Lectures in Open Economy Macroeconomics[1]

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Contents

Chapter 1

A First Look at the Data

In discussions of business cycles in small open economies, a critical distinction is between developed and emerging economies. The group of developed economies is typically defined by countries with high income per capita, and the group of emerging economies is composed of middle income countries. Examples of developed small open economies are Canada and Belgium, and examples of small open emerging economies are Argentina and Malaysia.

A striking difference between developed and emerging economies is that observed business cycles in emerging countries are about twice as volatile as in developed countries. Table 1.1 illustrates this contrast by displaying key business-cycle properties in Argentina and Canada. The volatility of detrended output is 4.6 in Argentina and only 2.8 in Canada. Another remarkable difference between developing and developed countries suggested by the table is that the trade balance-to-output ratio is much more countercyclical in emerging countries than in developed countries. Periods of economic boom (contraction) are characterized by relatively larger trade

Table 1.1: Business Cycles in Argentina and Canada

Variable	σ_x	$corr(x_t, x_{t-1})$	$corr(x_t, GDP_t)$
GDP			
Argentina	4.6	0.79	1
Canada	2.8	0.61	1
Consumption			
Argentina	5.4		0.96
Canada	2.5	0.70	0.59
Investment			
Argentina	13.3		0.94
Canada	9.8	0.31	0.64
TB/GDP			
Argentina	2.3		-0.84
Canada	1.9	0.66	-0.13
Hours			
Argentina	4.1		0.76
Canada	2.0	0.54	0.80
Productivity			
Argentina	3.0		0.48
Canada	1.7	0.37	0.70

Source: Mendoza (1991), Kydland and Zarazaga (1997). Standard deviations are measured in percentage points from trend. For Argentina, data on hours and productivity are limited to the manufacturing sector.

deficits (surpluses) in emerging countries than in developed countries. A third difference between Canadian and Argentine business cycles is that in Argentina consumption appears to be more volatile than output at business-cycle frequencies, whereas the reverse is the case in Canada. Two additional differences between the business cycle in Argentina and Canada are that in Argentina the correlation of the domestic components of aggregate demand (consumption and investment) with GDP are twice as high as in Canada, and that in Argentina hours and productivity are less correlated with GDP than in Canada.

One dimension along which business cycles in Argentina and Canada are similar is the procyclicality of consumption, investment, hours, and productivity. In both countries, these variables move in tandem with output.

The differences between the business cycles of Argentina and Canada turn out to hold much more generally between emerging and developed countries. Table 1.2 displays average business cycle facts in developed and emerging economies. The table displays average second moments of detrended data for 13 small emerging countries and 13 small developed countries (the list of countries appears at the foot of the table). For all countries, the time series are at least 40 quarters long. The data is detrended using a band-pass filter that leaves out all frequencies above 32 quarters and below 6 quarters. The data shown in the table is broadly in line with the conclusions drawn from the comparison of business cycles in Argentina and Canada. In particular, emerging countries are significantly more volatile and display a much more countercyclical trade-balance share than developed countries. Also, consumption is more volatile than output in emerging countries but

Table 1.2: Business Cycles: Emerging Vs. Developed Economies

Moment	Emerging Countries	Developed Countries
σ_y	2.02	1.04
$\sigma_{\Delta y}$	1.87	0.95
ρ_y	0.86	0.9
$\rho_{\Delta y}$	0.23	0.09
σ_c/σ_y	1.32	0.94
σ_i/σ_y	3.96	3.42
$\sigma_{tb/y}$	2.09	0.71
$\rho_{tb/y,y}$	-0.58	-0.26
$\rho_{c,y}$	0.74	0.69
$\rho_{i,y}$	0.87	0.75

Note: Average values of moments for 13 small emerging countries and 13 small developed countries. Emerging countries: Argentina, Brazil, Ecuador, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Slovak Republic, South Africa, Thailand, and Turkey. Developed Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland. Data are detrended using a band-pass filter including frequencies between 6 and 32 quarters with 12 leads and lags.

Source: Aguiar and Gopinath (2004).

less volatile than output in developed countries.

Chapter 2

An Endowment Economy

The purpose of this chapter is to build a canonical dynamic, general equilibrium model of the small open economy capable of capturing some of the empirical regularities of business cycles in small emerging and developed countries documented in chapter 1. The model developed in this chapter is simple enough to allow for a full characterization of its equilibrium dynamics using pen and paper.

2.1 The Model Economy

Consider an economy populated by a large number of infinitely lived households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{2.1}$$

where c_t denotes consumption and U denotes the single-period utility function, which is assumed to be strictly increasing and strictly concave.

Each period, households receive an exogenous and stochastic endowment and have the ability to borrow or lend in a risk-free real bond that pays a constant interest rate. The evolution of the debt position of the representative household is given by

$$d_t = (1+r)d_{t-1} + c_t - y_t, \tag{2.2}$$

where d_t denotes the debt position assumed in period t, r denotes the interest rate, assumed to be constant, and y_t is an exogenous and stochastic endowment of goods. This endowment process represents the sole source of uncertainty in this economy. The above constraint states that the change in the level of debt, $d_t - d_{t-1}$, has two sources, interest services on previously acquired debt, rd_{t-1}, and excess expenditure over income, $c_t - y_t$. Households are subject to the following borrowing constraint that prevents them from engaging in Ponzi games:

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1+r)^j} \leq 0. \tag{2.3}$$

This limit condition states that the household's debt position must be expected to grow at a rate lower than the interest rate r. The optimal allocation of consumption and debt will always feature this constraint holding with strict equality. This is because if the allocation $\{c_t, d_t\}_{t=0}^{\infty}$ satisfies the no-Ponzi-game constraint with strict inequality, then one can choose an alternative allocation $\{c'_t, d'_t\}_{t=0}^{\infty}$ that also satisfies the no-Ponzi-game constraint and satisfies $c'_t \geq c_t$, for all $t \geq 0$, with $c'_{t'} > c_{t'}$ for at least one date $t' \geq 0$. This alternative allocation is clearly strictly preferred to the original one because the single period utility function is strictly increasing.

The household chooses processes for c_t and d_t for $t \geq 0$, so as to maximize (2.1) subject to (2.2) and (2.3). The optimality conditions associated with this problem are (2.2), (2.3) holding with equality, and the following Euler condition:

$$U'(c_t) = \beta(1+r)E_t U'(c_{t+1}). \tag{2.4}$$

The interpretation of this expression is simple. If the household sacrifices one unit of consumption in period t and invests it in financial assets, its period-t utility falls by $U'(c_t)$. In period $t+1$ the household receives the unit of goods invested plus interests, $1+r$, yielding $\beta(1+r)E_t U'(c_{t+1})$ utils. At the optimal allocation, the cost and benefit of postponing consumption must equal each other in the margin.

We make two additional assumptions that greatly facilitates the analysis. First we require that the subjective and pecuniary rates of discount, β and $1/(1+r)$, be equal to each other, that is,

$$\beta(1+r) = 1.$$

This assumption eliminates long-run growth in consumption. Second, we assume that the period utility index is quadratic and given by

$$U(c) = -\frac{1}{2}(c - \bar{c})^2, \tag{2.5}$$

with $c < \bar{c}$.[1] This particular functional form makes it possible to obtain a closed-form solution of the model. Under these assumptions, the Euler condition (2.4) collapses to

$$c_t = E_t c_{t+1}, \tag{2.6}$$

which says that consumption follows a random walk; at each point in time, households expect to maintain a constant level of consumption.

We now derive an intertemporal resource constraint by combining the household's sequential budget constraint (2.2) and the no-Ponzi-scheme constraint (2.3) holding with equality—also known as the transversality condition. Begin by expressing the sequential budget constraint in period t as

$$(1+r)d_{t-1} = y_t - c_t + d_t.$$

Lead this equation 1 period and use it to get rid of d_t:

$$(1+r)d_{t-1} = y_t - c_t + \frac{y_{t+1} - c_{t+1}}{1+r} + \frac{d_{t+1}}{1+r}.$$

Repeat this procedure s times to get

$$(1+r)d_{t-1} = \sum_{j=0}^{s} \frac{y_{t+j} - c_{t+j}}{(1+r)^j} + \frac{d_{t+s}}{(1+r)^s}.$$

Apply expectations conditional on information available at time t and take the limit for $s \to \infty$ using the transversality condition (equation (2.3) hold-

[1]After imposing this assumption, our model becomes essentially Hall's (1978) permanent income model of consumption.

ing with equality) to get the following intertemporal resource constraint:

$$(1+r)d_{t-1} = E_t \sum_{j=0}^{\infty} \frac{y_{t+j} - c_{t+j}}{(1+r)^j}.$$

Intuitively, this equation says that the country's initial net foreign debt position must equal the expected present discounted value of current and future differences between output and absorption.

Now lead the Euler equation (2.6) one period to obtain $c_{t+1} = E_{t+1}c_{t+2}$. Take expectations conditional on information available at time t and use the las of iterated expectations to obtain $E_t c_{t+1} = E_t c_{t+2}$. Finally, using again the Euler equation (2.6) to replace $E_t c_{t+1}$ by c_t, we can write $c_t = E_t c_{t+2}$. Repeating this procedure j times, we can deduce that $c_t = E_t c_{t+j}$. Use this result to get rid of expected future consumption in the above intertemporal resource constraint to obtain (after slightly rearranging terms)

$$rd_{t-1} + c_t = \frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}. \tag{2.7}$$

This expression states that the optimal plan allocates the annuity value of the income stream $\frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}$ to consumption, c_t, and to debt service, rd_{t-1}. To be able to fully characterize the equilibrium in this economy, we assume that the endowment process follows an AR(1) process of the form,

$$y_t = \rho y_{t-1} + \epsilon_t,$$

where ϵ_t denotes an i.i.d. innovation and the parameter $\rho \in (-1, 1)$ defines the serial correlation of the endowment process. The larger is ρ, the more persistent is the endowment process. Given this autoregressive structure of the endowment, the j-period-ahead forecast of output in period t is given by

$$E_t y_{t+j} = \rho^j y_t.$$

Using this expression to eliminate expectations of future income from equation (2.7), we obtain

$$\begin{aligned} rd_{t-1} + c_t &= y_t \frac{r}{1+r} \sum_{j=0}^{\infty} \left(\frac{\rho}{1+r} \right)^j \\ &= \frac{r}{1+r-\rho} y_t. \end{aligned}$$

Solving for c_t, we obtain

$$c_t = \frac{r}{1+r-\rho} y_t - rd_{t-1}. \tag{2.8}$$

Because ρ is less than unity, we have that a unit increase in the endowment leads to a less-than-unit increase in consumption.

Two key variables in open economy macroeconomics are the trade balance and the current account. The trade balance is given by the difference between exports and imports of goods and services. In the present model, the trade balance is given by the difference between output and consumption. Formally, letting tb_t denote the trade balance in period t, we have that $tb_t \equiv y_t - c_t$. The current account is defined as the sum of the trade balance and net investment income on the country's net foreign asset position. Formally, letting ca_t denote the current account in period t, we have that

$ca_t \equiv -rd_{t-1} + tb_t$. We can then write the equilibrium levels of the trade balance and the current account as:

$$tb_t = rd_{t-1} + \frac{1-\rho}{1+r-\rho} y_t$$

and

$$ca_t = \frac{1-\rho}{1+r-\rho} y_t.$$

Note that the current account inherits the stochastic process of the underlying endowment shock. Because the current account equals the change in the country's net foreign asset position, i.e., $ca_t = -(d_t - d_{t-1})$, it follows that the equilibrium evolution of the stock of external debt is given by

$$d_t = d_{t-1} - \frac{1-\rho}{1+r-\rho} y_t.$$

According to this expression, external debt follows a random walk and is therefore nonstationary. A temporary increase in the endowment produces a gradual but permanent decline in the stock of foreign liabilities. The long-run behavior of the trade balance is governed by the dynamics of external debt. Thus, an increase in the endowment leads to a permanent deterioration in the trade balance.

2.2 Response to Output Shocks

Consider the response of our model economy to an unanticipated increase in output. Assume that $0 < \rho < 1$, so that endowment shocks are positively serially correlated. Two polar cases are of interest. In the first case, the endowment shock is assumed to be purely transitory, $\rho = 0$. According to equation (2.8), when innovations in the endowment are purely temporary only a small part of the changes in income—a fraction $r/(1+r)$—is allocated to current consumption. Most of the endowment increase—a fraction $1/(1+r)$—is saved. The intuition for this result is clear. Because income is expected to fall quickly to its long-run level, households smooth consumption by eating a tiny part of the current windfall and leaving the rest for future consumption. In this case, the current account plays the role of a shock absorber. Households borrow to finance negative income shocks and save in response to positive shocks. It follows that the more temporary are endowment shocks, the more volatile is the current account. In the extreme case of purely transitory shocks, the standard deviation of the current account is given by $\sigma_y/(1+r)$, which is close to the volatility of the endowment shock itself for small values of r. More importantly, the current account is procyclical. That is, it improves during expansions and deteriorates during contractions. This prediction represents a serious problem for this model. For, as documented in chapter 1, the current account is countercyclical in small open economies, especially in developing countries.

