochastic processes. That the HP filter is a stationarity-inducing transformation for such processes follows directly from results of Robert G. King and Sergio T. Rebelo (1988). We also used the first-difference filter in our analysis. Since the results are not substantially different from those reported

here, we refer the reader to Christiano and Eichenbaum (1990a) for details. The parameters in Ψ_2 are

$$(16)\quad \Psi_2 = \{\sigma_{c^p}/\sigma_y, \sigma_{dk}/\sigma_y, \sigma_n, \sigma_n/\sigma_{y/n}, \sigma_g/\sigma_y, \mathrm{corr}(y/n, n)\}$$

where σ_x denotes the standard deviation of the variable x, with $x = \{c^p, y, dk, n, y/n, g\}$, and $\mathrm{corr}(y/n, n)$ denotes the correlation between y/n and n.

1. *The Unconditional Moments Underlying Our Estimator of* Ψ_1.—The procedure we used to estimate the elements of Ψ_1 can be described as follows. Our estimator of δ is, roughly, the rate of depreciation of capital implicit in the empirical capital-stock and investment series. The estimators of θ and γ are designed to allow the model to reproduce the average value of the capital:output ratio and hours worked observed in the data. The point estimates of ρ, $\bar{g}$, and σ_μ are obtained by applying ordinary least squares to data on g_t/z_t, where z_t is constructed using the estimated value of θ. Finally, our point estimates of λ and σ_λ are the mean growth rate of output and the standard deviation of the growth rate of z_t, respectively. We map these estimators into a GMM framework to get an estimate of the sampling distribution of our estimator of Ψ_1. We need that estimate for our diagnostic procedures.

To use GMM, we express the estimator of Ψ_1, $\hat{\Psi}_{1,T}$, as the solution to the sample analog of first-moment conditions. We now describe these conditions. According to our model, $\delta = 1 + (dk_t/k_t) - (k_{t+1}/k_t)$. Let δ^* denote the unconditional mean of the time series $[1 + (dk_t/k_t) - (k_{t+1}/k_t)]$; that is,

$$(17)\quad E\{\delta^* - [1 - (dk_t/k_t) - (k_{t+1}/k_t)]\} = 0.$$

We identify δ with a consistent estimate of the parameter δ^*.

The social planner's first-order necessary condition for capital accumulation requires that the time-t expected value of the marginal rate of substitution of goods in consumption equals the time-t expected value of the marginal return to physical investment in capital. Therefore,

$$(18)\quad E\{\beta^{-1} - [\theta(y_{t+1}/k_{t+1}) + 1 - \delta]c_t/c_{t+1}\} = 0.$$

This is the moment restriction that underlies our estimate of θ.

The first-order necessary condition for hours worked requires that, for all t, the marginal productivity of hours times the marginal utility of consumption equals the marginal disutility of working. This implies the condition $\gamma = (1-\theta)(y_t/n_t)/[c_t V'(N - n_t)]$ for all t. Let γ^* denote the unconditional expected value of the time series on the right side of that condition; that is,

$$(19)\quad E\{\gamma^* - (1-\theta)(y_t/n_t)/[c_t V'(N - n_t)]\} = 0.$$

We identify γ with a consistent estimate of the parameter γ^*.

Next, consider the random variable

$$\lambda_t = ln(z_t/z_{t-1}) = (1-\theta)^{-1}\Delta ln(y_t) - \Delta ln(n_t) - \theta(1-\theta)^{-1}\Delta ln(k_t).$$

Here Δ denotes the first-difference operator. Under the null hypothesis of balanced growth, $\lambda \equiv E\lambda_t$, the unconditional growth rate of output. Therefore,

$$(20)\quad E[\Delta \ln(y_t) - \lambda] = 0$$
$$E[(\lambda_t - \lambda)^2 - \sigma_\lambda^2] = 0.$$

The relations in (20) summarize the moment restrictions underlying our estimators of λ and σ_λ.

Our assumptions regarding the stochastic process generating government consumption imply the unconditional moment re-

strictions:

$$(21)\quad E\left[\ln(\bar{g}_t)-(1-\rho)\ln(\bar{g})-\rho\ln(\bar{g}_{t-1})\right]=0$$

$$E\left[\ln(\bar{g}_t)-(1-\rho)\ln(\bar{g})-\rho\ln(\bar{g}_{t-1})\right]\bar{g}_{t-1}=0$$

$$E\left\{\left[\ln(\bar{g}_t)-(1-\rho)\ln(\bar{g})-\rho\ln(\bar{g}_{t-1})\right]^2-\sigma_\mu^2\right\}=0.$$

These moment restrictions can be used to estimate ρ, $\bar{g}$, and σ_μ.

Equations (17)–(21) consist of eight unconditional moment restrictions involving the eight elements of Ψ_1. These can be summarized as

$$(22)\quad E\mathbf{H}_{1,t}(\Psi_1^0)=0 \qquad \text{for all } t\geq 0$$

where Ψ_1^0 is the true value of Ψ_1 and $\mathbf{H}_{1,t}(\Psi_1)$ is the 8×1 random vector which has as its elements the left sides of (17)–(21) before expectations are taken.

