4.1 Potential explanations of the “consumption” home bias (Lewis 1999 JEL):

Lewis 1999 [JEL]

We have seen that consumption growth rates are not highly correlated across countries. Output growth rates are. What’s more, there is a “consumption home bias” (consumption growth is correlated with output growth). Regressions of the type $\Delta c_i^t = \alpha \Delta y_i^t + \epsilon_i$ where $c_i^t$ is country $i$ idiosyncratic consumption and $y_i^t$ is country $i$ idiosyncratic output growth give a highly significant $\alpha$(> 0.6).

- if there are non tradeable goods, the correlation of tradeables consumption need not be perfectly correlated across countries. The presence of non tradeables lowers consumption correlations across countries.

- Cole and Obstfeld (1991): gains in consumption due to risk sharing can be estimated: they are small (0.04% of permanent consumption). Therefore even small transaction costs lead almost to autarky. But Obstfeld (1994) finds large gains.

- empirical evidence does suggest that restrictions on capital flows are important in affecting equilibrium consumption outcomes (emerging markets).
4.1.1 Non tradable goods and consumption home bias.

Can the presence of non traded goods reconcile theory and evidence?

Assume N countries, 2 periods, 2 states, complete markets.

Pure endowment economy. The representative consumer of country n maximises:

\[
U = u(C_{T,t}, C_{N,t}) + \beta \pi(S) \equiv \left[ \pi(1) u(C_{T,t+1}, C_{N,t+1}) + \pi(2) u(C_{T,t+1}, C_{N,t+1}) \right]
\]

Note that non traded cannot be traded by definition but claims indexed on traded goods and paid in traded goods can perfectly exist.

The budget constraint takes the following form:

\[
C_{T,t} + p_{N,t} C_{N,t} + \sum_{s} q(s) [C_{T,t+1}(s) + p_{N,t+1}(s) C_{N,t+1}(s)] = Y_{T,t} + p_{N,t} Y_{N,t} + \sum_{s} q(s) [Y_{T,t+1}(s) + p_{N,t+1}(s) Y_{N,t+1}(s)]
\]

First order conditions: for each period and state:

\[
\frac{\partial u(C_{T,t}, C_{N,t})}{\partial C_{N,t}} = p_{N,t}
\]

(intratemporal) the marginal rate of substitution equals the price.

\[
q(s) \frac{\partial u(C_{T,t}, C_{N,t})}{\partial C_{T,t}} = \pi(s) \beta \frac{\partial u(C_{T,t+1}, C_{N,t+1})}{\partial C_{T,t}}
\]

second euler equation implies that the date t marginal utility cost of a date t + 1 state s unit of nontradable must equal the expected marginal benefit

For two countries i and j, the first FOC implies:

\[
\frac{\pi(s) \beta \frac{\partial u(C_{T,t}, C_{N,t})}{\partial C_{T,t}}}{p_{N,t}} = \pi(s) \beta \frac{\partial u(C_{T,t+1}, C_{N,t+1})}{\partial C_{T,t}}
\]

As in the previous model (without non traded goods), this means that ex post growth rates in the marginal utility of tradables are equal. And in the case of separable CRRA utility, national growth rates of tradables consumption are perfectly correlated with each other. Could still test risk sharing, if we can measure tradable consumption.

For non tradables, this is different: there is no common relative price across countries and therefore there is a priori no reason why ex-post growth rates in the marginal utility of non tradables should be equal.

The allocation of world ressources is constrained efficient.

Note that nontradable output plays the role of a preference shifter.

4.1.2 Application of non tradability to international consumption correlations

In the case where the utility function is a CRRA of CES aggregate of non traded and traded goods Lewis (1996) [JPE] shows that it is still possible to test for complete asset market integration. Assume that utility follows:

\[
u(C_{T}, C_{N}) = \left[ \gamma^{1/\theta} C_{T}^{\frac{\theta+1}{\theta}} + (1 - \gamma)^{1/\theta} C_{N}^{\frac{\theta+1}{\theta}} \right]^{\theta/(1-\rho)} / (1 - \rho)
\]
this is a CRRA ($\rho$) utility over a CES aggregator between tradable and non tradable with elasticity of substitution $\theta > 0$. When $\theta = 1$, the function is additive. We can rewrite the FOC (4.1) and differentiate to obtain the Planner’s allocation:

$$\hat{C}_T = \frac{-\theta}{1 - (\phi(1 - \theta \rho))} \hat{\lambda} + \frac{(1 - \phi)(1 - \theta \rho)}{1 - (\phi(1 - \theta \rho))} \hat{Y}_N$$

and $\phi < 1$ is the share of tradable consumption contribution to flow utility.

The change in $C_T$ depends upon the change in the global factor $\lambda$. Recall that $\lambda = q(s)/\pi(s) \beta$ so fluctuations in $\lambda$ reflect aggregate tradable risk, as well as the change in domestic endowment of nontradable. Notice that tradable consumption, as expected, does not depend upon domestic tradable output.

The sign of the effect of $Y_N$ on $C_T$ depends upon the sign of $(1 - \theta \rho)$. This is because we have

$$sgn \left( \frac{\partial^2 u (C_T, C_N)}{\partial C_T \partial C_N} \right) = sgn (1 - \theta \rho)$$

when $\theta > 1$, the marginal utility of tradable decreases with nontradables, so that they are ‘substitutes’ when $\theta < 1$, the marginal utility of tradable increases with nontradables, so that they are ‘complements’

There are two effects:

1. if $Y_N$ increases, want to consume more tradable (because of intratemporal substitution), controlled by $\theta$

2. country is richer, so intertemporal substitution may amplify or dampen the effect.

Main implications:

- if the elasticity of substitution of tradable for nontradable is small ($\theta$ smaller than $\rho^{-1}$, $1 - \theta \rho > 0$), then the marginal utility of tradable increases with nontradable. So an increase in nontradable increases and it is optimal to allocate larger consumption of tradable to countries with larger endowment of nontradable. (limit case: complements: if $Y_N$ increases, want to increase $C_T$)

- if the elasticity of substitution of tradable for non tradable is large, so that $1 - \theta \rho < 0$, then the marginal utility of tradable decreases with nontradable consumption. If very substitute, an increase in $Y_N$ substitutes for an increase in $C_T$.