The other polar case emerges when shocks are highly persistent, $\rho \to 1$. In this case, households allocate all innovations in their endowments to current consumption, and, as a result, the current account is nil and the stock of debt remains constant over time. Intuitively, when endowment shocks are permanent, an increase in income today is not accompanied by the expectation of a future decline in income. Rather, households expect

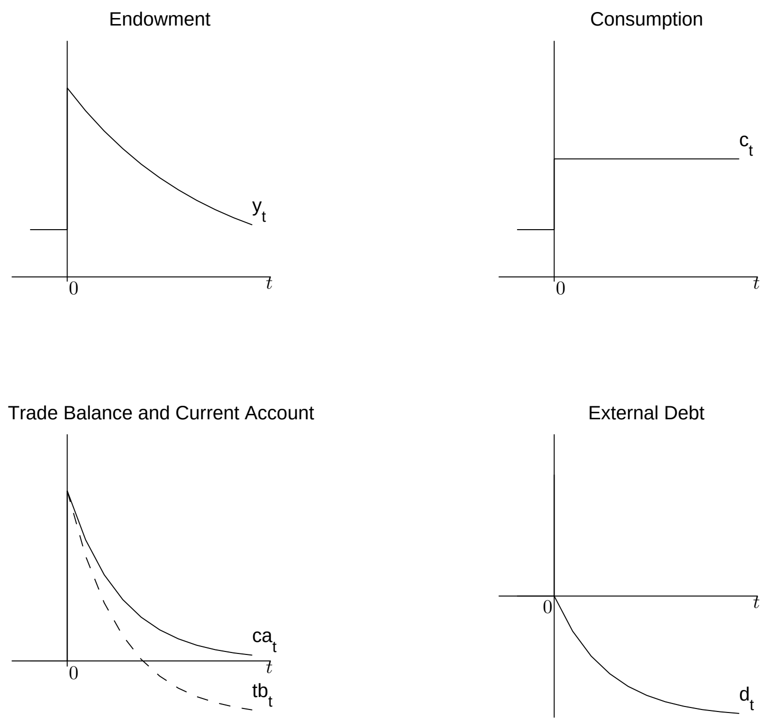


Figure 2.1: Response to a Positive Endowment Shock

the higher level of income to persist over time. As a result, households are able to sustain a smooth consumption path by consuming the totality of the current income shock.

The intermediate case of a gradually trend-reverting endowment process ($\rho \in (0, 1)$) is illustrated in figure 2.1. In response to the positive endowment shock, consumption experiences a once-and-for-all increase. This expansion in domestic absorption is smaller than the initial increase in income. As a result, the trade balance and the current account improve. After the initial increase, these two variables converge gradually to their respective long-run levels. Note that the trade balance converges to a level lower than the pre-shock level. This is because in the long-run the economy settles at a lower level of external debt, which requires a smaller trade surplus to be served.

Summarizing, in this model, which captures the essential elements of what has become known as the *intertemporal approach to the current account*, external borrowing is conducted under the principle: 'finance temporary shocks, adjust to permanent shocks.' A central failure of the model is the prediction of a procyclical current account. Fixing this problem is at the heart of what follows in this and the next two chapters.

2.3 Nonstationary Income Shocks

Suppose now that the rate of change of output, rather than its level, displays mean reversion. Specifically, let

$$\Delta y_t \equiv y_t - y_{t-1}$$

denote the change in endowment between periods $t-1$ and t. Suppose that Δy_t evolves according to the following autoregressive process:

$$\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t,$$

where ϵ_t is an i.i.d. shock with mean zero and variance σ_ϵ^2, and $\rho \in [0, 1)$ is a constant parameter. According to this process, the level of income is nonstationary, in the sense that a positive output shock ($\epsilon_t > 0$) produces a permanent future expected increase in the level of output. Faced with such

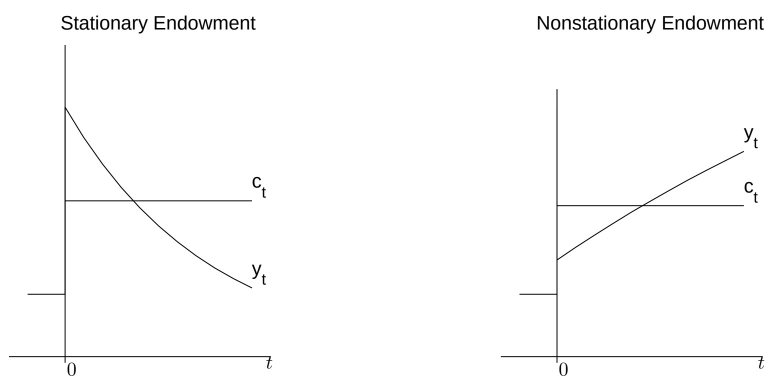


Figure 2.2: Stationary Versus Nonstationary Endowment Shocks

an income profile, consumption-smoothing households have an incentive to borrow against future income, thereby producing a countercyclical tendency in the current account. This is the basic intuition why allowing for a nonstationary output process can help explain the behavior of the trade balance and the current account at business-cycle frequencies. Figure 2.2 provides a graphical expression of this intuition. The following model formalizes this story.

As before, the model economy is inhabited by an infinitely lived representative household that chooses contingent plans for consumption and debt to maximize the utility function (2.5) subject to the sequential resource constraint (2.2) and the no-Ponzi-game constraint (2.3). The first-order conditions associated with this problem are the sequential budget constraint, the no-Ponzi-game constraint holding with equality, and the Euler equation (2.6). Using these optimality conditions yields the expression for consumption given in equation (2.7), which we reproduce here for convenience

$$c_t = -rd_{t-1} + \frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}.$$

Using this expression and recalling that the current account is defined as $ca_t = y_t - c_t - rd_{t-1}$, we can write

$$ca_t = y_t - \frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}.$$

Rearranging, we obtain

$$ca_t = -E_t \sum_{j=1}^{\infty} \frac{\Delta y_{t+j}}{(1+r)^j}.$$

This expression states that the current account equals the present discounted value of future expected income decreases. According to the autoregressive process assumed for the endowment, we have that $E_t \Delta y_{t+j} = \rho^j \Delta y_t$. Using this result in the above expression, we can write the current account as:

$$ca_t = \frac{-\rho}{1+r-\rho} \Delta y_t.$$

According to this formula, the current account deteriorates in response to a positive innovation in output. This implication is an important improvement relative to the model with stationary shocks. Recall that when the endowment level is stationary the current account increases in response to a positive endowment shock.

We note that the countercyclicality of the current account in the model with nonstationary shocks depends crucially on output changes being positively serially correlated, or $\rho > 0$. In effect, when ρ is zero (negative), the current account ceases to be countercyclical (is procyclical). The intuition behind this result is clear. For an unexpected increase in income to induce an increase in consumption larger than the increase in income itself, it is necessary that future income be expected to be higher than current income, which happens only if Δy_t is positively serially correlated.

Are implied changes in consumption more or less volatile than changes in output? This question is important because, as we saw in chapter 1, developing countries are characterized by consumption growth being more volatile than output growth. Formally, letting $\sigma_{\Delta c}$ and $\sigma_{\Delta y}$ denote the standard deviations of $\Delta c_t \equiv c_t - c_{t-1}$ and Δy_t, respectively, we wish to find out conditions under which $\sigma^2_{\Delta c}$ can be higher than $\sigma^2_{\Delta y}$ in equilibrium.[2] We start with the definition of the current account

$$ca_t = y_t - c_t - rd_{t-1}.$$

Taking differences, we obtain

$$ca_t - ca_{t-1} = \Delta y_t - \Delta c_t - r(d_{t-1} - d_{t-2}).$$

[2]Strictly speaking, this exercise is not comparable to the data displayed in chapter 1, because here we analyze changes in consumption and output, whereas in chapter 1 we reported statistics pertaining to the growth rates of consumption and output.

Noting that $d_{t-1} - d_{t-2} = -ca_{t-1}$ and solving for Δc_t, we obtain:

$$\begin{aligned}\Delta c_t &= \Delta y_t - ca_t + (1+r)ca_{t-1} \\ &= \Delta y_t + \frac{\rho}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\ &= \frac{1+r}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\ &= \frac{1+r}{1+r-\rho}\epsilon_t. \end{aligned} \tag{2.9}$$

It follows directly from the AR(1) specificaiton of Δy_t that $\sigma^2_{\Delta y}(1-\rho^2) = \sigma^2_\epsilon$. Then, we can write the standard deviation of consumption changes as

$$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}} = \left[\frac{1+r}{1+r-\rho}\right]\sqrt{1-\rho^2}.$$

The right-hand side of this expression equals unity at $\rho = 0$. This result confirms the one obtained earlier in this chapter, namely that when the *level* of income is a random walk, consumption and income move hand in hand, so their changes are equally volatile. The right hand side of the above expression is increasing in ρ at $\rho = 0$. It follows that there are values of ρ in the interval $(0, 1)$ for which the volatility of consumption changes is indeed higher than that of income changes. This property ceases to hold as Δy_t becomes highly persistent. This is because as $\rho \to 1$, the variance of Δy_t becomes infinitely large as changes in income become a random walk, whereas, as expression (2.9) shows, Δc_t follows an i.i.d. process with finite variance for all values of $\rho \in [0, 1)$.

2.4 Testing the Model

Hall (1978) was the first to explore the econometric implication of the simple model developed in this chapter. Specifically, Hall tested the prediction that consumption must follow a random walk. Hall's work motivated a large empirical literature devoted to testing the empirical relevance of the model described above. Campbell (1987), in particular, deduced and tested a number of theoretical restrictions on the equilibrium behavior of national savings. In the context of the open economy, Campbell's restrictions are readily expressed in terms of the current account. Here we review these restrictions and their empirical validity.

We start by deriving a representation of the current account that involves expected future changes in income. Noting that the current account in period t, denoted ca_t, is given by $y_t - c_t - rd_{t-1}$ we can write equation (2.7) as

$$-(1+r)ca_t = -y_t + rE_t \sum_{j=1}^{\infty}(1+r)^{-j} y_{t+j}.$$

Defining $\Delta x_{t+1} = x_{t+1} - x_t$, it is simple to show that

$$-y_t + rE_t \sum_{j=1}^{\infty}(1+r)^{-j} y_{t+j} = (1+r)\sum_{j=1}^{\infty}(1+r)^{-j} E_t \Delta y_{t+j}.$$

Combining the above two expression we can write the current account as

$$ca_t = -\sum_{j=1}^{\infty}(1+r)^{-j} E_t \Delta y_{t+j}. \tag{2.10}$$

Intuitively, this expression states that the country borrows from the rest of the world (runs a current account deficit) income is expected to grow in the future. Similarly, the country chooses to build its net foreign asset position (runs a current account surplus) when income is expected to decline in the future. In this case the country saves for a rainy day.

Consider now an empirical representation of the time series Δy_t and ca_t. Define

$$x_t = \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}.$$

Consider estimating a VAR system including x_t:

$$x_t = Dx_{t-1} + \epsilon_t.$$

Let H_t denote the information contained in the vector x_t. Then, from the above VAR system, we have that the forecast of x_{t+j} given H_t is given by

$$E_t[x_{t+j}|H_t] = D^j x_t.$$

It follows that

$$\sum_{j=1}^{\infty}(1+r)^{-j}E_t[\Delta y_{t+j}|H_t] = \begin{bmatrix} 1 & 0 \end{bmatrix} [I - D/(1+r)]^{-1}D/(1+r) \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}.$$

Let $F \equiv - \begin{bmatrix} 1 & 0 \end{bmatrix} [I - D/(1+r)]^{-1}D/(1+r)$. Now consider running a regression of the left and right hand side of equation (2.10) onto the vector x_t. Since x_t includes ca_t as one element, we obtain that the regression coefficient for the left-hand side regression is the vector $[0 \quad 1]$. The regression coefficients of the right-hand side regression is F. So the model implies the following restriction on the vector F:

$$F = [0 \quad 1].$$

Nason and Rogers (2006) perform an econometric test of this restriction. They estimate the VAR system using Canadian data on the current account and GDP net of investment and government spending. The estimation sample is 1963:Q1 to 1997:Q4. The VAR system that Nason and Rogers estimate includes 4 lags. In computing F, they calibrate r at 3.7 percent per year. Their data strongly rejects the above cross-equation restriction of the model. The Wald statistic associated with null hypothesis that $F = [0 \quad 1]$ is 16.1, with an asymptotic p-value of 0.04. This p-value means that if the null hypothesis was true, then the Wald statistic, which reflects the discrepancy of F from $[0 \quad 1]$, would take a value of 16.1 or higher only 4 out of 100 times.

Consider now an additional testable cross-equation restriction on the theoretical model. From equation (2.10) it follows that

$$E_t ca_{t+1} - (1+r)ca_t - E_t \Delta y_{t+1} = 0. \tag{2.11}$$

According to this expression, the variable $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ is unpredictable in period t. In particular, if one runs a regression of this variable on current and past values of x_t, all coefficients should be equal to zero.[3]

[3] Consider projecting the left- and right-hand sides of this expression on the information set H_t. This projection yields the orthogonality restriction $[0 \quad 1][D-(1+r)I]-[1 \quad 0]D = [0 \quad 0]$.

This restriction is not valid in a more general version of the model featuring private demand shocks. Consider, for instance, a variation of the model economy where the bliss point is a random variable. Specifically, replace $\bar{c}$ in equation (2.5) by $\bar{c} + \mu_t$, where $\bar{c}$ is still a constant, and μ_t is an i.i.d. shock with mean zero. In this environment, equation (2.11) becomes

$$E_t ca_{t+1} - (1+r)ca_t - E_t \Delta y_{t+1} = \mu_t.$$

Clearly, because in general μ_t is correlated with ca_t, the orthogonality condition stating that $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ be orthogonal to variables dated t or earlier, will not hold. Nevertheless, in this case we have that $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ should be unpredictable given information available in period $t-1$ or earlier.[4] Both of the orthogonality conditions discussed here are strongly rejected by the data. Nason and Rogers (2006) find that a test of the hypothesis that all coefficients are zero in a regression of $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ onto current and past values of x_t has a p-value of 0.06. The p-value associated with a regression featuring as regressors past values of x_t is 0.01.

[4]In particular, one can consider projecting the above expression onto Δy_{t-1} and ca_{t-1}. This yields the orthogonality condition $[0 \quad 1][D - (1+r)I]D - [1 \quad 0]D^2 = [0 \quad 0]$.

Chapter 3

An Economy with Capital

A theme of chapter 2 is that the simple endowment economy model driven by stationary endowment shocks fails to predict the countercyclicality of the trade balance. In this chapter, we show that allowing for capital accumulation can help resolve this problem.