2. *The Unconditional Moments Underlying Our Estimator of* Ψ_2.—Our estimator of the elements of Ψ_2 coincides with standard second-moment estimators. We find it convenient to map these into the GMM framework. The first-moment conditions we use are

$$(23)\quad E\left\{y_t^2(\sigma_x/\sigma_y)^2-x_t^2\right\}=0$$

$$x_t=[c_t^p, dk_t, g_t]$$

$$E\left[n_t^2-\sigma_n^2\right]=0$$

$$E\left\{(y/n)_t^2(\sigma_n/\sigma_{y/n})^2-n_t^2\right\}=0$$

$$E\left\{\left[\sigma_n^2/(\sigma_n/\sigma_{y/n})\right]\mathrm{corr}(y/n,n)-(y/n)_t n_t\right\}=0.$$

In (23) we have used the fact that, by construction, HP-filtered data have a zero mean.

Equations (23) consist of six unconditional moment restrictions involving the six elements of Ψ_2. These restrictions can be summarized as

$$(24)\quad E\mathbf{H}_{2,t}(\Psi_2^0)=0 \qquad \text{for all } t\geq 0$$

where Ψ_2^0 is the true value of Ψ_2 and $\mathbf{H}_{2,t}(\Psi_2)$ is the 6×1 vector-valued function which has as its elements the left sides of (23) before expectations are taken.

In order to discuss our estimator, it is convenient to define the 14×1 parameter vector $\Psi=[\Psi_1\ \Psi_2]'$ and the 14×1 vector-valued function $\mathbf{H}_t=[\mathbf{H}_{1,t}'\ \mathbf{H}_{2,t}']'$. With this notation, the unconditional moment restrictions (22) and (24) can be written as

$$(25)\quad E\mathbf{H}_t(\Psi^0)=0 \qquad \text{for all } t\geq 0$$

where $\Psi^0=[\Psi_1^0\ \Psi_2^0]'$, the vector of true values of Ψ. Let $\mathbf{g}_T$ denote the 14×1 vector-valued function

$$(26)\quad \mathbf{g}_T(\Psi)=(1/T)\sum_{t=0}^{T}\mathbf{H}_t(\Psi).$$

Our model implies that $\mathbf{H}_t(\Psi^0)$ is a stationary and ergodic stochastic process. Since $\mathbf{g}_T(\cdot)$ has the same dimension as Ψ, it follows from L. Hansen (1982) that the estimator Ψ_T, defined by the condition $\mathbf{g}_T(\hat{\Psi}_T)=0$, is consistent for Ψ^0.

Let $\mathbf{D}_T$ denote the matrix of partial derivatives

$$(27)\quad \mathbf{D}_T=\frac{\partial\mathbf{g}_T(\Psi)}{\partial\Psi'}$$

evaluated at $\Psi=\Psi_T$. It then follows from results in L. Hansen (1982) that a consistent estimator of the variance-covariance matrix of $\hat{\Psi}_T$ is given by

$$(28)\quad \mathrm{Var}(\hat{\Psi}_T)=[\mathbf{D}_T]^{-1}\mathbf{S}_T[\mathbf{D}_T']^{-1}/T.$$

Here, $\mathbf{S}_T$ is a consistent estimator of the spectral density matrix of $\mathbf{H}_t(\Psi^0)$ at frequency zero.[9]

B. *Testing*

Now we describe how a Wald-type test statistic described in Eichenbaum et al. (1984) and Whitney K. Newey and Kenneth D. West (1987) can be used to assess formally the plausibility of the model's implications for subsets of the second moments of the data. Our empirical analysis concentrates on assessing the model's implications for the labor-market moments, $[\text{corr}(y/n), \sigma_n/\sigma_{y/n}]$.[10] Here, we describe our procedure for testing this set of moments, a procedure which can be used for any finite set of moments.

Given a set of values for Ψ_1, our model implies particular values for $[\text{corr}(y/n), \sigma_n/\sigma_{y/n}]$ in population. We represent this relationship by the function f that maps $\mathbb{R}^8$ into $\mathbb{R}^2$:

$$(29) \qquad f(\Psi_1) = [f_1(\Psi_1), f_2(\Psi_1)].$$

Here, $f_1(\Psi_1)$ and $f_2(\Psi_2)$ denote the model's implication for $\text{corr}(y/n, n)$ and $\sigma_n/\sigma_{y/n}$ in population, conditional on the model parameters, Ψ_1. The function $f(\cdot)$ is highly nonlinear in Ψ_1 and must be computed using numerical methods. We use the spectral technique described in Christiano and Eichenbaum (1990b).