- tradable consumption should not respond to domestic tradable output $Y_T$, only world tradable output $Y_{wT}$.

Lewis tests the implications of the model with nontradables, following Obstfeld regression tests. Table 2 in her paper indicates that nontradables, when entered directly, do not appear significantly. This is perhaps not surprising if we believe that preferences are separable ($\theta \rho = 1$). But we still reject risk sharing: the coefficient on domestic tradable output is significant.

Lewis then incorporates the durable/non durable distinction. The problem, she notes, is that the consumption of nondurable tradable may be correlated with the services on durable tradable (such as rental cars, household appliances). The later are typically nontradable, even if the durable are themselves tradables. So the test may reject risk sharing when in fact it is simply picking the correlation between nondurable tradable and nontradable services on durable goods. This can be dealt with by running a regression of the type:

$$\Delta \log (C_{T,t} - Y_{D,t}) = \nu_t + \psi_1 \Delta \log (Y_{N,t}) + \psi_2 \Delta \log (Y_{D,t}) + \psi_3 \Delta \log (Y_{T,t} - Y_{D,t}) + \varepsilon_t$$

where $Y_{D,t}$ is a proxy for the services of durables (she uses the expenditure on durables), $\nu_t$ is a global consumption shock at date $t$, $Y_{N,t}$ is non tradable consumption. Under efficient risk sharing $\psi_3 = 0$. This is the case because because idiosyncratic risks to tradable output growth have been fully diversified across countries and should only affect consumption in traded goods through their effect on $\nu_t$ (common global non diversifiable risk). Lewis find that $\psi_3$ is significantly positive for her full sample. $\psi_1$ also fails to enter significantly. But if she splits the sample between countries with severe foreign restrictions and those without such restrictions, she finds that $\psi_3$ is significantly positive only for the former case so that constrained efficient risk sharing cannot be rejected for countries without capital controls.
**TABLE 2**

**RISK-SHARING TESTS WITH DISAGGREGATE CONSUMPTION**

**(A) SUMMARY STATISTICS**

(Annualized Percentage Growth Rates)

<table>
<thead>
<tr>
<th>SUMMARY STATISTIC</th>
<th>Nondurable</th>
<th>Tradable</th>
<th>Tradable</th>
<th>Tradable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.67</td>
<td>7.86</td>
<td>3.00</td>
<td>4.94</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>16.17</td>
<td>31.09</td>
<td>31.68</td>
<td>23.98</td>
</tr>
<tr>
<td>Expenditure shares</td>
<td>.593</td>
<td>.064</td>
<td>.343</td>
<td>...</td>
</tr>
</tbody>
</table>

**(B) PANEL REGRESSIONS**

\[
\ln(T'_i/T'_{i-3}) = \theta_0(t) + \sum_{i=1}^{I} \theta_i \ln(Z_i/Z'_{i-3}) + \beta X_i + u_i
\]

<table>
<thead>
<tr>
<th>Z^i</th>
<th>(\theta_i)</th>
<th>Consumption Variance Explained by Z (%)^*</th>
<th>(\beta)</th>
<th>Consumption Variance Explained by Output (%)^†</th>
</tr>
</thead>
<tbody>
<tr>
<td>None (separable utility)</td>
<td>...</td>
<td>...</td>
<td>.587†</td>
<td>71.5</td>
</tr>
<tr>
<td>2. Nontradables</td>
<td>.017</td>
<td>.550†</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.057)</td>
<td>(.071)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Durables</td>
<td>.169†</td>
<td>.412†</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.033)</td>
<td>(.052)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. All:</td>
<td>.029</td>
<td></td>
<td>.580†</td>
<td>31.9</td>
</tr>
<tr>
<td>Nontradables</td>
<td>(.052)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durables</td>
<td>.170†</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.041)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE.** Data are taken from the 5-year benchmark studies used in the Penn World Tables for disaggregated consumption expenditures in current international dollars. The data are converted into 1985 dollars using the ratio of the U.S. current dollar consumption to the U.S. 1985 dollar consumption in the Penn World Tables. The sample years are 1970, 1975, 1980, and 1985 for 46 countries (not all countries have data in each year, as described in App. A). All equations are estimated allowing for conditional heteroskedasticity. Standard errors are in parentheses.

* Calculated as \(\text{var} \left[ \ln(T'_i/T'_{i-3}) \right] / \text{var} \left[ \ln(T_i/T_{i-3}) \right] - \theta_0(0)\).

† Calculated as \(\text{var}(\beta Y) / \text{var}(\ln(T_i/T_{i-3}) - \theta_0(0))\).

‡ Significantly different from zero at the 95 percent confidence level.

Figure 1 Lewis, JPE 1996
4.1. Potential explanations of the “consumption” home bias (Lewis 1999 JEL):

\[
\ln(T_i^j/T_{i-3}^j) - \theta_0^j(t)D(j, t) + \theta_0^j(t)[1 - D(j, t)] + D(j, t) \sum_{i=1}^{4} \theta_0^j\ln(Z_i^j/Z_{i-3}^j)
\]

\[
+ [1 - D(j, t)] \sum_{i=1}^{4} \theta_i^j\ln(Z_i^j/Z_{i-3}^j) + \beta^jD(j, t)X_i^j + \beta^j[1 - D(j, t)]X_i^j + u_i^j
\]

<table>
<thead>
<tr>
<th>(Z^j)</th>
<th>(\theta_0^j)</th>
<th>(\theta_0^*)</th>
<th>Marginal Significance Level</th>
<th>(\beta^*)</th>
<th>Marginal Significance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>None (separable utility)</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>.751*</td>
<td>.721*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(.091)</td>
<td>(.097)</td>
</tr>
<tr>
<td>Nontradables</td>
<td>.660*</td>
<td>.596*</td>
<td>.762</td>
<td>.104</td>
<td>.078</td>
</tr>
<tr>
<td></td>
<td>(.219)</td>
<td>(.112)</td>
<td></td>
<td>(.181)</td>
<td>(.145)</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nontradables</td>
<td>−.182</td>
<td>.030</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.287)</td>
<td>(.104)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durables</td>
<td>.184</td>
<td>.768*</td>
<td>.037</td>
<td>.622*</td>
<td>−.007</td>
</tr>
<tr>
<td></td>
<td>(.141)</td>
<td>(.237)</td>
<td></td>
<td>(.115)</td>
<td>(.190)</td>
</tr>
</tbody>
</table>

**Note.**—The data are the same as in table 2 for consumption and output and table 3 for capital market restrictions. Standard errors are in parentheses.