3.1 The Basic Framework

Consider a small open economy populated by a large number of infinitely lived households with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t), \tag{3.1}$$

where c_t denotes consumption, $\beta \in (0,1)$ denotes the subjective discount factor, and U denotes the period utility function, assumed to be strictly increasing, strictly concave, and twice continuously differentiable. Households seek to maximize this utility function subject to the following three constraints:

$$b_t = (1+r)b_{t-1} + y_t - c_t - i_t, \tag{3.2}$$

$$y_t = \theta_t F(k_t),$$

$$k_{t+1} = k_t + i_t,$$

and

$$\lim_{j\to\infty} \frac{b_{t+j}}{(1+r)^j} \geq 0, \tag{3.3}$$

where b_t denotes real bonds bought in period t yielding the constant interest rate $r > 0$, y_t denotes output in period t, k_t denotes the stock of physical capital, and i_t denotes investment. The function F describes the production technology and is assumed to be strictly increasing, strictly concave, and to satisfy the Inada conditions. The variable θ_t denotes an exogenous non-stochastic productivity factor. For the sake of simplicity, we assume that the capital stock does not depreciate. Later in these notes, we relax this assumptions of a no depreciation and of deterministic productivity.

The Lagrangian associated with the household's problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t) + \lambda_t \left[(1+r)b_{t-1} + k_t + \theta_t F(k_t) - c_t - k_{t+t} - b_{t+1} \right] \right\}.$$

The first-order conditions corresponding to this problem are

$$U'(c_t) = \lambda_t,$$

$$\lambda_t = \beta(1+r)\lambda_{t+1},$$

$$\lambda_t = \beta\lambda_{t+1}[1 + \theta_{t+1}F'(k_{t+1})],$$

$$b_t = (1+r)b_{t-1} + \theta_t F(k_t) - c_t - k_{t+1} + k_t,$$

and the transversality condition

$$\lim_{t\to\infty} \frac{b_t}{(1+r)^t} = 0.$$

As in the endowment-economy model of chapter 2, we assume that

$$\beta(1+r) = 1,$$

to avoid inessential long-run dynamics. This assumption together with the first two of the above optimality conditions implies that consumption is constant over time,

$$c_{t+1} = c_t; \quad \forall t \geq 0.$$

The above optimality conditions can be reduced to the following two expressions:

$$r = \theta_{t+1}F'(k_{t+1}) \tag{3.4}$$

and

$$c_t = rb_{t-1} + \frac{r}{1+r}\sum_{j=0}^{\infty} \frac{\theta_{t+j}F(k_{t+j}) - k_{t+j+1} + k_{t+j}}{(1+r)^j}, \tag{3.5}$$

for $t \geq 0$. Equilibrium condition (3.4) states that households invest in physical capital until the marginal product of capital equals the rate of return on foreign bonds. It follows from this equilibrium condition that next period's level of physical capital, k_{t+1}, is an increasing function of the future expected level productivity, θ_{t+1}, and a decreasing function of the opportunity cost of holding physical capital, r. Formally,

$$k_{t+1} = \kappa(\theta_{t+1}; r); \quad \kappa_1 > 0, \kappa_2 < 0.$$

Equilibrium condition (3.5) says that consumption equals the interest flow on a broad definition of wealth, which includes not only initial financial wealth, b_{-1}, but also the present discounted value of the differences between output and investment, $\sum_{j=0}^{\infty} \frac{\theta_{t+j}F(k_{t+j})-k_{t+j+1}+k_{t+j}}{(1+r)^j}$. To obtain equilibrium condition (3.5), we follow the same steps as in the derivation of its counterpart for the endowment economy, equation (2.7).

3.1.1 A Permanent Productivity Shock

Suppose that up until period -1 inclusive, the technology factor θ_t was constant and equal to $\bar{\theta}$. Suppose further that before period 0 the economy was in a steady state in which consumption and the capital stock were constant and equal to $\bar{c}$ and $\bar{k} \equiv \kappa(\bar{\theta}, r)$, respectively. Because in this steady state the capital stock is constant and because the depreciation rate of capital is assumed to be zero, we have that investment is constant and equal to zero. The intertemporal resource constraint (3.5) then implies that bond holdings are constant and equal to $\bar{b} \equiv (\bar{c} - \bar{y})/r$, where $\bar{y} \equiv \bar{\theta}F(\bar{k})$ denotes the steady-state level of output. Because the current account equals the change in the net international asset position ($ca_t = b_{t+1} - b_t$), we have that in the steady state the current account equals zero, $\bar{ca} = 0$. Finally, the trade balance is constant and large enough to seve the external debt, that is, $tb_t = \bar{tb} \equiv -r\bar{b}$.

Suppose now that in period 0 there is a permanent, unexpected increase in the technology factor to $\theta' > \bar{\theta}$. That is, $\theta_t = \bar{\theta}$ for $t \leq 1$ and $\theta_t = \theta'$ for $t \geq 0$.

In response to the permanent technology shock, consumption experiences a permanent increase. That is, $c_t = c' > \bar{c}$ for all $t \geq 0$. To see this, consider the following suboptimal paths for consumption and investment: $c_t^s = \theta' F(\bar{k}) + r\bar{b}$ and $i_t^s = 0$ for all $tt \geq 0$. Clearly, because $\theta' > \bar{\theta}$, the consumption path c_t^s is strictly preferred to the pre-shock path, given by $\bar{c} \equiv \bar{\theta} F(\bar{k}) + r\bar{b}$. To show that the proposed allocation is feasible, let us plug the consumption and investment paths c_t^s and i_t^s into the sequential budget constraint (3.2) to obtain the sequence of asset positions $b_t^s = \bar{b}$ for all $t \geq 0$. Obviously, $\lim_{t\to\infty} \bar{b}/(1+r)^t = 0$, so the proposed suboptimal allocation satisfies the no-Ponzi-game condition (3.3). We have established the existence of a feasible consumption path that is strictly preferred to the pre-shock consumption allocation. It follows that the optimal consumption path must also be strictly preferred to the pre-shock consumption path. This result together with the fact that the optimal consumption path is constant starting in period 0—i.e., $c_t = c_{t+1}$ for $t \geq 0$—implies that consumption must experience a permanent increase in period 0.

Because k_0 was chosen in period -1, when households expected θ_0 to be equal to $\bar{\theta}$, we have that $k_0 = \bar{k}$. In period 0, investment experiences a once-and-for-all increase that brings the level of capital up from $\bar{k}$ to a level

$k' \equiv \kappa(\theta', r) > \bar{k}$. Thus, $k_t = k'$ for $t \geq 1$, $i_0 = k' - \bar{k} > 0$, and $i_t = 0$, for $t \geq 1$. Plugging this path for the capital stock into the intertemporal resource constraint (3.5) and evaluating that equation at $t = 0$ we get

$$c_0 = r\bar{b} + \frac{r}{1+r}\left[\theta' F(\bar{k}) - k' + \bar{k}\right] + \frac{1}{1+r}\theta' F(k').$$

The trade balance in period 0 is given by $tb_0 = \theta' F(\bar{k}) - c0 - i_0$. Recalling that $i_0 = k' - \bar{k}$ and usig the above expression to eliminate c_0, we obtain

$$tb_0 = -r\bar{b} - \frac{1}{1+r}\left[\theta' F(k') - \theta' F(\bar{k}) + (k' - \bar{k})\right].$$

Now recall that before period zero, the trade balance is simply equal to $-r\bar{b}$. We can therefore write

$$tb_0 = \bar{tb} - \frac{1}{1+r}\left[\theta' F(k') - \theta' F(\bar{k}) + (k' - \bar{k})\right].$$

Note that the expression within square brackets is unambiguously positive. This implies that in response to the permanent technology shock the trade balance deteriorates in period zero. In period 1 the trade balance improves. To see this, write $tb_1 = y_1 - c_1 - i_1$ and note that between periods 0 and 1 output increases (from $\theta' F(\bar{k})$ to $\theta' F(k')$), consumption is unchanged, and investment falls (from $k' - \bar{k} > 0$ to $k' - k' = 0$. The trade balance remains constant after period 1. The current account deteriorates in period 0 by the same magnitude as the trade balance. To see this, note that $ca_0 = r\bar{b} + tb_0$ and that $\bar{b}$ is predetermined in period 0. In period 1, the current account returns permanently to zero. To see this, it is useful to establish the behavior of net foreign asset holdings from period 1 onward. Evaluating the intertemporal budget constraint (3.5) at any period $t \geq 1$, we readily find that $b_t = (c' - \theta' F(k'))$ for $t \geq 0$. Thtat is net foreign asset holdings are constant from $t = 0$ onward. This means that the current account, given by $ca_t = b_t - b_{t-1}$, equals zero for periods $t \geq 1$. Finally, the identity $b_0 = \bar{b} + ca_0$ and the fact that the current account deteriorates in period 0 imply that net foreign asset holdings fall in period 0.

The assumed persistence of the productivity shock plays a significant role in inducing an initial deterioration of the trade balance and the current account. A permanent increase in productivity induces a strong response in domestic absorption. First, the increase in current and future expected income induced by the permanent technological improvement induces households to expand consumption expenditure. At the same time, the increase in the expected productivity of capital perceived in period 0 induces higher investment in physical capital in that period.

Another important factor in generating a decline in the trade balance in response to a positive productivity shock is the assumed absence of capital adjustment costs. Note that in response to the increase in future expected productivity, the entire adjustment in investment occurs in period zero. Indeed, investment falls to zero in period 1 and remains nil thereafter. In the presence of costs of adjusting the stock of capital, investment spending is spread over a number of periods, dampening the increase in domestic absorption in the date the shock occurs. We will study the role of adjustment costs in more detail shortly.

3.1.2 A Temporary Productivity shock

To stress the importance of persistence in productivity movements in inducing a deterioration of the trade balance in response to a positive output shock, it is worth analyzing the effect of a purely temporary shock. Specifically, suppose that up until period -1 inclusive the productivity factor θ_t was constant and equal to $\bar{\theta}$. Suppose also that in period -1 people assigned a zero probability to the event that θ_0 would be different from $\bar{\theta}$. In period 0, however, a zero probability event happens. Namely, $\theta_0 = \theta' > \bar{\theta}$. Furthermore, suppose that everybody correctly expects the productivity shock to be purely temporary. That is, everybody expects $\theta_t = \bar{\theta}$ for all $t > 0$. In this case, equation (3.4) implies that the capital stock, and therefore also investment, are unaffected by the productivity shock. That is, $k_t = \bar{k}$ for all $t \geq 0$, where $\bar{k}$ is the level of capital inherited in period 0. This is intuitive. The productivity of capital unexpectedly increases in period zero. As a result, households would like to have more capital in that period. But k_0 is fixed in period zero. Investment in period zero can only increase the future stock of capital. But agents have no incentives to have a higher capital stock in the future, because its productivity is expected to go back down to its historic level $\bar{\theta}$ right after period 0.

The positive productivity shock in period zero does produce an increase in output in that period, from $\bar{\theta}F(\bar{k})$ to $\theta' F(\bar{k})$. That is

$$y_0 = y_{-1} + (\theta' - \bar{\theta})F(\bar{k}),$$

where $y_{-1} \equiv \bar{\theta}F(\bar{k})$ is the pre-shock level of output. This output effect induces higher consumption. In effect, using equation (3.5) we have that

$$c_0 = c_{-1} + \frac{r}{1+r}(\theta' - \bar{\theta})F(\bar{k}),$$

where $c_{-1} \equiv rb_{-1} + \bar{\theta}F(\bar{\theta})$ is the pre-shock level of consumption. Basically, households invest the entire increase in output in the international financial market and increase consumption by the interest flow associated with that financial investment.

Combining the above two expressions and recalling that investment is unaffected by the temporary shock, we get that the trade balance in period 0 is given by

$$tb_0 - tb_{-1} = (y_0 - y_{-1}) - (c_0 - c_{-1}) - (i_0 - i_{-1}) = \frac{1}{1+r}(\theta' - \bar{\theta})F(\bar{k}) > 0.$$

This expression shows that the trade balance improves on impact. The reason for this counterfactual response is simple: 'Firms have no incentive to invest, as the increase in the productivity of capital is short lived, and consumers save most of the purely temporary increase in income in order to smooth consumption over time.

Comparing the results obtained under the two polar cases of permanent and purely temporary productivity shocks, we can derive the following principle:

Principle I: The more persistent are productivity shocks, the more likely is the trade balance to experience an initial deterioration.

We will analyze this principle in more detail later, in the context of a model featuring a more flexible notion of persistence.

3.2 Capital Adjustment Costs

Consider now an economy identical to the one analyzed thus far, except that now changes in the stock of capital come at a cost. Capital adjustment costs—in a variety of forms—are a regular feature of small open economy models because they help dampen the volatility of investment over the business cycle (see, e.g., Mendoza, 1991; and Schmitt-Grohé, 1998, among many others). Suppose that the sequential budget constraint is of the form

$$b_t = (1+r)b_{t-1} + \theta_t F(k_t) - c_t - i_t - \frac{1}{2}\frac{i_t^2}{k_t}.$$

Here, capital adjustment costs are given by $i_t^2/(2k_t)$ and are a strictly convex function of investment. Note that the level of this function vanishes at the steady-state value of investment, $i_t = 0$. This means that capital adjustment costs are nil in the steady state. Note further that the slope of the adjustment-cost function, given by i_t/k_t, also vanishes in the steady state. As will be cler shortly, this feature implies that in the steady state the relative price of capital goods in terms of consumption goods is unity.

As in the economy without adjustment costs, we assume that physical capital does not depreciate, so that the law of motion of the capital stock is given by

$$k_{t+1} = k_t + i_t.$$

Households seek to maximize the utility function given in (3.1) subject to the above two restrictions and the no-Ponzi-game constraint (3.3). The Lagrangian associated with this optimization problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t) + \lambda_t \left[(1+r)b_{t-1} + \theta_t F(k_t) - c_t - i_t - \frac{1}{2}\frac{i_t^2}{k_t} - b_t + q_t(k_t + i_t - k_{t+1}) \right] \right\}.$$

The variables λ_t and q_t denote Lagrange multipliers on the sequential budget constraint and the law of motion of the capital stock, respectively. We continue to assume that $\beta(1+r) = 1$. The first-order conditions associated with the household problem are:

$$1 + \frac{i_t}{k_t} = q_t, \tag{3.6}$$

$$(1+r)q_t = \theta_{t+1}F'(k_{t+1}) + \frac{1}{2}(i_{t+1}/k_{t+1})^2 + q_{t+1}, \tag{3.7}$$

$$k_{t+1} = k_t + i_t,$$

$$c_t = rb_{t-1} + \frac{r}{1+r}\sum_{j=0}^{\infty} \frac{\theta_{t+j}F(k_{t+j}) - i_{t+j} - \frac{1}{2}(i_{t+j}^2/k_{t+j})}{(1+r)^j},$$

and

$$c_{t+1} = c_t.$$

In deriving these optimality conditions, we combined the sequential budget constraint with the transversality condition to obtain a single intertemporal budget constraint and made use of the fact that the Lagrange multiplier λ_t is constant for $t \geq 0$.

