Econ 815 - Part 3

Derivatives

1. Consider an example. Suppose you have a call option with strike price 10, and the current price is 5. If this is the situation at expiration, then obviously the option is worthless. Now suppose this is the situation 6 months before expiration. Is the option still worthless? No, because the price might rise by more than 5 during the next 6 months.

2. It's because I defined x to be the total dollar value in the stock (i.e., number of shares times price per share). If instead I had defined x to be the number of shares, then you're right, there would be an S there.

3. The key idea here is that with continuous trading, there are really only two one-step ahead states (e.g., up or down). As a result, you really only need two linearly independent assets to span the state space. Any extra assets are 'redundant', and can be priced by no arbitrage. You can either use the stock and the call option to form a riskless portfolio (i.e, replicate the payoff on the riskless asset), or use the stock and the bond to replicate the call option payoff. BS used the first one in their paper. (Your language is a bit confusing, since if the stock and option were *perfectly* correlated, you could not combine them into a portfolio that was riskless).

4. The volatility smile is a plot of 'implied volatilities' for alternative strike prices. (Remember, the implied volatility is inferred from observed option prices, i.e., it's the volatility, σ^2, that makes the BS predicted price equal the observed price). Since the options all pertain to the same underlying stock, the Black-Scholes formula predicts that this should be a flat line. In practice, observed option prices tend to be higher than the Black-Scholes prediction for deep in the money and deep out of the money option (ie, those with really high and low strick prices).

Lucas Model

1. You don't need to worry about ARCH/GARCH effects. They concern autocorrelated conditional variances. That is, the fact that if things are volatile right now, they are more likely to be volatile tomorrow. Volatility is not i.i.d. In contrast, martingales concern the MEAN, not the variance. If something is a martingale, it just means that the current value is the best predictor of tomorrow's value, or alternatively, the expected CHANGE is zero. It's quite possible for something to be a martingale, with no expected change in the price, while at the same time there is predictability in the variance.

2. It's because dividends play a dual role in the Lucas model. They are the underlying cash flows that make the asset valuable. At the same time, in equilibrium, they represent the consumption that people care about, and so influence the risk premium. Roughly speaking, the current price of the stock is the expectation of the product of the marginal utility of consumption and the payoff, which is the dividend. If D is expected to go up, it increases the expected payoff, which would make the stock more valuable. But at the same time, it LOWERS the marginal utility of consumption, due to diminishing marginal utility. For the log utility case these offsetting effects exactly cancel, and the stock price becomes independent of expected future dividend changes.

3. In terms of economics, it tells us nothing new. The discrete-state Markov chain example just shows you how to COMPUTE asset prices, without having to do a Taylor series approximation. Of course, there's no such thing as a free lunch, so the Markov chain strategy requires a different sort of approximation, i.e., you must 'discretize' the state space.

4. The determination of interest rates is actually pretty complicated. People write books about it. However, in the risk-free case, it basically boils down to 3 main determinants: (1) Impatience, (2) Intertemporal Substitution/Consumption-Smoothing, and (3) Precautionary Saving. Impatience is perhaps the easiest to grasp, although perhaps the most controversial. The classical economists simply asserted based on 'common sense', that people preferred present payoffs to future payoffs. Why? That's a question for people like Arthur, who study evolution. (Who knows, maybe you'll get eaten by a tiger before you receive the payoff). It's not at all obvious why impatience can be summarized in a single parameter, like δ. Some people think it can't. Anyway, the latter two effects both relate to the *curvature* of the utility function. If $U''(\cdot) < 0$, then people prefer *smooth* consumption profiles over their lives. If for example incomes are expected to be higher in the future (due to ongoing growth), then people will attempt to borrow on the basis of their anticipated higher future income. Of course, when *everyone* tries to borrow, all that happens is that the interest rate rises. How much it rises depends on the 'intertemporal elasticity of substitution', which is just the reciprocal of γ in the CRRA case. If the elasticity of substitution is really low, people really do not like uneven consumption paths, so they will have a strong desire to borrow. Therefore the interest rate needs to rise by a lot in order to choke off the excess demand for borrowing. For the 3rd effect, remember that the basic intertemporal Euler equation equates current marginal utility to EXPECTED future marginal utility. From Jensen's inequality, the influence of uncertainty on MARGINAL utility depends on the THIRD derivative of the utility function. If it's positive, people are said to exhibit 'prudence', and they will save more in response to uncertain future income. (Uncertainty increases $E[U'(\cdot)]$ above $U'[E(\cdot)]$, so current consumption falls in order raise $U'(\cdot)$ today).