* Significantly different from zero at the 95 percent confidence level.

Figure 2 Lewis, JPE 1996
4.2 International Diversification and Nontraded Goods

Previous explanation about consumption bias, not the equity bias. We have looked at models with AD securities, but we can map that back into ‘real’ securities. Suppose that we look at a world with $n$ countries trading shares in the random future output of tradable—denoted $V_{T,t}$ and the random output of nontradable, denoted $V_{N,t}$.

Remark 1 We can trade shares in the foreign nontradable, only if they are converted back into tradable so what this means is that $V_{N,t}$ is the price of a claim to $p_{N,t+1}(s)Y_{N,t+1}(s)$ tomorrow.

Remark 2 Early models ruled out trading claims to foreign non-tradable (pesenti and vanWincoop) this is incorrect.

4.2.1 When tradable and nontradable are separable

If consumption is separable and CRRA in tradable, it is possible to show the following results (Stockman and Dellas 1989, JIE)

- the equilibrium is constrained efficient:
  \[
  \frac{C_{m,T,t+1}(s)}{C_{m,T,t}} = \frac{C_{n,T,t+1}(s)}{C_{n,T,t}} = \frac{Y_{w,T,t+1}(s)}{Y_{w,T,t}}.
  \]

- tradable consumption is pooled:
  \[C_T = \mu Y_{w,T}^\

- the portfolio of tradable is fully diversified: each country holds a share of the world tradable portfolio
- the portfolio of nontradable is fully concentrated.
- the share equals the country shares of world tradable endowment

It is simply a matter of retracing our steps for the Lucas model. To make sure that nontradable are not traded internationally, note that the value of a nontradable claim is:

\[
V_{N,t} = \sum_s \pi(s) \beta \frac{\partial u(C_{T,t+1}, C_{N,t+1})/\partial C_T}{\partial u(C_{T,t}, C_{N,t})/\partial C_T} p_{N,t+1}(s) Y_{N,t+1}(s)
\]

and that under separability and CRRA:

\[
V_{N,t} = \sum_s \pi(s) \beta \left( \frac{Y_{w,T,t+1}}{Y_{w,T,t}} \right)^{-p} p_{N,t+1}(s) Y_{N,t+1}(s)
\]

so the pricing kernel being common to all countries, they value identically the claim to foreign nontradable and there is no trade needed. (i.e. the value are consistent with the Euler equation of each country).

Implications:

- all nontradable equity is held domestically. if we assume that nontradable output represent about 50% of output so that nontradable equity represents about 50% of domestic equity, can explain up to 50% of the bias
- tradable portfolio is a constant share of the world tradable portfolio.

This does not explain the puzzle away:
• empirical evidence suggests that the share of domestic equity in domestic portfolio is much larger than the share of tradable + fraction of domestic tradable wealth in total wealth.

• it is a bit extreme to assume that tradable and nontradable are nonseparable. What happens if they are not is less clear (harder problem).

Hard problem because we have to characterize the competitive equilibrium with bonds and equity and it need not be constrained Pareto efficient. We already know that in that case consumption of non-tradable will affect the consumption growth of tradable.

4.2.2 When tradable and nontradable are not separable

Baxter Jermann and King (1998 JIE) argue that even when \( \theta \rho \) is not equal to one, the tradable portfolio is a world portfolio. They prove their result only for infinitesimal changes. For the general case, Serrat (2001 Econometrica) argues that this is not the case. But Kollman (2005) shows that Serrat result on portfolio allocations is incorrect. Conclusion: we have precious few results in this area.

Recall the above expression for \( \hat{C}_T \):

\[
\hat{C}_T = \frac{-\theta}{1 - (\phi (1 - \theta \rho))} \lambda + \frac{(1 - \phi) (1 - \theta \rho)}{1 - (\phi (1 - \theta \rho))} \hat{Y}_N
\]

Aggregate across countries and use the resource constraint that

\[
\sum_j C_T^j = Y_T^w
\]

to obtain:

\[
\hat{Y}_T^w = \left( \sum_j \theta_j \eta_j^\lambda \right) \lambda + \left( \sum_j \theta_j \eta_j^N \hat{Y}_N^j \right) = \left( \sum_j \theta_j \eta_j^\lambda \right) \lambda + \eta^N \hat{Y}_N
\]

where \( \theta_j = C_T^j / Y_T^w \). We see that \( \lambda \) depends upon the aggregate supply of tradable and nontradable. Substituting, we obtain:

\[
\hat{C}_T^j = \sum_j \theta_j \eta_j^\lambda \hat{Y}_T^w - \sum_j \theta_j \eta_j^\lambda \eta_j^N \hat{Y}_N^j + \eta_j^N \hat{Y}_N^j
\]

If all the elasticities are the same, this simplifies to:

\[
\hat{C}_T^j = \hat{Y}_T^w + \eta^N \left( \hat{Y}_N^j - \hat{Y}_N^w \right)
\]

so that the consumption of tradable depends upon the relative supply of nontradable.