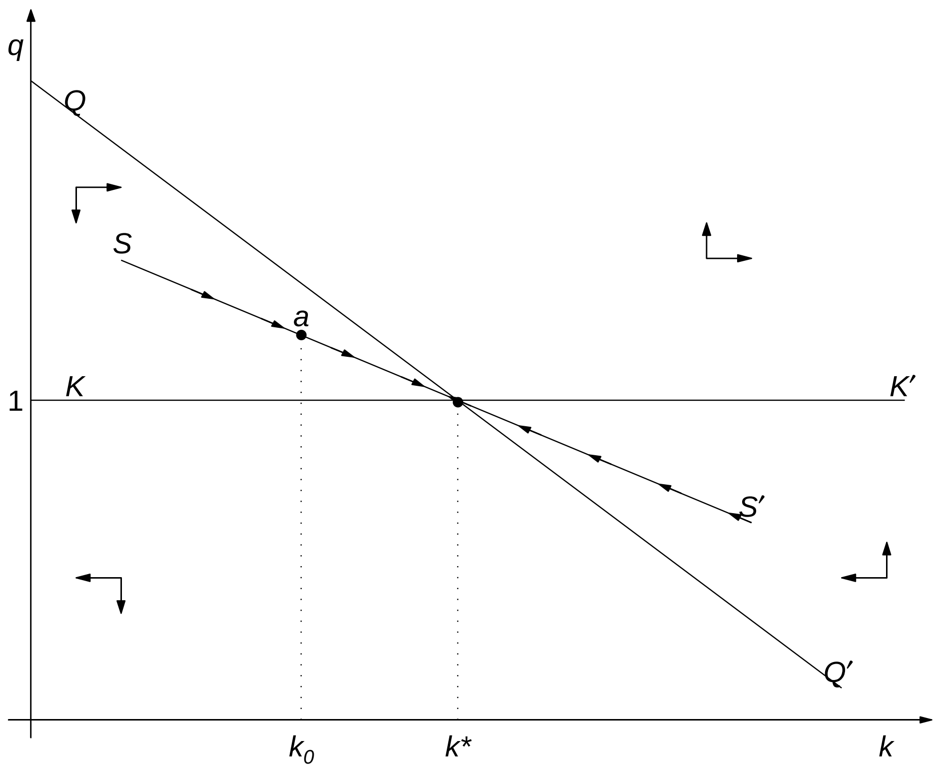
The Lagrange multiplier q_t represents the shadow relative price of capital in terms of consumption goods, and is known as Tobin's q. Optimality condition (3.6) equates the marginal cost of producing a unit of capital, $1 + i_t/k_t$, on the left-hand side, to the marginal revenue of selling a unit of capital, q_t, on the right-hand side. As q_t increases, agents have incentives to devote more resources to the production of physical capital. In turn, the increase in investment raises the marginal adjustment cost, i_t/k_t, which tends to restore the equality between the marginal cost and marginal revenue of capital goods. Equation (3.7) compares the rate of return on bonds (left-hand side) to the rate of return on physical capital (right-hand side). Consider first the rate of return of investing in physical capital. Adding one unit of capital to the existing stock costs q_t. This unit yields $\theta_{t+1}F'(k_{t+1})$ units of output next period. In addition, an extra unit of capital reduces tomorrow's adjustment costs by $(i_{t+1}/k_{t+1})^2/2$. Finally, the unit of capital can be solde next period at the price q_{t+1}. The sum of these three sources of income form the right hand side of (3.7). Alternatively, instead of using q_t units of goods to buy one unit of capital, the agent can engage in a financial investment by purchasing q_t units of bonds in period t with a gross return of $(1 + r)q_t$. This is the left-hand side of equation (3.7). At the optimum both strategies must yield the same return.

3.2.1 Dynamics of the Capital Stock

Eliminating i_t from the optimality conditions, we obtain the following two first-order, nonlinear difference equations in k_t and q_t:

$$k_{t+1} = q_t k_t \tag{3.8}$$

Figure 3.1: The Dynamics of the Capital Stock



$$q_t = \frac{\theta_{t+1}F'(q_t k_t) + (q_{t+1}-1)^2/2 + q_{t+1}}{1+r}. \tag{3.9}$$

The perfect foresight solution to these equations is depicted in figure 3.1. The horizontal line $\overline{KK'}$ corresponds to the pairs (k_t, q_t) for which $k_{t+1} = k_t$ in equation (3.8). That is,

$$q = 1. \tag{3.10}$$

Above the locus $\overline{KK'}$, the capital stock grows over time, and below the locus $\overline{KK'}$ the capital stock declines over time. The locus $\overline{QQ'}$ corresponds to the pairs (k_t, q_t) for which $q_{t+1} = q_t$ in equation (3.9). That is,

$$rq = \theta F'(qk) + (q-1)^2/2. \tag{3.11}$$

Jointly, equations (3.10) and (3.11) determine the steady-state value of the capital stock, which we denote by k^*, and the steady-state value of Tobin's q, 1. The value of k^* is implicitly determined by the expression $r = \theta F'(k^*)$. This is the same value obtained in the economy without adjustment costs. This is not surprising, because, as noted earlier, adjustment costs vanish in the steady state. For q_t near unity, the locus $\overline{QQ'}$ is downward sloping. Above and to the right of $\overline{QQ'}$, q increases over time and below and to the left of $\overline{QQ'}$, q decreases over time.

The system (3.8)-(3.9) is saddle-path stable. The locus $\overline{SS'}$ represents the converging saddle path. If the initial capital stock is different from its long-run level, both q and k converge monotonically to their steady states along the saddle path.

3.2.2 A Permanent Technology Shock

Suppose now that in period 0 the technology factor θ_t increases permanently from $\bar{\theta}$ to $\theta' > \bar{\theta}$. It is clear from equation (3.10) that the locus $\overline{KK'}$ is not affected by the productivity shock. Equation (3.11)shows that the locus $\overline{QQ'}$ shifts up and to the right. It follows that in response to a permanent increase in productivity, the long-run level of capital experiences a permanent increase. The price of capital, q_t, on the other hand, is not affected in the long run.

Consider now the transition to the new steady state. Suppose that the steady-state value of capital prior to the innovation in productivity is k_0 in figure 3.1. Then the new steady-state values of k and q are given by k^* and 1. In the period of the shock, the capital stock does not move. The price of installed capital, q_t, jumps to the new saddle path, point a in the figure. This increase in the price of installed capital induces an increase in investment, which in turn makes capital grow over time. After the initial impact, q_t decreases toward 1. Along this transition, the capital stock increases monotonically towards its new steady-state k^*.

The equilibrium dynamics of investment in the presence of adjustment costs are quite different from those arising in the absence thereof. In the frictionless environment, investment jumps up by $k^* - k_0$ in period zero. Under capital adjustment costs, the initial increase in investment is smaller, as the capital stock adjusts gradually to its long-run level.[1]

The different behavior of investment with and without adjustment costs has consequences for the equilibrium dynamics of the trade balance. In effect, because investment is part of domestic absorption, and because investment tends to be less responsive to productivity shocks in the presence of adjustment costs, it follows that the trade balance is less likely to deteriorate in response to a positive innovation in productivity in the environment with frictions. The following principle therefore emerges:

Principle II: The more pronounced are capital adjustment costs, the less likely is the trade balance to experience an initial deterioration in response

[1] It is straightforward to see that the response of the model to a purely temporary productivity shock is identical to that of the model without adjustment costs. In particular, capital and investment display a mute response.

to a positive productivity shock.

In light of principles I and II derived in this chapter it is natural to ask what the model would predict for the behavior of the trade balance in response to productivity shocks when one introduces realistic degrees of capital adjustment costs and persistence in the productivity-shock process. We address this issue in the next chapter.

Chapter 4

The Real Business Cycle Model

In the previous two chapters, we arrived at the conclusion that a model driven by productivity shocks can explain the observed countercyclicality of the current account. We also established that two features of the model are important in making this prediction possible. First, productivity shocks must be sufficiently persistent. Second, capital adjustment costs must not be too strong. In this chapter, we extend the model of the previous chapter by allowing for three features that add realism to the model's implied dynamics. Namely, endogenous labor supply and demand, uncertainty in the technology shock process, and capital depreciation. The resulting theoretical framework is known as the Real Business Cycle model, or, succinctly, the RBC model.

4.1 The Model

Consider an economy populated by an infinite number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \theta_t \, U(c_t, h_t), \tag{4.1}$$

$$\theta_0 = 1, \tag{4.2}$$

$$\theta_{t+1} = \beta(c_t, h_t)\theta_t \qquad t \geq 0, \tag{4.3}$$

where $\beta_c < 0$, $\beta_h > 0$. This preference specification was conceived by Uzawa (1968) and introduced in the small-open-economy literature by Mendoza (1991). The reason why we adopt this type of utility function here is that it gives rise to a steady state of the model that is independent of initial conditions. In particular, under these preferences the steady state is independent of the initial net foreign asset position of the economy. This property is desirable for a purely technical reason. Namely, it makes it possible to rely on linear approximations to characterize equilibrium dynamics.

The period-by-period budget constraint of the representative household is given by

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t), \tag{4.4}$$

where d_t denotes the household's debt position at the end of period t, r_t denotes the interest rate at which domestic residents can borrow in period t, y_t denotes domestic output, c_t denotes consumption, i_t denotes gross investment, and k_t denotes physical capital. The function $\Phi(\cdot)$ is meant to capture capital adjustment costs and is assumed to satisfy $\Phi(0) = \Phi'(0) = 0$. As pointed out earlier, small open economy models typically include capital adjustment costs to avoid excessive investment volatility in response to variations in the foreign interest rate. The restrictions imposed on Φ ensure that in the steady state adjustment costs are nil and the relative price of capital goods in terms of consumption goods is unity.

Output is produced by means of a linearly homogeneous production function that takes capital and labor services as inputs,

$$y_t = A_t F(k_t, h_t), \tag{4.5}$$

where A_t is an exogenous and stochastic productivity shock. This shock represents the single source of aggregate fluctuations in the present model. The stock of capital evolves according to

$$k_{t+1} = i_t + (1 - \delta)k_t, \tag{4.6}$$

where $\delta \in (0, 1)$ denotes the rate of depreciation of physical capital.

Households choose processes $\{c_t, h_t, y_t, i_t, k_{t+1}, d_t, \theta_{t+1}\}_{t=0}^{\infty}$ so as to maximize the utility function (4.1) subject to (4.2)-(4.6) and a no-Ponzi constraint of the form

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1 + r_s)} \leq 0. \tag{4.7}$$

Letting $\theta_t \eta_t$ and $\theta_t \lambda_t$ denote the Lagrange multipliers on (4.3) and (4.39), the first-order conditions associated with the household's maximization problem are (4.3)-(4.7) holding with equality and:

$$\lambda_t = \beta(c_t, h_t)(1 + r_t)E_t\lambda_{t+1} \tag{4.8}$$

$$\lambda_t = U_c(c_t, h_t) - \eta_t\beta_c(c_t, h_t) \tag{4.9}$$

$$\eta_t = -E_tU(c_{t+1}, h_{t+1}) + E_t\eta_{t+1}\beta(c_{t+1}, h_{t+1}) \tag{4.10}$$

$$-U_h(c_t, h_t) + \eta_t\beta_h(c_t, h_t) = \lambda_tA_tF_h(k_t, h_t) \tag{4.11}$$

$$\lambda_t[1+\Phi'(k_{t+1}-k_t)] = \beta(c_t, h_t)E_t\lambda_{t+1}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{4.12}$$

These first-order conditions are fairly standard, except for the fact that the marginal utility of consumption is not given simply by $U_c(c_t, h_t)$ but rather by $U_c(c_t, h_t) - \beta_c(c_t, h_t)\eta_t$. The second term in this expression reflects the fact that an increase in current consumption lowers the discount factor ($\beta_c < 0$). In turn, a unit decline in the discount factor reduces utility in period t by η_t. Intuitively, η_t equals the expected present discounted value of utility from period $t + 1$ onward. To see this, iterate the first-order condition (4.10) forward to obtain: $\eta_t = -E_t\sum_{j=1}^{\infty}\left(\frac{\theta_{t+j}}{\theta_{t+1}}\right)U(c_{t+j}, h_{t+j})$. Similarly, the marginal disutility of labor is not simply $U_h(c_t, h_t)$ but instead $U_h(c_t, h_t) - \beta_h(c_t, h_t)\eta_t$.

We assume free capital mobility. The world interest rate is assumed to be constant and equal to r. Equating the domestic and world interest rates, yields

$$r_t = r. \tag{4.13}$$

The law of motion of the productivity shock is given by the following first-order autoregressive process:

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}, \tag{4.14}$$

where the parameter $\rho \in (-1, 1)$ measures the serial correlation of the technology shock and the innovation ϵ_t is assumed to be i.i.d. white noise distributed $N(0, \sigma_\epsilon^2)$.

A competitive equilibrium is a set of processes $\{d_t, c_t, h_t, y_t, i_t, k_{t+1}, \eta_t, \lambda_t, r_t, A_t\}$ satisfying (4.39)-(4.14), given the initial conditions A_0, d_{-1}, and k_0 and the exogenous process $\{\epsilon_t\}$.