5. The equity premium puzzle refers to the difference between the average returns on the stock market and the average riskfree return. In the data, it's around $.04 - .07$ for most countries. Since this is just one moment, we can always pick a value of γ that matches the observed equity premium. The reason it's called a 'puzzle' is that you need really high values of γ to match the data. (Assuming, of course, we're using the standard complete markets, representative agent set-up with time-additive CRRA preferences). However, there are two serious problems with using a high γ to explain the equity premium. First, a high γ will generate a very high riskfree rate, higher than what we see in the data. (Remember, *really* high γ's start causing the riskfree rate to decline, since precautionary saving eventually dominates intertemporal substitution). Second, really high γ's seem to contradict other (e.g., introspective) evidence we have about peoples willingness to bear risk. (We did some calculations in class).

Hansen-Jagannathan Bounds

1. The usual strategy in asset pricing is to start with a preference specification, and then see what its predictions for prices are. If they aren't consistent with the observed data, that suggests your preference specifications (or something else), are misspecified. Hansen-Jagannathan go the other direction. They start with observed asset returns, and then ask - "What can we say about preferences?" They work in terms of a so-called 'stochastic discount factor'. A particular assumption about preferences will imply a particular specification for the SDF, but HJ proceed more nonparametrically, and instead derive a bound on the mean and variance of *any* empirically consistent stochastic discount factor. It's quite analogous to the mean/variance frontier we discussed in Sharpe's CAPM model. It can be related to the equity premium puzzle because remember that the basic pricing equation we had was $E(r) = r_f + \gamma \text{cov}(r, g)$, where in the Lucas model g is the growth rate of (per capita) consumption. The basic problem with the Lucas model is that $\text{cov}(r, g)$ is quite small, so the model doesn't generate much of an excess return. HJ basically replace g with and unobserved m (i.e., the stochastic discount factor), and infer its properties from observed r's.

2. If an asset has stricly positive payoffs, then it must have a strictly positive price, otherwise you could combine assets to generate a stictly positive payoff which costs you no up-front money. m must be strictly positive in order to rule out these sort of arbitrage possibilities. Remember that from the perspective of an individual agent, m is just the intertemporal elasticity of substitution (across both

dates and states). If markets are complete, then Pareto Optimality implies that everyone has the same marginal rates of substitution across dates and states (if they didn't there would be further gains from trade). Hence, everyone's m is the same (i.e., m is 'unique').

3. From a practical/empirical standpoint, we're mainly interested in the *minimum* variance stochastic discount factor, simply because standard models generate m's that are nowhere near volatile enough. We just want to reach the bar. Of course, if you can jump way over it, great.

4. Yes, with the clarification that the dots show model-implied stochastic discount factors, rather than different 'prices of risk' (which are closely related). (In the rep agent/CRRA case, the price of risk is just γ, whereas the stochastic discount factor is $\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$.

Rational Bubbles

1. Calling anything 'rational' is unfortunate, in my opinion. It's a loaded word. In this particular context, it refers to the fact that investors are adhering to the so-called Rational Expectations Hypothesis. Their expectations are correct (on average). Due to the presence of expectational feedback in most asset pricing equations, bubbles can emerge whenever expectations of a price rise cause prices to rise today.

2. π is the prob that the bubble continues. Notice the (mean) growth rate of B_t is $1/(\beta\pi)$. Hence, the smaller is π (ie, the more likely the bubble collapses), the faster must prices rise before it collapses. What matters is the mean price increase. It has to equal $1/\beta$. If there's some probability that the growth rate is less than this, then there must be some probability that it grows faster.

3. Yes!

Harrison and Kreps

1. Yes, exactly. Since switches between states 0 and 1 are random, so are prices. Trading occurs whenever the state switches, and so do prices. Since there are only two agents, trading simply consists of an exchange of the outstanding shares from one agent to the other. There's no sense in which trading volume increases when the volatility of price changes increases, as seen in the data. So the statement that trading volume is constant is not quite accurate. It's not accurate period-by-period, because the state might not change. Volume kind of switches on and off.

2. There's an old saying, 'progress not perfection'. The fact that the model generates at least *some* speculative trading volume is progress. Most (rational) models generate none. But the fact that the model cannot match the correlations between prices and volume *is* a problem, but it gives graduate students something to work on!

Scheinkman

1. Asset markets are inherently forward-looking. Prices are determined by what they're expected to be tomorrow. As in any forward-looking problem, the way to solve it is to work backwards. (Great saying from Pascal - "Life must be lived forwards, but understood backwards" (or something like that).

2. In Scheinkman's model, q indexes belief heterogeneity. If $q = 0$, beliefs are always identical, and so no speculative trading occurs. The bigger q is, the more likely the overconfident agents will have different beliefs.

Grossman

1. Yes, by observing the price, agents can effectively infer the average value of all signals, which from a statistical standpoint, dominates any individual's signal (ie, same mean, but lower variance).

2. I'm assuming you mean intuition behind Grossman-Stiglitz (1980), not Grossman (1976). In the first article I talked about (Grossman (1976)), price perfectly aggregated information. If information was costly to obtain, there could be no equilibrium. In the Grossman-Stiglitz paper, info is costly, but the price cannot perfectly reveal it to the uninformed (because of the assumption that the supply of the asset is random. In Grossman (1976), the supply was *not* random).