Baxter Jermann and King show that the representative household of country \( j \) can diversify his portfolio so that his consumption satisfies the above expression. To do this they note that in general, the consumption allocation is of the form:

\[
dC_T^j = \alpha_T (dY_T^w) + \alpha_N Y_N dY_N^w + \alpha^N Y_N dY_N^j
\]

Baxter Jermann and King consider a world of ‘small shocks’. They look for assets that can deliver the terms on the RHS so that consumption equals desired consumption. Consider three assets:

1. claim on country’s nontradable: pays \( pY_N \) tomorrow. With a small displacement, will pay: \( d(pY_N) \):

\[
d(pY_N) = pdY_N + Y_N dp
\]

but by definition of the elasticity of substitution

\[
\frac{dp}{p} = \frac{1}{\theta} \frac{d(C_T/Y_N)}{(C_T/Y_N)}
\]
so that
\[ dn_j = d(pY_N) = p \left( 1 - \frac{1}{\theta} \right) dY_N + \frac{pY_N}{Y_T} dC_T \]
\[ = \frac{pY_N}{Y_T} \alpha_d dY_w + \frac{pY_N}{Y_T} \alpha_N dY_N^w + \left[ p \left( 1 - \frac{1}{\theta} \right) + \frac{pY_N}{Y_T} \alpha^N \right] dY_N \]
\[ = \Lambda^T dY_w + \Lambda^N dY_N^w + \Lambda^N dY_N \]
so that it pays some combination of \( dY_w, dY_N^w \) and \( dY_N \).

2. Similarly, the world portfolio of nontradable equity pays off:
\[ dN = \Lambda^T dY_w^w + \Lambda^N dY_N^w \]

3. A claim to the world portfolio of traded good pays off:
\[ dT = dY^w \]
(\text{unit factor loading}).

This implies that we can construct the basic securities as:
\[
\begin{align*}
    dY^w_i &= dT \\
    dY^w_N &= \frac{\Lambda^T}{\Lambda_N} dN - \frac{\Lambda^T}{\Lambda_N} dT \\
    dY^j_N &= \left( \frac{\Lambda^N \Lambda^T}{\Lambda^N \Lambda_N} - \frac{\Lambda^T}{\Lambda_N} \right) dT - \left( \frac{1}{\Lambda_N} \Lambda^N \right) dN + \frac{1}{\Lambda_n} dn_j
\end{align*}
\]

The country’s optimal expenditures on traded and nontraded is \( e^j = C^j_T + p^j Y^j_T \) and fluctuates with:
\[
\begin{align*}
    de^j &= dC^j_T + d(p^j Y^j_N) \\
    &= (\alpha^T + \Lambda^T) dY^w_T + (\alpha^N + \Lambda^N) dY^w_N + (\alpha^N + \Lambda^N) dY_N \\
    &= \varepsilon^T dY^w_T + \varepsilon^N dY^w_N + \varepsilon^n dY^j_N
\end{align*}
\]
we can support this optimal consumption by substituting the payoff with the supporting portfolios:
\[ de^j = v^T dT + v^N dN + v^n dn_j \]

This indicates that it is possible to replicate the optimal allocations with only \( N+1 \) assets:

- a world tradable claim
- a world nontradable claim
- a domestic where nontradable claim for \( N-1 \) countries.

The weights satisfy the following property:

- domestic nontradable output:
\[ v^n_j = 1 + \frac{\alpha^n}{\Lambda^n} \]

Recall that \( \alpha^n \) represents the response of tradable consumption to nontradable endowment. We can check that when \( \alpha^n = 0 \) (separability), \( v^n_j = 1 \). If there is non-separability, then the rest of the world will go short in the nontradable equity \( (\alpha^n > 0) \) when \( \theta \rho < 1 \), so that it provides added income when nontradable output is high.
• world tradable:

\[ v^T_j = \alpha^T + \frac{\Lambda^T_A}{\Lambda^N_A} \alpha^n + \frac{\Lambda^T_N}{\Lambda^N_N} (\frac{\Lambda^N_A}{\Lambda^N_N} \alpha^n - \alpha^N) \]

when countries are identical, they show further that \( v^T_j = 1 \). That is, it is never optimal to exhibit home bias in the tradable portfolio, despite nonseparability in tradable and nontradable.

Note that this result only obtains for infinitesimal changes. The general result is less obvious. Some recent work by Serrat (2001) Econometrica claims that this is not true in the general case: Serrat finds that in the case where tradable and nontradable are complements, it is optimal to tilt the portfolio of tradable towards home assets when the domestic endowment of tradable is sufficiently correlated with the endowment of nontradable. This is a difficult paper to read, and more importantly, Kollman (2005) shows that Serrat equilibrium portfolio allocation is incorrect. Clearly, more work is warranted on this issue.

4.3 Obstfeld and Rogoff (2000)

Obstfeld and Rogoff (hereby OR) show that the introduction of costs to international trade, mainly in the form of transport costs\(^2\), goes a long way in explaining many of the pervasive empirical puzzles in international finance.

4.3.1 Home bias in trade puzzle

Interaction of elasticity of substitution and moderate trade cost generates a big home bias. The transportation cost is modelled as an iceberg cost.

Consider a two country endowment economy in which the representative consumer maximizes:

\[ C = \left( \frac{C_H}{p_H} + \frac{C_F}{p_F} \right)^{\frac{1}{\theta}} \]

\[ P_F = \frac{P_*^F}{1 - \tau} \]

\[ P_H = (1 - \tau) P^*_H \]

If we define

\[ p = \frac{P_F}{P_H} \]

Then:

\[ p^* = p (1 - \tau)^2 \]

Utility maximization gives

\[ \frac{C_H}{C_F} = p^\theta \]

\[ \frac{C_H^*}{C_F^*} = p^*^\theta \]

If we assume also (to illustrate) that the two countries are exactly symmetric then

\[ \frac{C_H}{C_F} = \frac{C_H^*}{C_F^*} = (1 - \tau)^{-\theta} = p^\theta \]

So that

\[ \frac{C_H}{p C_F} = \frac{p^* C_F^*}{C_H^*} = (1 - \tau)^{1-\theta} \]

\(^2\)Transport costs seem to be the largest component of overall trade costs.
This is a non linear relationship. It is therefore quite easy to get high values for the home bias even with low values for the friction. Empirically plausible values for $\theta$ are 5 to 6. It is difficult to come with numbers for transport cost, tariffs, etc... on average.