We parameterize the model following Mendoza (1991), who uses the following functional forms for preferences and technology:

$$U(c, h) = \frac{\left[c - \omega^{-1} h^{\omega}\right]^{1-\gamma} - 1}{1 - \gamma}$$

$$\beta(c, h) = \left[1 + c - \omega^{-1} h^{\omega}\right]^{-\psi_1}$$

$$F(k, h) = k^{\alpha} h^{1-\alpha}$$

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0.$$

The assumed functional forms for the period utility function and the discount factor imply that the marginal rate of substitution between consumption and leisure depends only on labor. In effect, combining equations (4.9) and (4.11) yields

$$h_t^{\omega-1} = A_t F_h(k_t, h_t). \tag{4.15}$$

Table 4.1: Calibration of the Small Open RBC Economy

γ	ω	ψ_1	α	ϕ	r	δ	ρ	σ_ϵ
2	1.455	.11	.32	0.028	0.04	0.1	0.42	0.0129

The right-hand side of this expression is the marginal product of labor, which in equilibrium equals the real wage rate. The left-hand side is the marginal rate of substitution of leisure for consumption. The above expression thus states that the labor supply depends only upon the wage rate and in particular that it is independent of the level of wealth.

We also follow Mendoza (1991) in assigning values to the structural parameters of the model. Mendoza calibrates the model to the Canadian economy. The time unit is meant to be a year. The parameter values are shown in table 4.1. The values assigned to the parameters γ, ω, δ, α, and r are quite standard in the real-business-cycle literature. The value of ω of 1.455 implies, by equation (4.15), a relatively high labor supply elasticity of $1/(\omega - 1) = 2.2$. The calibrate value of δ implies that capital goods depreciates at a rate of 10 percent per year. The assumed value of α implies a share of labor income in GDP of 68 percent. The calibrated value of r of 4 percent is in line with the average real rate of return of broad measures of the stock market in developed countries over the postwar period.

It is of interest to explain in more detail the calibration of the parameter ψ_1 defining the elasticity of the discount factor with respect to the composite $c - h^\omega/\omega$. This parameter determines the stationarity of the model and the speed of convergence of the net external debt position to the steady state. The value assigned to ψ_1 is picked to match the average Canadian trade-balance-to-GDP ratio of about 2 percent. To see how in steady state this ratio is linked to the value of ψ_1, use equation (4.12) in steady state to get

$$\frac{k}{h} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}.$$

It follows from this expression that the steady-state capital-labor ratio is independent of the parameter ψ_1. Given the capital-labor ratio, equilibrium condition (4.15) implies that the steady-state value of hours is also independent of ψ_1 and given by

$$h = \left[(1-\alpha)\left(\frac{k}{h}\right)^{\alpha}\right]^{\frac{1}{\omega-1}}.$$

Given the steady-state values of hours and the capital-labor ratio, we can find directly the steady-state values of capital, investment ($i = \delta k$), and output ($y = k^{\alpha}h^{1-\alpha}$), independently of ψ_1. Now note that in the steady state the trade balance, which we denote by tb, is given by $y - c - i$. This expression and equilibrium condition (4.8) imply the following steady-state condition relating the trade balance to ψ_1: $[1+y-i-tb-h^{\omega}/\omega]^{-\psi_1}(1+r) = 1$, which uses the specific functional form assumed for the discount factor. This expression can be solved for the trade balance-to-output ratio to obtain:

$$\frac{tb}{y} = 1 - \frac{i}{y} - \frac{\left[(1+r)^{1/\psi_1} + \frac{h^{\omega}}{\omega} - 1\right]}{y}.$$

Recalling that y, h, nd i are all independent of ψ_1, it follows that this expression can be solved for ψ_1 given tb/y, α, r, δ, and ω. Clearly, the larger is the assumed steady-state trade-balance-to-output ratio, the larger is ψ_1.

Note that in our assumed specification of the endogenous discount factor, the parameter ψ_1 governs both the steady-state trade-balance-to-output ratio and the stationarity of the equilibrium dynamics. This dual role may create a conflict. On the one had, one may want to set ψ_1 at a small level so as to ensure stationarity without affecting the predictions of the model at business-cycle frequency. On the other hand, matching the observed average trade-balance-to-output ratio might require a value of ψ_1 that does affect the behavior of the model at business-cycle frequency. For this reason, it might be useful to consider a two-parameter specifications of the discount factor, such as $\beta(c_t, h_t) = \bar{\beta}[1 + (c_t - c) - \omega^{-1}(h_t^\omega - h^\omega)]^{-\psi_1}$, where c and h denote the steady-state values of c_t and h_t, and $\bar{\beta} > 0$ is a parameter. With this specification, one can fix the parameter ψ_1 at a small value, just to ensure stationarity, and set the parameter $\bar{\beta}$ at a value such that the implied steady-state trade-balance-to-output ratio is in line with average value of this ratio observed in the data.

Finally, the parameters ϕ, σ_ϵ, and ρ are calibrated to match the standard deviation of investment, the standard deviation of output, and the serial correlation of output in Canada as shown in table 1.1.

Approximating Equilibrium Dynamics

We look for solutions to the equilibrium conditions (4.39)-(4.14) where the vector $x_t \equiv \{d_{t-1}, c_t, h_t, y_t, i_t, k_t, \eta_t, \lambda_t, r_t, A_t\}$ fluctuates in a small neighborhood around its nonstochastic steady-state level. In any such solution the stock of debt is bounded, we have that the transversality condition $\lim_{j\to\infty} E_t d_{t+j}/(1+r)^j = 0$ is always satisfied. We can therefore focus on bounded solutions to the system (4.39)-(4.6) and (4.8)-(4.14) of ten equations in ten variables given by the elements of the vector x_t. The system can be written as

$$E_t f(x_{t+1}, x_t) = 0.$$

This expression describes a system of nonlinear stochastic difference equations. Closed form solutions to this type of system are not typically available. We therefore must resort to an approximate solution.

There are a number of techniques that have been devised to solve dynamic systems like the one we are studying. The technique we will employ here consists in applying a first-order Taylor expansion (i.e., linearizing) the system of equations around the nonstochastic steady state. The resulting linear system can be readily solved using well-established techniques.

Before linearizing the equilibrium conditions, we introduce a convenient variable transformation. It is useful to express some variables in terms of percent deviations from their steady-state values. For any such variable, which we denote generically by w_t, we define its log deviation from steady state, $\hat{w}_t \equiv \log(w_t/w)$, where w denotes the steady-state value of w_t. Note that for small deviations of w_t from w it is the case that $\widehat{w}_t \approx (w_t - w)/w$. That is, $\widehat{w}_t$ is approximately equal to the proportional deviation of w_t from its steady-state level. Some other variables are more naturally expressed in levels. This is the case, for instance, with net interest rates or variables that can take negative values, such as the trade balance. For this type of variable, we define $\hat{w}_t \equiv w_t - w$.

The linearized version of the equilibrium system can then be written as

$$A\widehat{x}_{t+1} = B\widehat{x}_t,$$

where A and B are square matrices conformable with x_t. Appendix A displays the linearized equilibrium conditions of the RBC model studied here.

The vector $\hat{x}_t$ contains 10 variables. Of these 10 variables, 3 are state variables, namely, $\widehat{k}_t$, $\widehat{d}_{t-1}$, and $\widehat{A}_t$. State variables are variables whose values in any period $t \geq 0$ are either predetermined (i.e., determined before t) or determined in t but in an exogenous fashion. In our model, $\hat{k}_t$ and $\hat{d}_{t-1}$ are endogenous state variables, and $\hat{A}_t$ is an exogenous state variable. The remaining 7 elements of $\hat{x}_t$, that is, $\hat{c}_t$, $\hat{h}_t$, $\hat{\lambda}_t$, $\hat{\eta}_t$, $\widehat{r}_t$, $\widehat{\imath}_t$, and $\widehat{y}_t$, are co-state variables. Co-states are endogenous variables whose values are not predetermined in period t. All the coefficients of the linear system, that is, the elements of A and B, are known functions of the deep structural parameters of the model to which we assigned values when we calibrated the model. The linearized system has three known initial conditions $\widehat{k}_0$, $\widehat{d}_{-1}$, and $\widehat{A}_0$. To determine the initial value of the remaining seven variables, we impose a terminal condition requiring that at any point in time the system be expected to converge to the nonstochastic steady state. Formally, the terminal condition takes the form

$$\lim_{j\to\infty} |E_t\widehat{x}_{t+j}| = 0.$$

Table 4.2: Empirical and Theoretical Second Moments

Variable	Canadian Data			Model		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.61	1	3.1	0.61	1
c	2.5	0.7	0.59	2.3	0.7	0.94
i	9.8	0.31	0.64	9.1	0.07	0.66
h	2	0.54	0.8	2.1	0.61	1
$\frac{tb}{y}$	1.9	0.66	-0.13	1.5	0.33	-0.012
$\frac{ca}{y}$				1.5	0.3	0.026

Note. Empirical moments are taken from Mendoza (1991). Standard deviations are measured in percentage points.

Appendix B shows in some detail how to solve linear stochastic systems like the one describing the dynamics of our linearized equilibrium conditions. That appendix also shows how to compute second moments and impulse response functions. Matlab code to compute second moments and impulse response functions implied by the model studied here is available online at `http://www.columbia.edu/~mu2166/closing.htm`.

4.2 The Model's Performance

Table 4.2 displays some unconditional second moments of interest implied by our model. It should not come as a surprise that the model does very well at replicating the volatility of output, the volatility of investment, and the serial correlation of output. For we picked values for the parameters σ_ϵ, ϕ, and ρ to match these three moments. But the model performs relatively well along other dimensions. For instance, it correctly implies a volatility ranking featuring investment volatility above output volatility and output volatility above consumption volatility. Also in line with the data is the model's prediction of a countercyclical trade balance-to-output ratio. This prediction is of interest because the parameters ϕ and ρ governing the degree of capital adjustment costs and the persistence of the productivity shock, which, as we established in the previous chapter, are the key determinants of the cyclicality of the trade-balance-to-output ratio, were set independently of the cyclical properties of the trade balance.

On the downside, the model predicts too little countercyclicality in the trade balance and overestimates the correlations of both hours and consumption with output. Note in particular that the implied correlation between hours and output is exactly unity. This prediction is due to the assumed functional form for the period utility index. In effect, equilibrium condition (4.15), equating the marginal product of labor to the marginal rate of substitution between consumption and leisure, can be written as $h_t^\omega = (1-\alpha)y_t$. The log-linearized version of this condition is $\omega\widehat{h}_t = \widehat{y}_t$, which implies that $\widehat{h}_t$ and $\widehat{y}_t$ are perfectly correlated.

Figure 4.1. displays the impulse response functions of a number of variables of interest to a technology shock of size 1 in period 0. The model predicts an expansion in output, consumption, investment, and hours. Interestingly, the increase in domestic absorption (i.e., the increase in $c_t + it_t$) that takes place following the expansionary technology shock is larger than the increase in output, resulting in a deterioration of the trade balance.

Figure 4.1: Responses to a One-Percent Productivity Shock

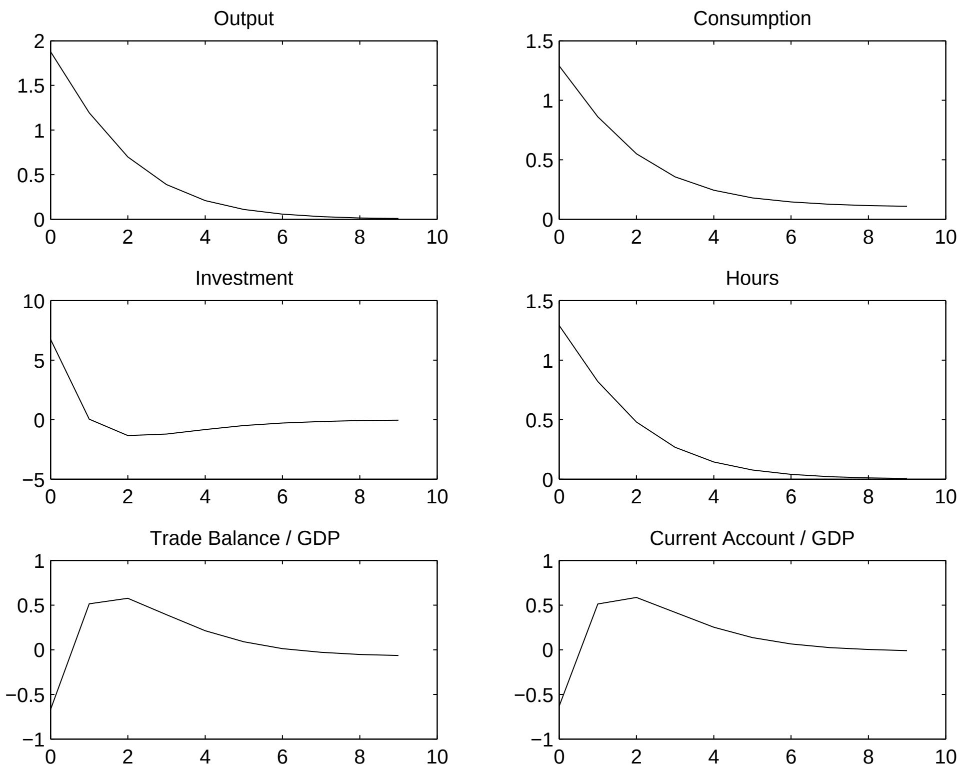
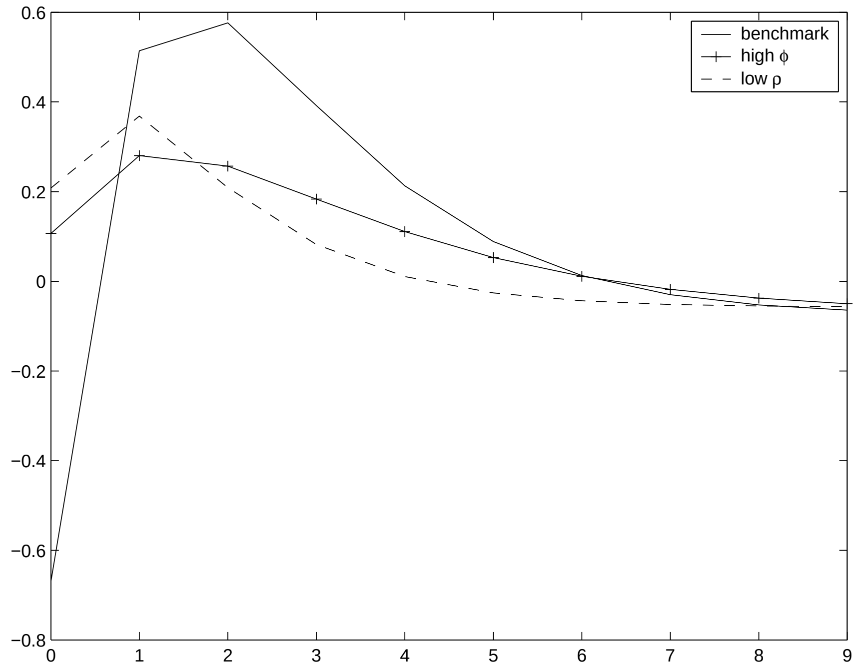


Figure 4.2: Response of the Trade-Balance-To-Output Ratio to a Positive Technology Shock



4.2.1 The Roles of Persistence and Capital Adjustment Costs

In the previous chapter, we deduced that the negative response of the trade balance to a positive technology shock was not a general implication of the neoclassical model. In particular, Principles I and II of the previous chapter state that two conditions must be met for the model to generate a deterioration in the external accounts in response to a mean-reverting improvement in total factor productivity. First, capital adjustment costs must not be too stringent. Second, the productivity shock must be sufficiently persistent. To illustrate this conclusion, figure 4.2 displays the impulse response function of the trade balance-to-GDP ratio to a technology shock of unit size in period 0 under three alternative parameter specifications. The solid line reproduces the benchmark case from figure 4.1. The broken line depicts an economy where the persistence of the productivity shock is half as large as in the benchmark economy ($\rho = 0.21$). In this case, because the productivity shock is expected to die out quickly, the response of investment is relatively weak. In addition, the temporariness of the shock induces households to save most of the increase in income to smooth consumption over time. As a result, the expansion in aggregate domestic absorption is modest. At the same time, because the size of the productivity shock is the same as in the benchmark economy, the initial responses of output and hours are identical in both economies (recall that, by equation (4.15), h_t depends only on k_t and A_t). The combination of a weak response in domestic absorption and an unchanged response in output, results in an improvement in the trade balance when productivity shocks are not too persistent.