Let $\mathbf{A}$ be the 2×14 matrix composed of zeros and ones with the property that

$$(30) \qquad \mathbf{A}\Psi = [\text{corr}(y/n, n), \sigma_n/\sigma_{y/n}]'$$

and let

$$(31) \qquad F(\Psi) = f(\Psi_1) - \mathbf{A}\Psi.$$

Under the null hypothesis that the model is correctly specified,

$$(32) \qquad F(\Psi^0) = 0.$$

If our data sample were large, then $\hat{\Psi}_T = \Psi^0$ and (32) could be tested by simply comparing $F(\hat{\Psi}_T)$ with a 2×1 vector of zeros. However, $F(\hat{\Psi}_T)$ need not be zero in a small sample, because of sampling uncertainty in $\hat{\Psi}_T$. To test (32), then, we need the distribution of $F(\hat{\Psi}_T)$ under the null hypothesis. Taking a first-order Taylor-series approximation of $F(\hat{\Psi}_T)$ about Ψ^0 yields

$$(33) \qquad F(\hat{\Psi}_T) \cong F(\Psi^0) + F'(\Psi^0)[\hat{\Psi}_T - \Psi^0]'.$$

It follows that a consistent estimator of the variance-covariance matrix of $F(\hat{\Psi}_T)$ is given by

$$(34) \qquad \text{Var}[F(\hat{\Psi}_T)] = [F'(\hat{\Psi}_T)] \times \text{Var}(\hat{\Psi}_T)[F'(\hat{\Psi}_T)]'.$$

An implication of results in Eichenbaum et al. (1984) and Newey and West (1987) is that the test statistic

$$(35) \qquad J = F(\hat{\Psi}_T)'\text{Var}[F(\hat{\Psi}_T)]^{-1}F(\hat{\Psi}_T)$$

[9] Let $\mathbf{S}_0 = \sum_{k=-\infty}^{\infty} E[\mathbf{H}_{t+k}(\Psi^0)][\mathbf{H}_t(\Psi^0)]'$ denote the spectral density matrix of $\mathbf{H}_t(\Psi^0)$ at frequency zero. Our estimator of $\mathbf{S}_0$, $\mathbf{S}_T$, uses the damped, truncated covariance estimator discussed by Eichenbaum and Hansen (1990). The results we report were calculated by truncating after six lags. Strictly speaking, HP-filtered data do not satisfy the Eichenbaum and Hansen (1990) assumption that $\mathbf{S}_0$ be nonsingular. This is because our model implies that data need to be differenced only once to induce stationarity, while the results of King and Rebelo (1988) show that the HP filter differences more than once. We think this is not a serious problem from the perspective of asymptotic-distribution theory. This is because our numerical results would have been essentially unchanged had we worked with a version of the HP filter in which the extra unit roots were replaced by roots arbitrarily close to 1. Then, the Eichenbaum and Hansen (1990) analysis would apply without caveat. What the small-sample properties are in the presence of unit roots in the data-generating process remains an open and interesting question.

[10] Our formal test does not include σ_n/σ_y because this is an exact function of $[\text{corr}(y/n, n), \sigma_n/\sigma_{y/n}]$. To see this, let $b = \sigma_n/\sigma_{y/n}$ and $c = \text{corr}(y/n, n)$. Then, after some algebraic manipulation,

$$\sigma_n/\sigma_y = b/(1+2cb+b^2)^{1/2}.$$

is asymptotically distributed as a chi-square random variable with two degrees of freedom. We used this fact to test the null hypothesis (32).

C. *The Data*

Next we describe the data used in our analysis. In all of our empirical work, private consumption, c_t^p, was measured as quarterly real expenditures on nondurable consumption goods plus services plus the imputed service flow from the stock of durable goods. The first two time series came from the U.S. Department of Commerce's *Survey of Current Business* (various issues). The third came from the data base documented in Flint Brayton and Eileen Mauskopf (1985). Government consumption, g_t, was measured by real government purchases of goods and services minus real investment of government (federal, state, and local).[11]

A measure of government investment was provided to us by John C. Musgrave of the U.S. Bureau of Economic Analysis. This measure is a revised and updated version of the measure discussed in Musgrave (1980). Gross investment, dk_t, was measured as private-sector fixed investment plus government fixed investment plus real expenditures on durable goods.

The capital-stock series, k_t, was chosen to match the investment series. Accordingly, we measured k_t as the sum of the stock of consumer durables, producer structures and equipment, government and private residential capital, and government nonresidential capital.

Gross output, y_t, was measured as c_t^p plus g_t plus dk_t plus time-t inventory investment. Given our consumption series, the difference between our measure of gross output and the one reported in the *Survey of Current Business* is that ours includes the imputed service flow from the stock of consumer durables but excludes net exports.

We used two different measures of hours worked and average productivity. Our first measure of hours worked corresponds to the one constructed by G. Hansen (1984) which is based on the household survey conducted by the U.S. Department of Labor. A corresponding measure of average productivity was constructed by dividing our measure of gross output by this measure of hours. For convenience, we refer to this measure of n_t and $(y/n)_t$ as *household* hours worked and *household* productivity.

A potential problem with our measure of household average productivity is that gross output covers more sectors than does the household hours data (for details, see appendix 1 of Christiano and Eichenbaum [1988]). In order to investigate the quantitative impact of this problem, we considered a second measure of hours worked and productivity which covers the same sectors: output per hour of all persons in the nonagricultural business sector (CITIBASE mnemonic LBOUTU) and per capita hours worked by wage and salary workers in private nonagricultural establishments as reported by the U.S. Department of Labor (Bureau of Labor Statistics, IDC mnemonic HRSPST). For convenience, we refer to this measure of n_t and $(y/n)_t$ as *establishment* hours worked and *establishment* productivity.