### 4.3.2 Feldstein-Horioka

Using a standard two-period consumption smoothing model for a small open economy, the authors introduce transport costs for goods while maintaining free and costless trade in securities. They show that moderate transport costs will generate significant international differences in real interest rates despite full capital market integration. The model also contains a prediction that countries running a current account surplus should have lower real interest rates than countries running deficits, i.e. it also uncovers a negative link between real interest rates and the current account to GDP ratio.

As the authors emphasize, their point is easy to illustrate in the two-period consumption model. The representative agent in the small open economy set-up solves the following problem:

$$\max_{C_1, C_2} U(C_1) + \beta U(C_2)$$

with $C$ depending on consumption of the home and foreign good as before:

$$C = \left( C_H^{\frac{\theta+1}{\theta}} + C_F^{\frac{\theta+1}{\theta}} \right)^\frac{\theta}{\theta+1}$$

The small open economy is endowed only with good $H$. The foreign good must be imported. First period budget constraint

$$P_{H1}Y_{H1} + B = P_{H1}C_{H1} + P_{F1}C_{F1} = P_1C_1$$

where $Y_i$ $i = 1, 2$ are the endowments at each date and

$$P = \left( C_H^{1-\theta} + C_F^{1-\theta} \right)^{\frac{1}{\theta}}$$

Therefore

$$C_H = \left( \frac{P_H}{P} \right)^{-\theta} C$$

$$C_F = \left( \frac{P_F}{P} \right)^{-\theta} C$$

The second period budget constraint is:

$$P_{H2}Y_{H2} - (1 + r^*) B = P_{H2}C_{H2} + P_{F2}C_{F2} = P_2C_2$$

Intertemporal budget constraint

$$P_1 C_1 + \frac{P_2 C_2}{1 + r^*} = P_{H1}Y_{H1} + \frac{P_{H2}Y_{H2}}{1 + r^*}$$

or, since $1 + r = \frac{(1 + r^*)P_1}{P_2}$

$$C_1 + \frac{C_2}{1 + r} = \frac{P_{H1}Y_{H1}}{P_1} + \left( \frac{1}{1 + r} \right) \frac{P_{H2}Y_{H2}}{P_2}$$

Due to international arbitrage $P_F = \frac{P^*_F}{P^*_H}$ since $F$ is always imported.

When good $H$ is exported (it must be in at least one of the 2 periods) $P_H = P^*_H (1 - \tau)$. But if total domestic spending is high enough relative to income in a given period, then it is possible that $H$ is imported rather than exported ($C_H > Y_H$). In that case $P_H = P^*_H / (1 - \tau)$. There are also cases where $C_H = Y_H$ in which case $P_H$ will be between $P^*_H (1 - \tau)$ and $P^*_H / (1 - \tau)$. 
4.4. Finance and equity home bias

Thus the effective real interest rate of the home country will depend on the current account. The model implies that we should observe high real interest rates in deficit countries and low real interest rates in surplus countries. Real interest rates increasingly respond to current account imbalances: the larger the initial deficit, the higher the real interest rate; the larger the initial surplus the lower the real interest rate.

One of the empirical exercises in the paper consists of testing the above implication for a sample of OECD countries from 1975 to 1998. The authors find strong and highly significant negative correlations between current account positions and real interest rates.

4.3.3 Equity home bias

Take the same class of model with iceberg costs and where the utility function is of the form:

\[ EU = E \left[ \frac{C^{\theta-1}_F + C^{\theta-1}_H}{1 - \theta} \right] \]

where \( \theta \) is the coefficient of substitution (also inverse of the degree of relative risk aversion). In that case it is possible to show that the equilibrium portfolio shares are

\[ x_H = \frac{1}{1 + (1 - \tau)^{\theta-1} Y_H} \]

\[ x^*_H = \frac{(1 - \tau)^{\theta-1} Y_H}{1 + (1 - \tau)^{\theta-1} Y_H^*} \]

\[ x_F = \frac{1}{1 + (1 - \tau)^{\theta-1} Y_F^*} \]

\[ x^*_F = \frac{(1 - \tau)^{\theta-1} Y_F^*}{1 + (1 - \tau)^{\theta-1} Y_F^*} \]

The interaction of the transport costs and the coefficient of substitution give rise to a strong home bias without assuming any friction on the asset market.

4.3.4 International consumption correlations

They argue that incomplete asset markets and nominal rigidity takes care of it.

4.3.5 PPP puzzle

Models should incorporate nominal rigidities, monopoly power, pricing to market and trade bands.

4.3.6 The exchange rate disconnect puzzle

Integration of good market is incomplete.

4.4 Finance and equity home bias

4.4.1 Lewis, JEL 1999

Partial equilibrium CAPM: equity returns are exogenous to the model and the portfolio decision of the investor is static. This is different from the general equilibrium consumption CAPM that we studied before where we solved simultaneously for equity returns and intertemporal asset allocation decisions.
Wealth is denoted by $W$.
Investors maximize the following utility function:

$$U = U\left(E_t(W_{t+1}), \frac{1}{2}Var_t(W_{t+1})\right)$$

with $U_x > 0$ and $U_Y < 0$.

We define the coefficient of relative risk aversion as:

$$\rho = -U_x W/U_Y$$

We call $\chi_t = \left(\chi_t^h, \chi_t^f\right)'$ the portfolio weights and $r_{t+1} = \left(r_{t+1}^h, r_{t+1}^f\right)'$ the vector of real returns. Investors choose the portfolio weights to maximize their utility function.

$$E_t(W_{t+1}) = W_t (1 + \chi_t^f E_t r_{t+1})$$
$$Var_t(W_{t+1}) = W_t^2 \chi_t^f Var_t(r_{t+1}) \chi_t$$

where $Var_t(r_{t+1})$ is the conditional variance covariance matrix of the vector of real returns.