The crossed line depicts the case of high capital adjustment costs. Here the parameter ϕ equals 0.084, a value three times as large as in the benchmark case. In this environment, high adjustment costs discourage firms from increasing investment spending by as much as in the benchmark economy. As a result, the response of aggregate domestic demand is weaker, leading to an improvement in the trade balance-to-output ratio.

4.3 Alternative Ways to Induce Stationarity

In the RBC model analyzed thus far households have endogenous discount factors. We will refer to that model as the 'internal discount factor model,' or IDF model. The inclusion of an endogenous discount factor responds to the need to obtain stationary dynamics up to first order. Had we assumed a constant discount factor, the log-linearized equilibrium dynamics would have contained a random walk component. Two problems emerge when the linear approximation possesses a unit root. First, one can no longer claim that the linear system behaves like the original nonlinear system—which is ultimately the focus of interest—when the underlying shocks have sufficiently small supports. Second, when the variables of interest contain random walk elements, it is impossible to compute unconditional second moments, such as standard deviations, serial correlations, correlations with output, etc., which are the most common descriptive statistics of the business cycle.

In this section, we analyze and compare alternative ways of inducing stationarity in small open economy models. Our analysis follows closely Schmitt-Grohé and Uribe (2003), but expands their analysis by including a model with an internal interest-rate premium.

4.3.1 External Discount Factor (EDF)

Consider an alternative formulation of the endogenous discount factor model where domestic agents do not internalize the fact that their discount factor depends on their own levels of consumption and effort. Specifically, suppose that the discount factor depends not upon the agent's own consumption and effort, but rather on the average per capita levels of these variables.

Formally, preferences are described by (4.1), (4.2), and

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t \qquad t \geq 0, \tag{4.16}$$

where $\tilde{c}_t$ and $\tilde{h}_t$ denote average per capita consumption and hours, which the individual household takes as given.

The first-order conditions associated with the household's maximization problem are (4.2), (4.39)-(4.7), (4.16) holding with equality, and:

$$\lambda_t = \beta(\tilde{c}_t, \tilde{h}_t)(1 + r_t)E_t\lambda_{t+1} \tag{4.17}$$

$$\lambda_t = U_c(c_t, h_t) \tag{4.18}$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{4.19}$$

$$\lambda_t[1+\Phi'(k_{t+1}-k_t)] = \beta(\tilde{c}_t, \tilde{h}_t)E_t\lambda_{t+1}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{4.20}$$

In equilibrium, individual and average per capita levels of consumption and effort are identical. That is,

$$c_t = \tilde{c}_t \tag{4.21}$$

and

$$h_t = \tilde{h}_t. \tag{4.22}$$

A competitive equilibrium is a set of processes $\{d_t,\ c_t, h_t,\ \tilde{c}_t, \tilde{h}_t, y_t,\ i_t,$ $k_{t+1},\ \lambda_t, r_t, A_t\}$ satisfying (4.39)-(4.7), (4.13), (4.14), (4.17)-(4.22) all holding with equality, given A_0, d_{-1}, and k_0 and the stochastic process $\{\epsilon_t\}$.

Note that the equilibrium conditions include one fewer Euler equation (equation (4.10)), and one fewer variable, η_t, than the internal discount-factor model of subsection 4.1. The smaller size of the external-discount-factor model slightly speeds up the computation of equilibrium dynamics using perturbation techniques. Remarkably, the reverse is true if the model is solved using value-function iterations over a discretized state space. The reason is that in the value-function formulation, the aggregate variables $\tilde{c}_t$ and $\tilde{h}_t$ add two state variables. And each of these state variables comes at a cost in terms of computational speed.[1]

We evaluate the model using the same functional forms and parameter values as in the IDF model.

4.3.2 External Debt-Elastic Interest Rate (EDEIR)

Under an external debt-elastic interest rate, stationarity is induced by assuming that the interest rate faced by domestic agents, r_t, is increasing in the aggregate level of foreign debt, which we denote by $\tilde{d}_t$. Specifically, r_t is given by

$$r_t = r + p(\tilde{d}_t), \tag{4.23}$$

where r denotes the world interest rate and $p(\cdot)$ is a country-specific interest rate premium. The function $p(\cdot)$ is assumed to be strictly increasing.

Preferences are given by equation (4.1). Unlike in the previous model, preferences are assumed to display a constant subjective rate of discount.

[1]A similar comment applies to the computation of equilibrium in a model with an interest-rate premium that depends on the aggregate level of external al debt to be discussed in the next subsection.

Formally,

$$\theta_t = \beta^t,$$

where $\beta \in (0,1)$ is a constant parameter.

The representative agent's first-order conditions are (4.39)-(4.7) holding with equality and

$$\lambda_t = \beta(1+r_t)E_t\lambda_{t+1} \tag{4.24}$$

$$U_c(c_t, h_t) = \lambda_t, \tag{4.25}$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t). \tag{4.26}$$

$$\lambda_t[1+\Phi'(k_{t+1}-k_t)] = \beta E_t\lambda_{t+1}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2}-k_{t+1})\right]. \tag{4.27}$$

Because agents are assumed to be identical, in equilibrium aggregate per capita debt equals individual debt, that is,

$$\tilde{d}_t = d_t. \tag{4.28}$$

A competitive equilibrium is a set of processes $\{d_t, \tilde{d}_t, c_t, h_t, y_t, i_t, k_{t+1}, r_t, \lambda_t, A_t\}$ satisfying (4.39)-(4.7), (4.14), and (4.23)-(4.28) all holding with equality, given A_0, d_{-1}, $\tilde{d}_{-1} = d_{-1}$, and k_0, and the process $\{\epsilon_t\}$.

We adopt the same forms for the functions U, F, and Φ as in the IDF of subsection 4.1. We use the following functional form for the risk premium:

$$p(d) = \psi_2\left(e^{d-\bar{d}} - 1\right),$$

Table 4.3: Calibration of the Model with an External Debt Elastic Interest Rate

β	$\bar{d}$	ψ_2
0.96	0.7442	0.000742

where ψ_2 and $\bar{d}$ are constant parameters.

We calibrate the parameters γ, ω, α, ϕ, r, δ, ρ, and σ_ϵ using the values shown in table 4.1. We set the subjective discount factor equal to the world interest rate; that is,

$$\beta = \frac{1}{1+r}.$$

The parameter $\bar{d}$ equals the steady-state level of foreign debt. To see this, note that in steady state, the equilibrium conditions (4.23) and (4.24) together with the assumed form of the interest-rate premium imply that $1 = \beta\left[1 + r + \psi_2\left(e^{d-\bar{d}} - 1\right)\right]$. The fact that $\beta(1+r) = 1$ then implies that $d = \bar{d}$. It follows that in the steady state the interest rate premium is nil. We set $\bar{d}$ so that the steady-state level of foreign debt equals the one implied by the model with an internal endogenous discount factor studied in section 4.1. Finally, we set the parameter ψ_2 to ensure that the model analyzed here and the model of section 4.1 generate the same volatility of the current-account-to-GDP ratio. The resulting values of β, $\bar{d}$, and ψ_2 are given in Table 4.3.

4.3.3 Internal Debt-Elastic Interest Rate (IDEIR)

The model with an internal debt-elastic interest rate assumes that the interest rate faced by domestic agents is increasing in the individual debt position, d_t. In all other aspects, the model is identical to the model featuring an external debt-elastic interest rate. Formally, in the IDEIR model the interest rate is given by

$$r_t = r + p(d_t), \tag{4.29}$$

where, as before, r denotes the world interest rate and $p(\cdot)$ is a household-specific interest-rate premium. Note that the argument of the interest-rate premium function is the households own net debt position. This means that in deciding its optimal expenditure and savings plan, the household will take into account the fact that a change in the debt position alters the marginal cost of funds. The only optimality condition that changes relative to the case with an external premium is the Euler equation for debt accumulation, which now takes the form

$$\lambda_t = \beta[1 + r + p(d_t) + p'(d_t)d_t]E_t\lambda_{t+1}. \tag{4.30}$$

This expression features the derivative of the premium with respect to debt because households internalize the fact that as their net debt increases, so does the interest rate they face in financial markets.

A competitive equilibrium is a set of processes $\{d_t, c_t, h_t, y_t, i_t, k_{t+1}, r_t, \lambda_t, A_t\}$ satisfying (4.39)-(4.7), (4.14), (4.25)-(4.27), (4.29), and (4.30), all holding with equality, given A_0, d_{-1}, and k_0, and the exogenous process $\{\epsilon_t\}$.

We assume the same functional forms and parameter values as in the EDEIR model of section 4.3.2. We note that in the model analyzed in here the steady-state level of debt is no longer equal to $\bar{d}$. To see this, recall that $\beta(1 + r) = 1$ and note that the steady-state version of equation (4.30) imposes the following restriction on d,

$$(1+d)e^{d-\bar{d}} = 1,$$

which does not admit the solution $d = \bar{d}$, except in the special case in which $\bar{d} = 0$. We set $\bar{d} = 0.7442$, which is the value imposed in the EDEIR model of section 4.3.2. The implied steady-state level of debt is then given by $d = 0.4045212$. The fact that the steady-state debt is lower than $\bar{d}$ implies that the country premium is negative in the steady state. However, the marginal country premium, given by $\partial[\rho(d_t)d_t]/\partial d_t$, is nil in the steady state, as it is in the EDEIR economy of section 4.3.2. An alternative calibration strategy is to impose $d = \bar{d}$, and adjust β to ensure that equation (4.30) holds in steady state. In this case, the country premium vanishes in the steady state, but the marginal premium is positive and equal to $\psi_2\bar{d}$.

4.3.4 Portfolio Adjustment Costs (PAC)

In this model, stationarity is induced by assuming that agents face convex costs of holding assets in quantities different from some long-run level. Preferences and technology are as in the EDEIR model of section 4.3.2. However, in contrast to what is assumed in the EDEIR model, here the interest rate at which domestic households can borrow from the rest of the world is assumed to be constant and equal to the world interest rate. That is, equation (4.13) is assumed to hold. The sequential budget constraint of the household is given by

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t) + \frac{\psi_3}{2}(d_t - \bar{d})^2, \quad (4.31)$$

where ψ_3 and $\bar{d}$ are constant parameters defining the portfolio adjustment cost function. The first-order conditions associated with the household's maximization problem are (4.5)-(4.7), (4.25)-(4.27), (4.31) holding with equality and

$$\lambda_t[1 - \psi_3(d_t - \bar{d})] = \beta(1 + r_t)E_t\lambda_{t+1} \quad (4.32)$$

This optimality condition states that if the household chooses to borrow an additional unit, then current consumption increases by one unit minus the marginal portfolio adjustment cost $\psi_3(d_t - \bar{d})$. The value of this increase in consumption in terms of utility is given by the left-hand side of the above equation. Next period, the household must repay the additional unit of debt plus interest. The value of this repayment in terms of today's utility is given by the right-hand side of the above optimality condition. At the optimum, the marginal benefit of a unit debt increase must equal its marginal cost.

A competitive equilibrium is a set of processes $\{d_t, c_t, h_t, y_t, i_t, k_{t+1}, r_t, \lambda_t, A_t\}$ satisfying (4.5)-(4.7), (4.13), (4.14), (4.25)-(4.27), (4.31), and (4.32) all holding with equality, given A_0, d_{-1}, and k_0, and the exogenous process $\{\epsilon_t\}$.