All data, except those for $(y/n)_t$, were converted to per capita terms using an efficiency-weighted measure of the population. The data cover the period from the third quarter of 1955 through the fourth quarter of 1983 (1955:3–1983:4) (for further details on the data, see Christiano [1987b, 1988]).

III. Empirical Results

In this section, we report our empirical results. Subsection A discusses the results obtained using the household data while Subsection B presents results based on the establishment data. In each case, our results

[11] It would be desirable to include in g_t a measure of the service flow from the stock of government-owned capital, since government capital is included in our measure of k_t. Unfortunately, we know of no existing measures of that service flow. This contrasts with household capital, for which there are estimates of the service flow from housing and the stock of consumer durables. The first is included in the official measure of consumption of services, and the second is reported by Brayton and Mauskopf (1985).

TABLE 1—MODEL PARAMETER ESTIMATES (AND STANDARD ERRORS) GENERATED BY THE HOUSEHOLD DATA SET

	Model			
	Without government ($\alpha = 1$)		With government ($\alpha = 0$)	
Parameter	Divisible labor	Indivisible labor	Divisible labor	Indivisible labor
N	1,369	1,369	1,369	1,369
δ	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)
β	$1.03^{-0.25}$	$1.03^{-0.25}$	$1.03^{-0.25}$	$1.03^{-0.25}$
θ	0.339 (0.006)	0.339 (0.006)	0.344 (0.006)	0.344 (0.006)
γ	2.99 (0.03)	0.00285 (0.00003)	3.92 (0.05)	0.00374 (0.00005)
λ	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)
σ_ε	0.018 (0.001)	0.018 (0.001)	0.018 (0.001)	0.018 (0.001)
$\bar{g}$	186.0 (10.74)	186.0 (10.74)	190.8 (7.09)	190.8 (7.09)
ρ	0.96 (0.028)	0.96 (0.028)	0.96 (0.029)	0.96 (0.029)
σ_μ	0.020 (0.001)	0.020 (0.001)	0.021 (0.001)	0.021 (0.001)

Notes: Standard errors are reported in parentheses only for estimated parameters. Other parameters were set a priori.

are presented for four models. These correspond to versions of the model in Section II with V given by (3) or (4) and $\alpha = 1$ or 0. We refer to the model with V given by (3) and $\alpha = 1$ as our *base model.*

A. *Results for the Household Data*

Table 1 reports our estimates of Ψ_1 along with standard errors for the different models. (We report the corresponding equilibrium decision rules in Christiano and Eichenbaum [1990a].) Table 2 documents the implications of our estimates of Ψ_1 for various first moments of the data. To calculate these, we used the fully parameterized models to simulate 1,000 time series, each of length 113 (the number of observations in our data set). First moments were calculated on each synthetic data set. Table 2 reports the average value of these moments across synthetic data sets as well as estimates of the corresponding first moments of the data.

As can be seen, all of the models do extremely well on this dimension. This is not surprising, given the nature of our estimator of Ψ_1. Notice that the models predict the same mean growth rate for c_t^p, k_t, g_t, and y_t. This prediction reflects the balanced-growth properties of our models. This prediction does not seem implausible given the point estimates and standard errors reported in Table 2.[12] The models also pre-

[12]The large standard error associated with our estimate of the growth rate of g_t may well reflect a break in the data around 1970. For example, the sample

TABLE 2—SELECTED FIRST-MOMENT PROPERTIES, HOUSEHOLD DATA SET

Variable	Model: Without government ($\alpha = 1$), Divisible labor	Model: Without government ($\alpha = 1$), Indivisible labor	Model: With government ($\alpha = 0$), Divisible labor	Model: With government ($\alpha = 0$), Indivisible labor	U.S. data (1955:4–1983:4)
c_t^p / y_t	0.56 (0.012)	0.56 (0.012)	0.56 (0.010)	0.56 (0.010)	0.55 (0.003)
g_t / y_t	0.177 (0.007)	0.178 (0.007)	0.176 (0.006)	0.177 (0.006)	0.177 (0.003)
dk_t / y_t	0.260 (0.009)	0.260 (0.010)	0.264 (0.009)	0.264 (0.009)	0.269 (0.002)
k_{t+1} / y_t	10.54 (0.268)	10.54 (0.260)	10.68 (0.307)	10.68 (0.293)	100.62 (0.09)
n_t	315.60 (3.01)	314.24 (4.09)	315.19 (4.47)	314.12 (5.74)	320.02 (10.51)
$\Delta \log c_t^p$	0.0040 (0.0017)	0.0040 (0.0016)	0.0040 (0.0016)	0.0040 (0.0016)	0.0045 (0.0007)
$\Delta \log k_t$	0.0040 (0.0015)	0.0040 (0.0016)	0.0040 (0.0015)	0.0040 (0.0016)	0.0047 (0.0005)
$\Delta \log g_t$	0.0040 (0.0019)	0.0040 (0.0019)	0.0040 (0.0019)	0.0040 (0.0019)	0.0023 (0.0017)
$\Delta \log y_t$	0.0040 (0.0017)	0.0040 (0.0017)	0.0040 (0.0017)	0.0040 (0.0017)	0.0040 (0.0017)
$\Delta \log n_t$	0.00001 (0.0002)	0.00002 (0.0003)	0.00001 (0.0003)	0.00001 (0.0005)	0.00002 (0.0013)

Notes: Numbers in the columns under the "model" heading are averages, across 1,000 simulated data sets, each with 113 observations, of the sample average of the variables in the first column. Numbers in parentheses are the standard deviations across data sets. The last column reports empirical averages, with standard errors in parentheses.

dict that the unconditional growth rate of n_t will be zero. This restriction also seems reasonably consistent with the data.