First order condition:

$$E_t r_{t+1} = \rho \chi_t^f Var_t(r_{t+1})$$

Therefore

$$\chi_t = \frac{1}{\rho} E_t r_{t+1} [Var_t(r_{t+1})]^{-1}$$

These theoretical weights can be compared to the actual ones. For all the countries one observes “equity home bias”.

### 4.4.2 Tesar Werner (1995)

Journal of International Money and Finance

#### 4.4.2.1 Evidence on home bias

They reconstruct data on asset holdings for the US, Canada, Germany, Japan from data on capital flows.

They compute hedge excess returns from the point of view of US investors (forward contracts on foreign currency value of the investment after 30 days):

$$R_{t}^{FX} + s_{t+1} - s_t + (f_{t,t+1} - s_{t+1}) - r_s = R_{t}^{FX} - \tilde{r}_{t}^{FX}$$

Using monthly data between 1981 and 1990, they find that the value weighted world portfolio dominates any national portfolio both for investment in equities and investment in equities and bonds.

The Sharpe ratio

$$S = \frac{\mu - r}{\sigma}$$

also favors the world portfolio.

So their evidence suggests once more potential gains from increasing international diversification.

#### 4.4.2.2 Evidence on turnovers

International investment positions are small. They look at 2 potential explanations:

1) taxes
2) transaction costs
1) quantitatively does not fly
2) variable costs:

If these costs were substantial we would expect a buy and hold strategy. But in the data turnovers seem to higher for foreign equities than for domestic equities.

fixed costs?
Could it be that only institutional investors transact in foreign equities and that they transact much more than individual investors? Does not seem to be true either.

### 4.4.3 Baxter and Jermann 1997 [AER]

Argue that most human wealth is non-tradeable and is highly correlated with domestic equity. The optimal portfolio should hedge human capital by shorting domestic equity. Worsens the home equity bias.

Write the output as:

\[ Y_t = A_t K_t^{1-\alpha} L_t^\alpha \]

suppose that we only trade the claims to financial return:

\[
V_{K_t} = E_t \sum_{j=1}^{\infty} \frac{(1-\alpha)Y_{t+j}}{1+r_{t,t+j}} \\
= (1-\alpha) V_{Y_t}
\]

the claim to human capital is:

\[
V_{L_t} = E_t \sum_{j=1}^{\infty} \frac{\alpha Y_{t+j}}{1+r_{t,t+j}} \\
= \alpha V_{Y_t} \\
= \frac{\alpha}{1-\alpha} V_{K_t}
\]

and is perfectly correlated with the claim to capital: so the implicit exposure is \( \frac{\alpha}{1-\alpha} \). This implies that the optimal portfolio should \textit{short} the domestic equity and go long in foreign.

Caveat: our own Christian Julliard claims that returns to human capital are negatively correlated with returns to capital. Empirical question is whether there are big shocks to the capital/labor share.

### 4.4.4 Evidence on portfolio equity flows

- Let us assume that investors try to maximise the return of their portfolio while minimising its variance (partial equilibrium CAPM). Depending on how risk averse they are, they go for high returns, high variance or lower returns, lower variance. If capital markets were frictionless, countries would best diversify their risk by investing into other countries’ assets, which have low correlations with their own assets. Empirically the business cycles of neighbouring countries are highly correlated and this correlation decreases with distance. So, if capital market were frictionless, countries should \textit{ceteris paribus} invest in assets of countries which are far away. When one looks at the data however, one uncovers very strong geographical patterns in portfolio equity flows which point exactly towards the opposite: countries like to buy assets of their neighbours. This suggests the existence of strong informational asymmetries on asset markets. More than that, this seems to indicate that these frictions dominate the risk diversification motive.

- Portes-Rey (2001) shows (panel data) that gross cross-border equity flows are best explained by a model of the form:

\[
(Eqflows_{ij,t}) = \alpha M_{CAPi,t} + \beta M_{CAPj,t} + \gamma D_{ij} + Z_{i,t} + Z_{j,t} + \text{dummies} + \epsilon_{ij,t}
\]

Other variables proxying information asymmetries like international phone calls, number of branches of banks work very well (see table). Asymmetry of information (or familiarity) seems therefore to play a crucial role in capital flows.
They also run regressions on disaggregated asset data: they look separately at corporate bonds, government bonds and portfolio equity flows. They find that portfolio equity flows and corporate bond flows are described by the same type of model (distance plays a big role) whereas distance is not significant for transactions in government bonds. This evidence is consistent with a priori differences in information intensities across assets: trading in government bonds require information which is mostly common knowledge whereas there is a lot more scope for information asymmetries for equities and corporate bonds.

They also find that for the two years where benchmark surveys of asset holdings are available, there is a remarkably high correlations between transactions and holdings:

this suggests that the geographical distribution of holdings mirrors the geographical distribution of transactions.

4.4.5 Regional evidence

Cowal and Moskowitz

They find very striking results: there is a regional/local home bias even within the US. Active fund managers overweight proximate firms in their portfolios and earn substantial abnormal returns in local holdings.

4.4.6 How could we reconcile models and empirical evidence?

1. Potential explanations of the “consumption” home bias (Lewis 1998):

   • if there are non tradeable goods, the correlation of tradeables consumption need not be perfectly correlated across countries. The presence of non tradeables does lower consumption correlations across countries... but not enough.

   • Cole and Obsfeld (1991): gains in consumption due to risk sharing can be estimated: they are small (0.04% of permanent consumption). Therefore even small transaction costs lead almost to autarky. But Obsfeld (1994) finds big gains.

   • empirical evidence does suggest that restrictions on capital flows are important in affecting equilibrium consumption outcomes.

2. Potential explanation of the equity home bias.

   • domestic companies are multinationals (but their return is nevertheless highly correlated with the domestic stock index).

   • there is human capital as well (but it is highly correlated with domestic stock index!)

   • taxes and other barriers are too high: evidence does suggest that gvt restrictions are important for developing countries. But this is not the case for the US for example! Tesar and Wener (1992) shows that the turnover rate is higher on foreign equities than on domestic ones. There is no "buy and hold strategy”.

