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Econ 842 International Monetary Economics Prof. Kasa Spring 2014

## PROBLEM SET 2 (Due April 11)

A Stylized Currency Crisis Model: Consider the following game between a central bank and a group of speculators. There is a continuum of atomistic speculators indexed on [0, 1], each possessing a single unit of domestic currency. By assumption then, a single speculator can neither force a devaluation nor prevent one. The economy is characterized by the state of "fundamentals" indexed by  $\theta$ , with  $\theta$  being uniformly distributed on the interval [0, 1]. Higher values of  $\theta$  represent "stronger" fundamentals. Define the shadow exchange rate (ie, the rate prevailing in the absence of central bank intervention) by  $f(\theta)$ , where f is a continuous strictly increasing function.

Initially, the exchange rate (defined as the value of domestic currency) is pegged at  $\bar{e}$ , with  $\bar{e} \geq f(\theta)$  for all  $\theta$ . Each speculator must decide whether to sell or hold his unit of domestic currency. Selling the domestic currency (ie, attacking the peg) involves paying a transaction cost of  $\tau$ . However, if an attack is successful and the central bank abandons the peg, speculators receive a payoff of  $\bar{e} - f(\theta) \equiv D(\theta)$ , or a net return of  $D(\theta) - \tau$ . If the central bank withstands an attack, then each attacking speculators ends up with  $-\tau$ . For simplicity, assume that not attacking the peg involves no (opportunity) costs, so non-attackers always have a payoff of zero. To make things interesting, assume that  $\bar{e} - f(1) = D(1) < \tau$ , so that if fundamentals are known to be sufficiently strong speculators will refrain from attacking.

Turning to the central bank's payoffs, assume the central bank derives a value of v from keeping the exchange rate fixed (don't ask why), but also faces a cost of c when defending the peg. Assume that this cost is decreasing in the current state of fundamentals,  $\theta$ , and increasing in the number of attacking speculators, m. Hence, the central bank's net payoff from defending the peg is  $v - c(\theta, m)$ . Again to make things interesting, assume that c(0,0) > v, so that when fundamentals are really bad, the costs of defending the peg always exceed the benefits from maintaining it, even if nobody attacks. At the same time, suppose c(1,1) > v also, so that if enough speculators decide to attack, the central bank will decide to abandon the peg, even when fundamentals are very strong.

Given these assumptions, it is possible to define two threshold values of the fundamentals:

$$\begin{array}{ll} \underline{\theta}: & c(\underline{\theta}, 0) = v\\ \overline{\theta}: & D(\overline{\theta}) = \tau \end{array}$$

Hence,  $\underline{\theta}$  is the value of fundamentals for which the central bank is just indifferent between abandoning the peg and defending it in the absence of any speculation. Similarly, for  $\theta = \overline{\theta}$  speculators are just indifferent between attacking and not attacking. In what follows, assume  $\underline{\theta} < \overline{\theta}$ .

These two thresholds define three regions: For  $\theta \in [0, \underline{\theta})$  the peg is *unstable*, i.e., fundamentals are so bad that the central bank devalues even when nobody attacks. For  $\theta \in (\overline{\theta}, 1]$  the peg is *stable*, ie, fundamentals are so strong that it is a dominant strategy for speculators not to attack. Finally, for  $\theta \in [\underline{\theta}, \overline{\theta}]$  the peg is said to be *ripe for attack*, since if only a few speculators attack the peg will endure, while if a large number attacks the central bank will decide to abandon the peg. According to Obstfeld (EER, 1996), there are multiple equilibria when fundamentals are in this region.

Following Morris and Shin (AER, 1998), let's posit the following information structure. Suppose at the beginning of the game nature selects a value of  $\theta$  from the unit interval. This value is unobserved by speculators. However, conditional on this value, each speculator observes a private signal  $s_i$  of  $\theta$ , drawn uniformly from the interval  $[\theta - \varepsilon, \theta + \varepsilon]$ . Conditional on  $\theta$ , these signals are i.i.d.

across individuals. After observing their signals, speculators must decide simultaneously whether or not to attack the peg. The central bank observes  $\theta$  and the mass of attacking speculators, m, and then decides whether or not to abandon the peg. This game is played only once.

- (a) Explain informally why it is never common knowledge that the peg is stable, regardless of the actual value of  $\theta$ . (Hint: Define "nth-order knowledge", and let  $n \to \infty$ ).
- (b) Prove the following theorem.

**Theorem** (Morris and Shin, 1998) For the above model, there exists a unique equilibrium  $(s^*, \theta^*)$ , such that each speculator receiving the signal  $s \leq s^*$  attacks the peg, and the central bank abandons the peg if and only if  $\theta \leq \theta^*$ .

(Hints: (1) Work backwards. Derive the central bank's optimal strategy contingent on the mass of attacking speculators. Then analyze the game between speculators. (2) Show that the critical mass of speculators,  $a(\theta)$ , that induces the central bank to abandon the peg is a non-decreasing function of  $\theta$ , which is zero below  $\underline{\theta}$  and monotonically increasing above  $\underline{\theta}$ . Define a function  $\sigma(s)$  as the proportion of speculators who attack after observing a signal of s. Let  $m(\theta, \sigma)$  be the proportion of speculators who end up attacking the peg when the state is  $\theta$  and the aggregate attack strategy is  $\sigma$ . Note that  $m(\theta, \sigma) = \frac{1}{2\varepsilon} \int_{\theta-\varepsilon}^{\theta+\varepsilon} \sigma(s) ds$ . Hence, the event that the central bank abandons the peg is defined by the set,  $\mathcal{A}(\sigma) = \{\theta : m(\theta, \sigma) \ge a(\theta)\}$ . (3) Use  $\mathcal{A}(\sigma)$  to define the payoff function,  $h(\theta, \sigma)$ , of an attacking speculator, (4) Use  $h(\theta, \sigma)$  to define the expected payoff from attacking,  $u(s, \sigma)$ , given an observed signal of s. (5) Prove that there is a unique threshold,  $s^*$ , below which a speculator attacks. (6) Show that  $m(\theta, \sigma(s^*))$  is a decreasing function of  $\theta$ , and that it intersects  $a(\theta)$  at a unique interior value,  $\theta^*$ .)

(c) Show that greater "transparency" reduces the likelihood of a currency attack. That is, derive the comparative static result,  $\frac{d\theta^*}{d\varepsilon} > 0$ .