Table 3 displays estimates of a subset of the second moments of the household data, as well as the analog model predictions. All of the models do reasonably well at matching the estimated values of σ_{c^p} / σ_y, σ_{dk} / σ_y, σ_g / σ_y, and σ_y. Interestingly, introducing government into the analysis (i.e., moving from $\alpha = 1$ to $\alpha = 0$) actually improves the performance of the models with respect to σ_{c^p} / σ_y, σ_{dk} / σ_y, and σ_g / σ_y but has little impact on their predictions for σ_y. The models do not do well, however, at matching the volatility of hours worked relative to output (σ_n / σ_y). Not surprisingly, incorporating government into the analysis ($\alpha = 0$) generates additional volatility in n_t, as does allowing for indivisibilities in labor supply. Indeed, the quantitative impact of these two perturbations to the base model (divisible labor with $\alpha = 1$) is similar. Nevertheless, even when both effects are operative, the model still underpredicts the volatility of n_t relative to y_t. Similarly, allowing for nonconvexities in labor supply and introducing government into the analysis improves the

average of the growth rate of g_t between 1955:2 and 1970:1 is 0.0060, whereas between 1970:1 and 1984:1 it is −0.0018.

TABLE 3—SECOND-MOMENT PROPERTIES AFTER HP-DETRENDING MODELS ESTIMATED USING THE HOUSEHOLD DATA SET

Statistic	Model: Without government ($\alpha = 1$) Divisible labor	Model: Without government ($\alpha = 1$) Indivisible labor	Model: With government ($\alpha = 0$) Divisible labor	Model: With government ($\alpha = 0$) Indivisible labor	U.S. data (1955:4–1983:4)
σ_{c^p}/σ_y	0.57 (0.085)	0.53 (0.076)	0.49 (0.049)	0.46 (0.05)	0.44 (0.027)
σ_{dk}/σ_y	2.33 (0.16)	2.45 (0.17)	2.11 (0.16)	2.24 (0.17)	2.24 (0.062)
σ_n/σ_y	0.36 (0.004)	0.50 (0.006)	0.46 (0.02)	0.62 (0.03)	0.86 (0.060)
$\sigma_n/\sigma_{y/n}$	0.54 (0.01)	0.96 (0.03)	0.79 (0.07)	1.36 (0.14)	1.21 (0.11)
σ_g/σ_y	1.76 (0.24)	1.55 (0.21)	1.66 (0.20)	1.44 (0.16)	1.15 (0.23)
σ_y	0.020 (0.0026)	0.023 (0.003)	0.021 (0.003)	0.025 (0.003)	0.019 (0.001)
corr$(y/n, n)$	0.95 (0.014)	0.92 (0.022)	0.81 (0.058)	0.73 (0.074)	−0.20 (0.11)

Notes: All of the statistics in this table are computed after first logging and then detrending the data using the Hodrick-Prescott filter. Here, σ_i is the standard deviation of variable i detrended in this way and corr(x,w) is the correlation between detrended x and detrended w. Numbers in the columns under the "model" heading are averages, across 1,000 simulated data sets, each with 113 observations, of the sample average of the variables in the first column. Numbers in parentheses are the standard deviations across data sets. The last column reports results for U.S. data with associated standard errors, computed as discussed in the text, in parentheses.

model's performance with respect to the volatility of n_t relative to y_t/n_t. In fact, the model that incorporates both of these effects actually overstates the volatility of n_t relative to y_t/n_t.[13]

Next we consider the ability of the different models to account for the Dunlop–Tarshis observation. Table 3 shows that the prediction of the base model is grossly inconsistent with the observed correlation between average productivity and hours worked. Introducing nonconvexities in labor supply has almost no impact on the model's prediction for this correlation.[14] However, introducing government into the analysis ($\alpha = 0$) does reduce the prediction some, at least moving it in the right direction. But not nearly enough: the models with $\alpha = 0$ still substantially overstate the correlation

[13]These results differ in an important way from those of G. Hansen (1985). Using data processed with the HP filter, he reports that the indivisible labor model with $\alpha = 1$ implies a value of $\sigma_n/\sigma_{y/n}$ equal to 2.7 (Hansen, 1985 table 1). This is more than twice the corresponding empirical quantity. Our version of this model ($\alpha = 1$) underpredicts $\sigma_n/\sigma_{y/n}$ by more than 20 percent. The reason for the discrepancy is that Hansen models innovations to technology as having a transient effect on z_t, whereas we assume that their effect is permanent. Consequently, the intertemporal substitution effect of a shock to technology is considerably magnified in Hansen's version of the model.

[14]To gain intuition into this result, consider a static version of our model, with no capital, in which the wage is equated to the marginal product of labor in each period. In that model, introducing indivisibilities can be thought of as flattening the labor-supply schedule, thereby increasing the fluctuations of hours worked relative to the wage. However, as long as the only shocks are to technology, the correlation between hours worked and the wage will still be strongly negative, regardless of the slope of labor supply.