   • information costs?

   • Uncertainty in portfolio performance: statistical uncertainty on computations of means and variances casts doubts on the mean efficient frontier results.
4.5 Heathcote and Perri (2003): Explaining the diversification puzzle

Lewis, 1999: “Do individuals do a good job in hedging risk across countries?”. The answer from one-good international macro models is

- output-consumption correlations: NO.
  “In contrast to the data, the theory produces output fluctuations that are less highly correlated than consumption.” (Backus et al 1994)
- Country portfolios: NO
  “..portfolio of the typical investor is still very far from representing a truly diversified world portfolio” (Baxter and Jermann, 1997)

Heathcote and Perri propose that an extension of the one-good model can explain both puzzles. They show that a model with cross country risk sharing is consistent with relatively low consumption correlation (this is not new) and low international diversification (this is new).

The main idea is that international price movements (as in Cole and Obstfeld, 1991) interact with international diversification.

4.5.1 The one-good model

- preferences:
  \[
  E_0 \sum_{t=0}^{\infty} \beta^t (\log (c_t) + \mu \log (1 - l_t))
  \]
- subject to:
  \[
  y_t = A_t k_{t}^{\theta} l_{t}^{1-\theta} \\
  k_t = (1 - \delta) k_{t-1} + x_t \\
  y_t + y_t^* \geq c_t + c_t^* + x_t + x_t^*
  \]

4.5.1.1 Complete risk sharing

\[
\Delta c = c - c^* = 0 \\
corr (c, c^*) = 1 > corr (y, y^*) \approx corr (A, A^*)
\]

In the data (US vs rest of the world, 1981.1-2002.1):

\[
corr (c, c^*) = 0.13 < corr (y, y^*) = 0.31
\]

4.5.1.2 Country Portfolios

Assume stocks are traded internationally. The home budget constraint (ignoring asset trade) is:

\[
c = (1 - \theta) y + \lambda (\theta y - x) + \lambda^f (\theta y^* - x^*)
\]

where \(\lambda\) and \(\lambda^f\) represent the shares of domestic and foreign firms.

Combining with the foreign budget constraint and using the asset market clearing condition (\(\lambda + \lambda^* = 1\)), and the symmetry condition \(\lambda = \lambda^{f*}\), we obtain:

\[
\Delta c = (1 - \theta) \Delta y + (\lambda - \lambda^*) (\theta y - x) + \left(\lambda^f - \lambda^{f^*}\right) (\theta y^* - x^*) \\
= (1 - \theta) \Delta y + (2\lambda - 1) (\theta y - x) + (1 - 2\lambda) (\theta y^* - x^*) \\
= (1 - 2(1 - \lambda) \theta) \Delta y + (1 - 2\lambda) \Delta x
\]
1. Suppose that $\Delta x = 0$ (for simplicity, equivalent to an endowment economy), we obtain

$$\Delta c = (1 - 2 (1 - \lambda) \theta) \Delta y$$

so that we obtain full consumption risk sharing iff

$$1 - \lambda = \frac{1}{2\theta}$$

- If $\theta = 1$ (no labor income), then $1 - \lambda = 50\%$. This is Lucas, 1982 pooling result.
- If $\theta = 0.4$ (realistic labor income share), then $1 - \lambda = 1/0.8 = 125\%$. This is Baxter and Jermann: diversification would imply that the country goes short in domestic asset, since capital and labor income are perfectly positively correlated.
- In the data, (US v/s rest of the world, 1990s), $1 - \lambda < 30\%$.

2. More generally, if $\Delta x \neq 0$, then $\Delta c = 0$ requires

$$(1 - 2 (1 - \lambda) \theta) \Delta y = (2\lambda - 1) \Delta x$$

so that

$$\text{corr}(\Delta y, \Delta x) = \frac{1 - 2 (1 - \lambda) \theta}{2\lambda - 1} \text{var}\Delta y$$

In general, there is no portfolio that will deliver perfect risk sharing. The reason is that there is no portfolio that can hedge both output and investment shocks, unless both shocks are perfectly correlated.

### 4.5.2 The two-good model

- Two countries, with a representative agent in each with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log (c_t) + \mu \log (1 - l_t) \right)$$

- Two traded intermediate goods: $a$ (aluminium, US) and $b$ (bricks, Europe) produced from domestic capital and labor.
- Non traded final goods (consumption and investment) produced from intermediate goods
- Home bias in the production of final goods (equivalent to think about the home bias in consumption preferences)

#### 4.5.2.1 Intermediate goods firms

$$\max_{t, x_t} \sum_{t=0}^{\infty} \sum_{s'} Q_t d_t$$

where

$$d_t = q_{at} (y_t - \bar{w}_t l_t) - x_t$$

$$y_t = A_t \bar{k}_t^{\theta_1 - \theta}$$

$$k_t = (1 - \delta) k_{t-1} + x_t$$

$Q_t$ represents the pricing kernel for the domestic intermediate good firm. H-P assume that it is a weighted average of the home and foreign intertemporal rates of substitution, expressed in the same units:

$$Q_t = \beta^t \left( \frac{U_{c_1}}{U_{c_1}} + (1 - \gamma) \frac{1}{r_x} \frac{U_{c_1}}{U_{c_1}} \right)$$

where $r_x$ is the real exchange rate (price of consumption in country 2 relative to consumption in country 1).

$d_t$ represents the dividend of the intermediate goods firm, expressed in terms of the final good in country 1 (sells output at price $q_{at}$ with a wage $w_t$ expressed in terms of the intermediate good).
4.5.2.2 Final goods firms

The final good is produced with a CRS technology:

$$\max_{a_t, b_t} \{ \omega a_t \omega b_t - q_a a_t - q_b b_t \}$$

where $\omega > 1/2$ measures the degree of home bias toward domestic intermediaries. $(1 - \omega)$ is the import share. $G (a_t, b_t) = a_t^{1-\omega} b_t^{1-\omega}$ can be thought of as a preference aggregator, or as a production function for final goods.