Preferences and technology are parameterized as in the EDEIR model of section 4.3.2. The parameters γ, ω, α, ϕ, r, δ, ρ, and σ_ϵ take the values displayed in table 4.1. As in the EDEIR model of section 4.3.2, the subjective discount factor is assumed to satisfy $\beta(1 + r) = 1$. This assumption and equation (4.32) imply that the parameter $\bar{d}$ determines the steady-state level of foreign debt ($d = \bar{d}$). We calibrate $\bar{d}$ so that the steady-state level of foreign debt equals the one implied by models IDF, EDF, and EDEIR (see table 4.3). Finally, we assign the value 0.00074 to ψ_3, which ensures that this model and the IDF model of section 4.1 generate the same volatility in the current-account-to-GDP ratio. This parameter value is almost identical to that assigned to ψ_2 in the EDEIR model. This is because the log-linearized versions of models 2 and 3 are almost identical. Indeed, the models share all equilibrium conditions but the resource constraint (compare equations (4.39) and (4.31)), the Euler equations associated with the optimal choice of foreign bonds (compare equations (4.24) and (4.32)), and the interest rate faced by domestic households (compare equations (4.13) and (4.23)). The log-linearized versions of the resource constraints are the same in both models. The log-linear approximation to the domestic interest rate is given by $\widehat{1+r_t} = \psi_2 d(1+r)^{-1}\widehat{d}_t$ in the EDEIR model and by $\widehat{1+r_t} = 0$ in the PAC model. Using these results, the log-linearized versions of the Euler equation for debt can be written as $\widehat{\lambda}_t = \psi_2 d(1+r)^{-1}\widehat{d}_t + E_t\widehat{\lambda}_{t+1}$ in the EDEIR model and as $\widehat{\lambda}_t = \psi_3 d\widehat{d}_t + E_t\widehat{\lambda}_{t+1}$ in the PAC model. It follows that for small values of ψ_2 and ψ_3, satisfying $\psi_2 = (1+r)\psi_3$, the EDEIR and PAC models imply similar dynamics.

4.3.5 Complete Asset Markets (CAM)

All model economies considered thus far feature incomplete asset markets. In those models, agents have access to a single financial asset that pays a risk-free real rate of return. In the model studied in this subsection, agents have access to a complete array of state-contingent claims. As we will see, this assumption per se induces stationarity in the equilibrium dynamics.

Preferences and technology are as in the EDEIR model of section 4.3.2. The period-by-period budget constraint of the household is given by

$$E_t r_{t,t+1} b_{t+1} = b_t + y_t - c_t - i_t - \Phi(k_{t+1} - k_t), \tag{4.33}$$

where $r_{t,t+1}$ is a stochastic discount factor such that the period-t price of a random payment b_{t+1} in period $t+1$ is given by $E_t r_{t,t+1} b_{t+1}$.[2] Note that because $E_t r_{t,t+1}$ is the price in period t of an asset that pays 1 unit of good in every state of period $t+1$, it follows that $1/[E_t r_{t,t+1}]$ denotes the risk-free real interest rate in period t. Households are assumed to be subject to a no-Ponzi-game constraint of the form

$$\lim_{j\to\infty} E_t r_{t,t+j} b_{t+j} \geq 0, \tag{4.34}$$

at all dates and under all contingencies. The variable $r_{t,t+j} \equiv r_{t,t+1} r_{t,t+2} \cdots r_{t,t+j}$ represents the stochastic discount factor such that $E_t r_{t,t+j} b_{t+j}$ is the period-t price of a stochastic payment b_{t+j} in period $t+j$. The first-order condi-

[2]To clarify the nature of the stochastic discount factor $r_{t,t+1}$, define the current state of nature as S^t. Let $p(S^{t+1}|S^t)$ denote the price of a contingent claim that pays one unit of consumption in a particular state, S^{t+1} following the current state S^t. Then the current price of a portfolio composed of $b(S^{t+1}|S^t)$ units of contingent claims paying in states S^{t+1} following S^t is given by $\sum_{S^{t+1}|S^t} p(S^{t+1}|S^t) b(S^{t+1}|S^t)$. Now let $\pi(S^{t+1}|S^t)$ denote the probability of occurrence of state S^{t+1}, given information available at the current state S^t. Multiplying and dividing the expression inside the summation sign by $\pi(S^{t+1}|S^t)$ we can write the price of the portfolio as $\sum_{S^{t+1}|S^t} \pi(S^{t+1}|S^t) \frac{p(S^{t+1}|S^t)}{\pi(S^{t+1}|S^t)} b(S^{t+1}|S^t)$. Now let $r_{t,t+1} \equiv p(S^{t+1}|S^t)/\pi(S^{t+1}|S^t)$ be the price of a contingent claim that pays in state $S^{t+1}|S^t$ scaled by the inverse of the probability of occurrence of the state in which the claim pays. Also, let $b_{t+1} \equiv b(S^{t+1}|S^t)$. Then, we can write the price of the portfolio as $\sum_{S^{t+1}|S^t} r_{t,t+1} b_{t+1} \pi(S^{t+1}|S^t)$. But this expression is simply the conditional expectation $E_t r_{t,t+1} b_{t+1}$.

tions associated with the household's maximization problem are (4.5), (4.6), (4.25)-(4.27), (4.33), and (4.34) holding with equality and

$$\lambda_t r_{t,t+1} = \beta \lambda_{t+1}. \tag{4.35}$$

A difference between this expression and the Euler equations that arise in the models with incomplete asset markets studied in previous sections is that under complete markets in each period t there is one first-order condition for each possible state in period $t+1$, whereas under incomplete markets the above Euler equation holds only in expectations.

In the rest of the world, agents have access to the same array of financial assets as in the domestic economy. Consequently, one first-order condition of the foreign household is an equation similar to (4.35). Letting starred letters denote foreign variables or functions, we have

$$\lambda_t^* r_{t,t+1} = \beta \lambda_{t+1}^*. \tag{4.36}$$

Note that we are assuming that domestic and foreign households share the same subjective discount factor. Combining the domestic and foreign Euler equations—equations (4.35) and (4.36)—yields

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\lambda_{t+1}^*}{\lambda_t^*}.$$

This expression holds at all dates and under all contingencies. This means that the domestic marginal utility of consumption is proportional to its foreign counterpart. Formally,

$$\lambda_t = \xi \lambda_t^*,$$

where ξ is a constant parameter determining differences in wealth across countries. We assume that the domestic economy is small. This means that λ_t^* is exogenously determined.

Because we are interested only in the effects of domestic productivity shocks, we assume that λ_t^* is constant and equal to λ^*, where λ^* is a parameter. The above equilibrium condition then becomes

$$\lambda_t = \psi_4, \tag{4.37}$$

where $\psi_4 \equiv \xi \lambda^*$ is a constant parameter.

A competitive equilibrium is a set of processes $\{c_t, h_t, y_t, i_t, k_{t+1}, \lambda_t, A_t\}$ satisfying (4.5), (4.6), (4.14), (4.25)-(4.27), and (4.37), given , A_0, and k_0, and the exogenous process $\{\epsilon_t\}$.

The functions U, F, and Φ are parameterized as in the previous models. The parameters γ, β, ω, α, ϕ, δ, ρ, and σ_ϵ take the values displayed in tables 4.1 and 4.3. The parameter ψ_4 is set so as to ensure that the steady-state level of consumption is the same in this model as in the IDF, EDF, EDEIR, and PAC models.

4.3.6 The Nonstationary Case (NC)

For comparison with the models considered thus far, we consider a version of the small open economy model that displays no stationarity. In this model (a) the discount factor is constant; (b) the interest rate at which domestic agents borrow from the rest of the world is constant (and equal to the subjective discount factor); (c) agents face no frictions in adjusting the size of their portfolios; and (d) markets are incomplete in the sense that domestic households have only access to a single risk-free international bond. Under this specification, the deterministic steady state of the model depends on the assumed initial level net foreign debt. Also, up to first order, the equilibrium dynamics contain a random wal component in variables such as consumption, and net external debt.

A competitive equilibrium in the nonstationary model is a set of processes $\{d_t, c_t, h_t, y_t, i_t, k_{t+1}, r_t, \lambda_t, A_t\}$ satisfying (4.39)-(4.7), (4.13), (4.14), and (4.24)-(4.27) all holding with equality, given d_{-1}, and k_0, and the exogenous process $\{\epsilon_t\}$. We calibrate the model using the parameter values displayed in tables 4.1 and 4.3.

4.3.7 Quantitative Results

Table 4.4 displays a number of unconditional second moments of interest implied by the IDF, EDF, EDEIR, IDEIR, PAC, CAM, and NC models.[3] For all models, we compute the equilibrium dynamics by solving a log-linear approximation to the set of equilibrium conditions. The appendix

[3]The NC model is nonstationary up to first order, and therefore does not have well defined unconditional second moments.

shows the log-linear version of the IDF model of subsection 4.1. The Matlab computer code used to compute the unconditional second moments and impulse response functions for all models presented in this section is available at `www.columbia.edu/~mu2166`.

The main result of this section is that regardless of how stationarity is induced, the model's predictions regarding second moments are virtually identical. This result is evident from table 4.4. The only noticeable difference arises in the CAM model, the complete markets case, which as expected predicts less volatile consumption. The low volatility of consumption in the complete markets model introduces an additional difference between the predictions of this model and the IDF, EDF, EDEIR, IDEIR, and PAC models. Because consumption is smoother in the CAM model, its role in determining the cyclicality of the trade balance is smaller. As a result, the CAM model predicts that the correlation between output and the trade balance is positive, whereas the models featuring incomplete asset markets all imply that it is negative.

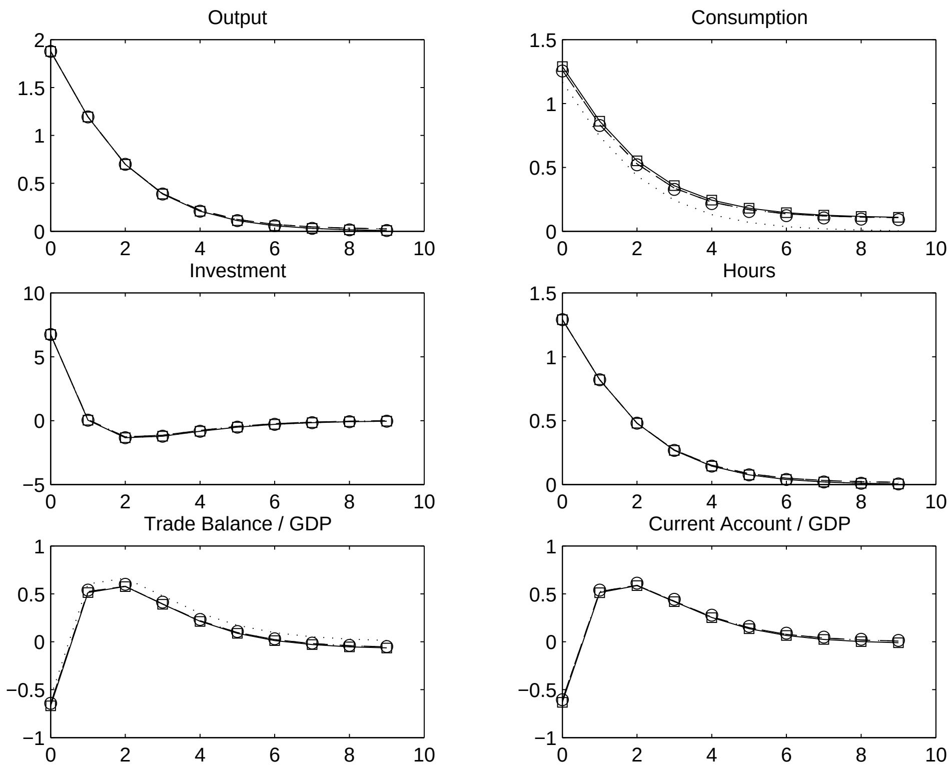
Figure 4.3 demonstrates that all of the models being compared imply virtually identical impulse response functions to a technology shock. Each panel shows the impulse response of a particular variable in the six models. For all variables but consumption and the trade-balance-to-GDP ratio, the impulse response functions are so similar that to the naked eye the graph appears to show just a single line. Again, the only small but noticeable difference is given by the responses of consumption and the trade-balance-to-GDP ratio in the complete markets model. In response to a positive technology shock, consumption increases less when markets are complete

Table 4.4: Implied Second Moments

	IDF	EDF	IDEIR	EDEIR	PAC	CAM
Volatilities:						
std(y_t)	3.1	3.1	3.1	3.1	3.1	3.1
std(c_t)	2.3	2.3	2.5	2.7	2.7	1.9
std(i_t)	9.1	9.1	9	9	9	9.1
std(h_t)	2.1	2.1	2.1	2.1	2.1	2.1
std($\frac{tb_t}{y_t}$)	1.5	1.5	1.6	1.8	1.8	1.6
std($\frac{ca_t}{y_t}$)	1.5	1.5	1.4	1.5	1.5	
Serial Correlations:						
corr(y_t, y_{t-1})	0.61	0.61	0.62	0.62	0.62	0.61
corr(c_t, c_{t-1})	0.7	0.7	0.76	0.78	0.78	0.61
corr(i_t, i_{t-1})	0.07	0.07	0.068	0.069	0.069	0.07
corr(h_t, h_{t-1})	0.61	0.61	0.62	0.62	0.62	0.61
corr($\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}}$)	0.33	0.32	0.43	0.51	0.5	0.39
corr($\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}$)	0.3	0.3	0.31	0.32	0.32	
Correlations with Output:						
corr(c_t, y_t)	0.94	0.94	0.89	0.84	0.85	1
corr(i_t, y_t)	0.66	0.66	0.68	0.67	0.67	0.66
corr(h_t, y_t)	1	1	1	1	1	1
corr($\frac{tb_t}{y_t}, y_t$)	-0.012	-0.013	-0.036	-0.044	-0.043	0.13
corr($\frac{ca_t}{y_t}, y_t$)	0.026	0.025	0.041	0.05	0.051	

Note. Standard deviations are measured in percent per year. IDF = Internal Discount Factor; EDF = External Discount Factor; IDEIR = Internal Debt-Elastic Interest Rate; EDEIR = External Debt-Elastic Interest Rate; PAC = Portfolio Adjustment Costs; CAM = Complete Asset Markets. NC = Nonstationary Case.

Figure 4.3: Impulse Response to a Unit Technology Shock in Models 1 - 5



Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashed line: Debt-elastic interest rate model; Dash-dotted line: Portfolio adjustment cost model; Dotted line: complete asset markets model; Circles: Model without stationarity inducing elements.

than when markets are incomplete. This in turn, leads to a smaller decline in the trade balance in the period in which the technology shock occurs.

4.3.8 The Perpetual-Youth Model

In this subsection, we develop a discrete-time, small open economy version of the perpetual youth model due to Blanchard (1985). Cardia (1991) represents an early adoption of the perpetual-youth model in the context of a small open economy. We treat this case separately because exact aggregation in the presence of aggregate uncertainty requires the adoption of a particular functional form for the period utility index. As we will see, however, the aggregate dynamics of the model are very much in line with those obtained from the other models studied earlier in this section.