TABLE 4—DIAGNOSTIC RESULTS WITH THE TWO DATA SETS

Data set	Statistic	U.S. data	Model: Without government ($\alpha = 1$) Divisible labor	Model: Without government ($\alpha = 1$) Indivisible labor	Model: With government ($\alpha = 0$) Divisible labor	Model: With government ($\alpha = 0$) Indivisible labor
A. Households	corr($y/n, n$)	−0.20	0.951	0.915	0.818	0.737
		(0.11)	(0.11)	(0.11)	(0.14)	(0.15)
			[10.56]	[10.23]	[7.10]	[6.17]
	$\sigma_n/\sigma_{y/n}$	1.21	0.543	0.959	0.785	1.348
		(0.11)	(0.11)	(0.11)	(0.12)	(0.12)
			[5.87]	[2.13]	[3.67]	[1.16]
	J		168.84	119.29	62.18	41.46
			{0}	{0}	{0}	{0}
B. Establishments	corr($y/n, n$)	0.16	0.946	0.915	0.659	0.575
		(0.08)	(0.08)	(0.08)	(0.22)	(0.22)
			[9.43]	[9.02]	[2.30]	[1.84]
	$\sigma_n/\sigma_{y/n}$	1.64	0.605	0.959	0.951	1.437
		(0.16)	(0.16)	(0.16)	(0.18)	(0.19)
			[6.45]	[4.23]	[3.75]	[1.07]
	J		131.35	100.53	14.55	3.48
			{0}	{0}	{0.0007}	{0.176}

Notes: All results are based on data detrended by the Hodrick-Prescott filter. The numbers in the "U.S. data" column are point estimates based on U.S. data for the statistic. The portion of this column in panel A is taken directly from Table 3. The numbers in parentheses are the associated standard-error estimates. The numbers in the columns under the "model" heading are the probability limits of the statistics implied by the indicated model at its estimated parameter values; the numbers in parentheses are the standard errors of the discrepancy between the statistic and its associated sample value, reported in the U.S. data column. This standard error is computed by taking the square root of the appropriate diagonal element of equation (34). The numbers in brackets are the associated t statistics. The J statistic is computed using equation (35), and the number in braces is the probability that a chi-square with two degrees of freedom exceeds the reported value of the associated J statistic.

between average productivity and hours worked.

Panel A in Table 4 reports the results of implementing the diagnostic procedures discussed in Section II. The last row of the panel (labeled "J") reports the statistic for testing the joint null hypothesis that the model predictions for both corr($y/n, n$) and $\sigma_n/\sigma_{y/n}$ are true. As can be seen, this null hypothesis is overwhelmingly rejected for every version of the model. Notice also that the t statistics (given in brackets in the table) associated with corr($y/n, n$) are all larger than the corresponding t statistics associated with $\sigma_n/\sigma_{y/n}$. This is consistent with our claim that the single most striking failure of existing RBC models lies in their implications for the Dunlop–Tarshis observation, rather than the relative volatility of hours worked and average productivity.

B. *Results Based on Establishment Data*

There are at least two reasons to believe that the negative correlation between hours worked and average productivity reported above is spurious and reflects measurement error. One potential source of distortion lies in the fact that gross output covers more sectors than household hours. The other potential source of distortion is that household hours data may suffer from classical

TABLE 5—MODEL PARAMETER ESTIMATES (AND STANDARD ERRORS) GENERATED BY THE ESTABLISHMENT DATA SET

	Model			
	Without government ($\alpha = 1$)		With government ($\alpha = 0$)	
Parameter	Divisible labor	Indivisible labor	Divisible labor	Indivisible labor
δ	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)
θ	0.339 (0.006)	0.339 (0.006)	0.344 (0.006)	0.344 (0.006)
γ	3.92 (0.03)	0.00353 (0.00003)	5.15 (0.05)	0.00463 (0.00005)
λ	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)
σ_ε	0.012 (0.0008)	0.012 (0.0008)	0.012 (0.0008)	0.012 (0.0008)
$\bar{g}$	144.9 (22.30)	144.9 (22.30)	148.9 (19.65)	148.9 (19.65)
ρ	0.98 (0.003)	0.98 (0.003)	0.98 (0.003)	0.98 (0.003)
σ_μ	0.016 (0.001)	0.016 (0.001)	0.016 (0.001)	0.016 (0.001)

Notes: Standard errors are reported (in parentheses) only for estimated parameters. Other parameters were set a priori.

measurement error. Classical measurement error in n will bias standard estimates of $\text{corr}(y/n, n)$ downward.

In order to investigate the quantitative impact of the coverage problem, we redid our analysis using establishment hours worked and establishment average productivity. An important virtue of these measures is that they cover the same sectors. With these data, the estimated value of $\text{corr}(y/n, n)$ becomes positive: 0.16 with a standard error of 0.08. This result is consistent with the view that the negative correlation reported in panel A of Table 4 reflects, in part, coverage problems with the household data. Interestingly, our estimate of $\sigma_n/\sigma_{y/n}$ is also significantly affected by the move to the new data set. This increases to 1.64 with a standard error of 0.16. Thus, while the models' performance with respect to the Dunlop–Tarshis observation ought to be enhanced by moving to the new data set, it ought to deteriorate with respect to the relative volatility of hours worked and output per hour. Therefore, the net effect of the new data set on overall inference cannot be determined a priori.