4.5.2.3 Consumers

solve

$$\max_{\lambda_t, \lambda_t^*, c_t} \sum_{t=0}^{\infty} \beta^t (\log c_t + \mu \log (1 - l_t))$$

$$c_t + P_t (\lambda_t - \lambda_{t-1}) + rx_t P_t^* (\lambda_t^* - \lambda_{t-1}^*)$$

$$= q_a w l_t + \lambda_{t-1} d_t + \lambda_{t-1}^* r x_t d^*_t$$

where $P_t$ is the price (in domestic goods) of a unit of the home tree, and $P_t^*$ is the price (in terms of the foreign tree) of a unit of the foreign tree.

4.5.2.4 Equilibrium

intermediate goods markets:

$$a_t + a_t^* = y_t$$
$$b_t + b_t^* = y_t^*$$

final goods markets

$$c_t + x_t = G (a_t, b_t)$$
$$c_t^* + x_t^* = G (a_t^*, b_t^*)$$

asset markets:

$$\lambda_t + \lambda_t^* = 1$$
$$\lambda_t^* + \lambda_t^{**} = 1$$

4.5.2.5 Results

Since final good production is cobbdouglas, we have:

$$q_a a = \omega G = \omega (c + x)$$
$$rx q_b b^* = (1 - \omega)(c + x)$$
$$q_a a^*/rx = (1 - \omega)(c^* + x^*)$$
$$q_b^* b^* = \omega (c^* + x^*)$$

Define

$$\Delta y = (q_a y - rx q_b y^*)$$
$$\Delta c = c - rx c^*$$
$$\Delta x = x - rx x^*$$
we obtain:

\[ q_a y = \omega (c + x) + (1 - \omega) \, rx \, (c^* + x^*) \]
\[ rx q_b^* y^* = (1 - \omega) (c + x) + \omega \, rx \, (c^* + x^*) \]

so that subtracting:

\[ \Delta y = (2\omega - 1) (\Delta c + \Delta x) \]

this equation relates differences in absorption to differences in output.

- If \( \omega = 1/2 \) (no home bias in trade), then \( \Delta y = 0 \) despite differences in absorption.
- if \( \omega = 1 \), then changes in absorption impact one to one changes in output. \( \Delta y = (\Delta c + \Delta x) \)

4.5.2.5.1 Suppose now that \( \Delta x = 0 \). Then, there is perfect risk sharing, regardless of \( \lambda \).

Combining the budget constraint

\[ \Delta c = (1 - 2 (1 - \lambda) \theta) \Delta y \]

one obtains:

\[ \Delta c = (1 - 2 (1 - \lambda) \theta) (2\omega - 1) \Delta c \]

since all these parameters are set, this implies that \( \Delta c = \Delta y = 0 \). This is Cole and Obstfeld (1991): the equilibrium terms of trade \( p = y^*/y \) (log preferences) and absorbs movements in output. There is no need for financial assets to obtain perfect risk sharing.

4.5.2.5.2 When investment is optimal. with optimal investment is undertaken, HP show that there exists an optimal share that delivers perfect risk sharing:

\[ 1 - \lambda = \frac{1 - \omega}{1 + \theta - 2\omega\theta} \]

why? combine the two equations:

\[ \Delta y = (2\omega - 1) (\Delta c + \Delta x) \]
\[ \Delta c = (1 - 2 (1 - \lambda) \theta) \Delta y + (1 - 2\lambda) \Delta x \]

and set \( \Delta c = 0 \) to obtain the result. One needs to show that this is also consistent with optimal portfolio choice (\( \lambda \) and \( \lambda^* \)).

- when \( \omega = 1/2 \), then \( 1 - \lambda = 1/2 \) so that there is full financial diversification.
- when \( \omega = 1 \), then \( 1 - \lambda = 0 \). No financial diversification

In general, diversification is decreasing in \( \omega \): financial diversification decreases with trade bias. Further, if \( \omega > 1/2 \), then diversification increases with \( \theta \), unlike Baxter and Jermann. For instance,

- with \( \theta \to 0 \), \( 1 - \lambda = 1 - \omega \), so it is between 0 and 1/2.
- with \( \theta = 1 \), \( 1 - \lambda = 1/2 \) so that when there is no labor income, there is full financial diversification.
4.5.3 Discussion

1. In the absence of investment, risk sharing obtains independently of financial asset trade. So optimal $\lambda$ is needed to take care of investment shocks. Consider the case where $x \uparrow$ but not $x^*$, $y$ or $y^*$.

   - no diversification ($\lambda = 1$)
     from the budget constraint,
     $$c = q_a (1 - \theta) y + \lambda (q_a \theta y - x) + (1 - \lambda) rx (q_a^* y^* - x^*)$$
     so that $\Delta c = -\Delta x$. The increase in investment is financed from an decrease in consumption, without any change in output or the real exchange rate.

   - Positive diversification ($\lambda < 1$). Now $\Delta c < -\Delta x$. Some of the investment is financed from abroad so that $\Delta c + \Delta x = \Delta y > 0$. Since by construction output is unchanged, we must have a decline in $rx$: a real appreciation that increases the purchasing power of domestic consumption.

   - Optimal diversification: $\lambda = \frac{1 - \theta}{1 - \theta + 2 \omega}$ in that case, we have $\Delta c = 0$. The movement in the real exchange rate absorbs the change in investment.

2. Why is international diversification small? Price movements (here the real exchange rate) do most of the job.

3. Why is diversification positively related to the trade share $(1 - \omega)$?
   With large trade shares, price movements needed are smaller and more diversification is needed to obtain the right price changes.

4. Why is diversification related to capital share?
   Relative price movement works off relative labor income, not capital income. The bigger the labor share, the larger the impact of relative price changes.

4.5.4 Empirical work

rewrite optimal share as
$$\frac{1}{1 - \lambda} = 2\theta + (1 - \theta) \frac{1}{1 - \omega}$$
so inverse of foreign asset share related to inverse of trade share.