To assess the net impact on the models' performance, we reestimated the structural parameters and redid the diagnostic tests discussed in Section II. The new parameter estimates are reported in Table 5. The data used to generate these results are the same as those underlying Table 1, with two exceptions. One has to do with the calculations associated with the intratemporal Euler equation, that is, the third element of $\mathbf{H}_t(\cdot)$. Here we used our new measure of average productivity, which is actually an index. This measure of average productivity was scaled so that the sample mean of the transformed index coincides with the sample mean of our measure of y_t divided by establishment hours. The other difference is that, apart

from the calculations involving y_t/n_t, we measured n_t using establishment hours.

The new second-moment implications, with the exception of those pertaining to σ_y, corr$(y/n, n)$, and $\sigma_n/\sigma_{y/n}$, are very similar to those reported in Table 3. The new values of σ_y are 0.013 (0.0017) and 0.014 (0.002) for the versions of the divisible-labor model without government ($\alpha = 1$) and with government ($\alpha = 0$), respectively, and 0.015 (0.0019) and 0.016 (0.002) for the versions of the indivisible-labor model without and with government. (Numbers in parentheses are standard deviations, across synthetic data sets.) The fact that these values are all lower than those in Table 3 primarily reflects our finding that the variance of the innovation to the Solow residual is lower with the establishment hours data.

The results of our diagnostic tests are summarized in panel B of Table 4. Notice that, for every version of the model, the J statistic in panel B is lower than the corresponding entry in panel A. Nevertheless, as long as government is not included (i.e., when $\alpha = 1$), both versions of the model are still rejected at essentially the zero-percent significance level. However, this is no longer true when government is added (when $\alpha = 0$). Then, we cannot reject the indivisible labor model at even the 15-percent significance level.

To understand these results, we first consider the impact of the new data set on inference regarding the correlation between hours worked and average productivity. Comparing the $\alpha = 0$ models in panels A and B of Table 4, we see a dramatic drop in the t statistics (the bracketed numbers there). There are two principal reasons for this improvement. The most obvious reason is that $\widehat{\text{corr}}(y/n, n)$ is positive in the new data set (0.16), while it is negative in the old data set (-0.20). In this sense, the data have moved toward the model. The other reason for the improved performance is that the new values of $\hat{\Psi}_{1,T}$ generate a smaller value for corr$(y/n, n)$. For example, in the indivisible-labor model with $\alpha = 0$, corr$(y/n, n)$ drops from 0.737 to 0.575. In part, this reflects the new values of $\hat{\rho}$ and $\hat{\gamma}$. Consider $\hat{\rho}$ first. With the household data set, $\hat{\rho}$ is 0.96 (after rounding) for all of the models; with the establishment data set, $\hat{\rho}$ is 0.98 (after rounding). As we emphasized in Section I, increases in ρ are associated with decreases in the correlation between y_t/n_t and n_t.[15] Next consider $\hat{\gamma}$. With the establishment data, the estimates of γ are consistently larger than we obtained with the household data.[16] For example, in the indivisible-labor model with $\alpha = 0$, $\hat{\gamma}$ was 0.00374; now $\hat{\gamma} = 0.00463$. As we noted in Section I, the impact of a change in γ on corr$(y/n, n)$ cannot be determined a priori. As it turns out, the increase in $\hat{\gamma}$ contributes to a drop in these statistics.[17]

We now examine the impact of the establishment data on inference regarding the relative volatility of hours worked and average productivity. Comparing panels A and B of Table 4, we see that in all cases but one, the t statistics rise. In the exceptional case, that is, the indivisible-labor model with $\alpha = 0$, the change is very small. Three factors influence the change in these t statistics. First, the point estimate of $\sigma_n/\sigma_{y/n}$ is larger with the establishment data. Other things equal, this hurts

[15]Consistent with this relationship, corr$(y/n, n) =$ 0.644 in the indivisible-labor model with $\alpha = 0$, when it is evaluated at the parameter values in Table 1 except with ρ set to 0.98.

[16]To see why the establishment data set generates a higher value of $\hat{\gamma}$, it is convenient to concentrate on the divisible-labor model. The parameter $\hat{\theta}$ is invariant to which data set or model is used. In practice, our estimator of $\hat{\gamma}$ is approximately $\hat{\gamma} \cong [(1-\hat{\theta})/(c/y)][(N/n)-1]$, where c/y denotes the sample average of $(c_t^p + \alpha g_t)/y_t$ and N/n denotes the sample average of N/n_t. Obviously, $\hat{\gamma}$ is a decreasing function of n. The value of n with the household data set is 320.4, and the implied value of n/N is 0.23. With the establishment data set, $n = 257.7$, and the implied value of n/N is 0.19. Our estimates of γ are different from the one used by Kydland and Prescott (1982). This is because Kydland and Prescott deduce a value of γ based on the assumption that $n/N = 0.33$. In defending this assumption, Prescott (1986 p. 15) says that "[Gilbert R.] Ghez and [Gary S.] Becker (1975) find that the household allocates approximately one-third of its productive time to market activities and two-thirds to nonmarket activities." We cannot find any statement of this sort in Ghez and Becker (1975).

[17]For example, in the indivisible labor model with $\alpha = 0$ evaluated at the parameter estimates in Table 1, but with γ increased to 0.0046, corr$(y/n, n) = 0.684$.

the empirical performance of all the models, except the indivisible-labor model with $\alpha = 0$. Second, these statistics are estimated less precisely with the establishment data, and this contributes to a reduction in the t statistics. Third, the new parameter estimates lead to an increase in each model's implied value of $\sigma_n / \sigma_{y/n}$. For example, the value of $\sigma_n / \sigma_{y/n}$ implied by the indivisible-labor model with $\alpha = 0$ rises to 1.437 from 1.348. In part, this reflects the new values of $\hat{\rho}$ and $\hat{\gamma}$. When we evaluate the indivisible-labor model with $\alpha = 0$ at the parameter estimates in Table 1, with ρ increased to its Table 5 value of 0.98, the value of $\sigma_n / \sigma_{y/n}$ equals 1.396. The analog experiment with γ increases the value of this statistic to 1.436.

Comparing panels A and B of Table 4, we see that inference about the importance of the role of government consumption appears to hinge sensitively on which data set is used. On the one hand, the household data suggest that the role of government consumption is minimal. This is because both the divisible-labor and indivisible-labor models are rejected, regardless of whether $\alpha = 0$ or 1. On the other hand, the establishment data suggest an important role for government consumption. While the divisible-labor model is rejected in both its variants, the indivisible-labor model cannot be rejected at conventional significance levels as long as $\alpha = 0$.

In Christiano and Eichenbaum (1990a), we argue that the sensitivity of inference to which data set is used is resolved once we allow for classical measurement error in hours worked. The basic idea is to assume, as in Prescott (1986), that the measurement errors in the logarithm of household and establishment hours worked are uncorrelated over time and with each other, as well as with the logarithm of true hours worked. In Christiano and Eichenbaum (1990a), we show how to estimate the parameters of the models considered here, allowing for this kind of measurement error. In addition, we did the diagnostic tests that we have discussed in this paper. The main findings can be briefly summarized as follows. First, allowing for measurement error, the indivisible-labor model cannot be rejected at conventional significance levels as long as government is incorporated into the analysis. This is true regardless of whether household or establishment hours data are used. Second, the divisible-labor model continues to be rejected for both data sets, regardless of whether government is included in the analysis. Therefore, with this model of measurement error, inference is not sensitive to which measure of hours worked is used. Regardless of whether household or establishment hours data are used, incorporating government into the analysis substantially improves the empirical performance of the indivisible-labor model.

In Christiano and Eichenbaum (1990a), we also present evidence that the plausibility of the divisible-labor model with government is affected by the choice of stationarity-inducing transformation. In particular, there is substantially less evidence against that model with $\alpha = 0$ when the diagnostic tests are applied to growth rates of the establishment hours data set and measurement error is allowed for.

IV. Concluding Remarks

Existing RBC theories assume that the only source of impulses to postwar U.S. business cycles are exogenous shocks to technology. We have argued that this feature of RBC models generates a strong positive correlation between hours worked and average productivity. Unfortunately, this implication is grossly counterfactual, at least for the postwar United States. This led us to conclude that there must be other quantitatively important shocks driving fluctuations in aggregate U.S. output. We have focused on assessing the importance of shocks to government consumption. Our results indicate that, when aggregate demand shocks arising from stochastic movements in government consumption are incorporated into the analysis, the model's empirical performance is substantially improved.

Two important caveats about our empirical results should be emphasized. One has to do with our implicit assumption that public and private capital are perfect substi-

tutes in the aggregate production function. Some researchers, including most prominently Aschauer (1989), have argued that this assumption is empirically implausible. To the extent that these researchers are correct, and to the extent that public-investment shocks are important, our assumption makes it easier for our model to account for the Dunlop–Tarshis observation. This is because these kinds of shocks have an impact on the model similar to technology shocks, and they contribute to a positive correlation between hours worked and productivity. The other caveat has to do with another implicit assumption: that all taxes are lump-sum. We chose this strategy in order to isolate the role of shocks to government consumption per se.

We leave to future research the important task of incorporating distortionary taxation into our framework. How this would affect our model's ability to account for the Dunlop–Tarshis observation is not clear. Recent work by R. Anton Braun (1989) and Ellen R. McGrattan (1991) indicates that randomness in marginal tax rates enhances the model on this dimension. However, some simple dynamic optimal-taxation arguments suggest the opposite. Suppose, for example, that it is optimal for the government to increase distortionary taxes on labor immediately in response to a persistent increase in government consumption. This would obviously reduce the positive employment effect of an increase in government consumption. Still, using a version of our divisible-labor model, V. V. Chari et al. (1991) show that this last effect is very small. In their environment, introducing government into the analysis enhances the model's overall ability to account for the Dunlop–Tarshis observation. In any event, if it were optimal for the government to increase taxes with a lag, we suspect that this type of distortionary taxation would actually enhance the model's empirical performance.

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