The Basic Intuition

The basic intuition behind why the assumption of finite lives by itself helps to eliminate the unit root in the aggregate net foreign asset position can be seen from the following simple example. Consider an economy in which debt holdings of individual agents follow a pure random walk of the form $d_{s,t} = d_{s,t-1} + \mu_t$. Here, $d_{s,t}$ denotes the net debt position in period t of an agent born in period s, and μ_t is a white noise common to all agents. This was exactly the equilibrium evolution of debt we obtained in the quadratic-preference, representative-agent economy of chapter 2. We now depart from the representative-agent assumption by introducing a constant and age-independent probability of death at the individual level. Specifically, assume that the population is constant over time and normalized to unity.

Each period, individual agents face a probability $1-\theta \in (0,1)$ of dying. In addition, to keep the size of the population constant over time, we assume that $1-\theta$ agents are born each period. Assume that those agents who die leave their outstanding debts unpaid and that newborns inherit no debts. Adding the left- and right-hand sides of the law of motion for debt over all agents alive in period t—i.e., applying the operator $(1-\theta)\sum_{s=t}^{-\infty}\theta^{t-s}$ on both sides of the expression $d_{s,t} = d_{s,t-1} + \mu_t$—yields $d_t = \theta d_{t-1} + \mu_t$, where d_t denotes the aggregate debt position in period t. In performing the aggregation, recall that $d_{t,t-1} = 0$, because agents are born free of debts. Clearly, the resulting law of motion for the aggregate level of debt is mean reverting at the survival rate θ. The key difference with the representative agent model is that here each period a fraction $1-\theta$ of the stock of debt simply disappears.

In what follows we derive this result in the context of a richer, more microfounded environment.

Households

Each agent maximizes the utility function

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}(\beta\theta)^t(x_{s,t}-\bar{x})^2$$

with

$$x_{s,t} = c_{s,t} - \frac{h_{s,t}^{\omega}}{\omega}, \tag{4.38}$$

where $c_{s,t}$ and $h_{s,t}$ denote consumption and hours worked in period t by an agent born in period s. The parameter $\beta \in (0,1)$ represents a subjective discount factor, and $\bar{x}$ is a parameter denoting a satiation point. Following the preference specification used in all of the models studied in this chapter, we assume that agents derive utility from a quasi-difference between consumption and leisure. But we depart from the preference specifications used earlier in this chapter by assuming a quadratic period utility index. As will become clear shortly, this assumption is essential to achieve aggregation in the presence of aggregate uncertainty.

Agents can borrow from foreign lenders by means of a bond paying a constant real interest rate. in the world financial market, the risk-free interest rate is given by r. The debts of deceased domestic agents are assumed to go unpaid. Foreign agents are assumed to lend to a large number of domestic agents so that the fraction of unpaid loans due to death is deterministic. To compensate foreign lenders for these losses, domestic agents therefore pay a constant premium over the world interest rate. Domestic agents can also lend at the rate r. In addition, they have access to an actuarily fair insurance market. the insurance contract in this market specifies that lenders receive a constant premium over the interest rate r while alive and must leave their assets to the insurance company in case of death. It follows that the gross interest rate on the domestic agent's asset position (whether this position is positive or negative) is given by $(1+r)/\theta$.

The agent's sequential budget constraint can be written as

$$d_{s,t} = \left(\frac{1+r}{\theta}\right) d_{s,t-1} + c_{s,t} - \pi_t - w_t h_{s,t}, \qquad (4.39)$$

where π_t and w_t denote, respectively, profits received from the ownership of stock shares and the real wage rate. To facilitate aggregation, we assume that agents do not trade shares and that the shares of the dead are passed to the newborn in an egalitarian fashion. Thus, share holdings are identical across agents. Agents are assumed to be subject to the following no-Madoff-game constraint

$$\lim_{j\to\infty} E_t \left(\frac{\theta}{1+r}\right)^j d_{s,t+j} \leq 0 \tag{4.40}$$

The first-order conditions associated with the agent's maximization problem are (4.38), (4.39), (4.40) holding with equality, and

$$-\left(x_{s,t} - \bar{x}\right) = \lambda_{s,t}, \tag{4.41}$$

$$h_{s,t}^{\omega-1} = w_t, \tag{4.42}$$

and

$$\lambda_{s,t} = \beta(1+r)E_t.\lambda_{s,t+1} \tag{4.43}$$

Note that $h_{s,t}$ is independent of s (i.e., it is independent of the agent's birth date). This means that we can drop the subscript s from $h_{s,t}$ and write

$$h_t^{\omega-1} = w_t, \tag{4.44}$$

Use equations (4.38) and (6.9) to eliminate $c_{s,t}$ from the sequential budget constraint (4.39). This yields,

$$d_{s,t} = \left(\frac{1+r}{\theta}\right) d_{s,t-1} - \pi_t - \left(1 - \frac{1}{\omega}\right) w_t h_{s,t} + \bar{x} + \left(x_{s,t} - \bar{x}\right)$$

To facilitate notation, we introduce the following auxiliary variable:

$$y_t \equiv \pi_t + \left(1 - \frac{1}{\omega}\right) w_t h_t - \bar{x}, \tag{4.45}$$

which is the same for all generations s because both profits and hours worked are independent of the age of the cohort. Then the sequential budget constraint becomes:

$$d_{s,t} = \left(\frac{1+r}{\theta}\right) d_{s,t-1} - y_t + (x_{s,t} - \bar{x})$$

Now iterate this expression forward and use the transversality condition (i.e., equation (4.40) holding with equality), to obtain

$$\left(\frac{1+r}{\theta}\right) d_{s,t-1} \quad = \quad E_t \sum_{j=0}^{\infty} \left(\frac{\theta}{1+r}\right)^j [y_{t+j} - (x_{s,t+j} - \bar{x})]$$

Using equations (4.41) and (4.43) to replace $E_t x_{s,t+j}$ yields

$$\left(\frac{1+r}{\theta}\right) d_{s,t-1} \quad = \quad E_t \sum_{j=0}^{\infty} \left(\frac{\theta}{1+r}\right)^j y_{t+j} - \frac{\beta(1+r)^2}{\beta(1+r)^2 - \theta}(x_{s,t} - \bar{x})$$

Solve for $x_{s,t}$ to obtain

$$x_{s,t} = \bar{x} + \frac{\beta(1+r)^2 - \theta}{\beta\theta(1+r)}(\tilde{y}_t - d_{s,t-1}), \tag{4.46}$$

where $\tilde{y}_t$ denotes a weighted average of current and future expected values of y_t. Specifically, $\tilde{y}_t$ is defined as

$$\tilde{y}_t \equiv \frac{\theta}{1+r} E_t \sum_{j=0}^{\infty} \left(\frac{\theta}{1+r} \right)^j y_{t+j},$$

and can be expressed recursively as

$$\tilde{y}_t = \frac{\theta}{1+r} y_t + \frac{\theta}{1+r} E_t \tilde{y}_{t+1} \tag{4.47}$$

We now aggregate individual variables by summing over generations born at time $s \leq t$. Notice that at time t there are alive $1-\theta$ people born in t, $(1-\theta)\theta$ people born in $t-1$, and, in general, $(1-\theta)\theta^s$ people born in period $t-s$. Let

$$x_t \equiv (1-\theta) \sum_{s=t}^{-\infty} \theta^{t-s} x_{s,t}$$

and

$$d_t \equiv (1-\theta) \sum_{s=t}^{-\infty} \theta^{t-s} d_{s,t}$$

denote the aggregate levels of $x_{s,t}$ and $b_{s,t}$, respectively. Now multiply (4.46) by $(1-\theta)\theta^{t-s}$ and then sum for $s = t$ to $s = -\infty$ to obtain the following expression for the aggregate version of equation (4.46):

$$x_t = \bar{x} + \frac{\beta(1+r)^2 - \theta}{\beta\theta(1+r)} (\tilde{y}_t - \theta d_{t-1}). \tag{4.48}$$

In performing this step, keep in mind that $d_{t,t-1} = 0$. That is, agents inherit no debts at birth.

Finally, aggregate the first-order condition (4.41) and the budget constraint(4.39) to obtain

$$-(x_t - \bar{x}) = \lambda_t \tag{4.49}$$

and

$$d_t = (1 + r)d_{t-1} - y_t + x_t - \bar{x} \tag{4.50}$$

We now turn to the economic units producing consumption and capital goods.

Firms Producing Consumption Goods

We assume the existence of competitive firms that hire capital and labor services to produce consumption goods. these firms maximize profits, which are given by:

$$A_t F(k_t, h_t) - w_t h_t - u_t k_t,$$

where the productivity factor A_t and the production function F are as described earlier in this section. The first-order conditions associated with the firm's profit-maximization problem are

$$A_t F_k(k_t, h_t) = u_t \tag{4.51}$$

and

$$A_t F_h(k_t, h_t) = w_t. \tag{4.52}$$

Firms Producing Capital Goods

We assume that there exist firms that buy consumption goods to transform them into investment goods, rent out capital, and pay dividends, π_t.

Formally, dividends in period t are given by

$$\pi_t = u_t k_t - i_t - \Phi(k_{t+1} - k_t) \tag{4.53}$$

The evolution of capital follows the law of motion given earlier in this chapter, which we reproduce here for convenience

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{4.54}$$

Because investment goods take one period to become productive capital and because of the presence of adjustment costs, current investment decisions of the firm have consequences not only for current profits but also for future profits. It follows that the optimization problem of the capital-producing firm is inherently dynamic. The firm must maximize some present discounted value of current and future expected profits. A problem that emerges at this point is what discount factor should the firm use. This issue does not have a clear answer for two reasons: first, the owners of the firm change over time. Recall that the shares of the dead are distributed in equal parts among the newborn. It follows that the firm cannot use as its discount factor the intertemporal marginal rate of substitution of the 'representative household.' For the representative household does not exist. Second, the firm operates in a financial environment characterized by incomplete asset markets. For this reason, it cannot use the price of state-contingent claims to discount future profits. For there is no market for such claims.

One must therefore introduce assumptions regarding the firm's discount-ing behavior. These assumptions will in general not be inocuous with respect to the dynamics of capital accumulation. With this in mind, we will assume that the firm uses the discount factor $\beta^j \lambda_{t+j}/\lambda_t$ to discount quantities of goods delivered in a particular state of period $t+j$ into period t. Note that this discount factor uses the average marginal utility of consumption of agents alive in period t and $t+j$, given by λ_t and λ_{t+j}, respectively.That is, at any given time, the firm converts goods into utils by multiplying the amount of goods by the average marginal utility of consumption of the shareholders alive at that time. Note also that we use as the subjective discount factor the parameter β and not $\beta\theta$. This is because the number of shareholders is constant over time (and equal to unity), unlike the size of a cohort born at a particular date, which declines at the mortality rate $1-\theta$.

The Lagrangean associated with the capital-producing optimization problem is then given by

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[u_{t+j} k_{t+j} - k_{t+j+1} + (1-\delta) k_{t+j} - \Phi(k_{t+j+1} - k_{t+j}) \right]$$

The first-order condition with respect to k_{t+1} is

$$\lambda_t [1 + \Phi'(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1} \left[u_{t+1} + 1 - \delta + \Phi'(k_{t+2} - k_{t+1}) \right] \quad (4.55)$$

Equilibrium

Equations (4.44), (4.45), (4.47)-(4.55) form a system of eleven equations in eleven unknowns: x_t, λ_t, h_t, w_t, u_t, π_t, i_t, k_t, d_t, y_t, $\tilde{y}_t$. Below, we reproduce the system of equilibrium conditions for convenience:

$$h_t^{\omega-1} = w_t,$$

$$y_t \equiv \pi_t + \left(1 - \frac{1}{\omega}\right) w_t h_t - \bar{x},$$

$$\tilde{y}_t = \frac{\theta}{1+r} y_t + \frac{\theta}{1+r} E_t \tilde{y}_{t+1},$$

$$x_t = \bar{x} + \frac{\beta(1+r)^2 - \theta}{\beta\theta(1+r)}(\tilde{y}_t - \theta d_{t-1}),$$

$$-(x_t - \bar{x}) = \lambda_t,$$

$$d_t = (1+r)d_{t-1} - y_t + x_t - \bar{x},$$

$$A_t F_k(k_t, h_t) = u_t,$$

$$A_t F_h(k_t, h_t) = w_t,$$

$$\pi_t = u_t k_t - i_t - \Phi(k_{t+1} - k_t),$$

$$k_{t+1} = (1-\delta)k_t + i_t,$$

$$\lambda_t[1 + \Phi'(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1}\left[u_{t+1} + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right],$$

It is of interest to consider the special case in which $\beta(1+r) = 1$. In this case, the evolution of per capita external debt is given by $d_t = \theta d_{t-1} + (1+r-\theta)/\theta\tilde{y}_t - y_t$. This expression shows that the stock of debt per capital does not follows a random walk as was the case in the representative-agent economy with constant subjective discount factor and world interest rate. In effect, the (autoregressive) coefficient on past external debt is $\theta \in (0,1)$.

The mean reverting property of aggregate external debt obtains in spite of the fact that individual debt positions follow a random walk. The reason why the aggregate level of external debt is trend reverting in equilibrium is the fact that each period a fraction $1-\theta \in (0,1)$ of the agents die and are replaced by newborns holding no financial assets. As a result, on average, the current aggregate level of debt is only a fraction θ of the previous period's level of debt. (This intuition also goes through when $\beta(1+r) \neq 1$, although in this case individual levels of debt display a trend in the deterministic equilibrium.) The forcing term $y_t - (1+r-\theta)/\theta \tilde{y}_t$ represents the difference between current and permanent income. In equilibrium, the current account absorbs these income deviations.

In the deterministic steady state, the aggregate level of debt is given by

$$d = \frac{\frac{1}{r} - \frac{\beta(1+r)^2-\theta}{r\beta(1+r)(1+r-\theta)}}{1 - \frac{\beta(1+r)^2-\theta}{r\beta(1+r)}} y$$

In the special case in which $\beta(1+r)$ equals unity, the steady-state aggregate stock of debt is nil. This is because in this case agents, all of whom are born with no debts, wish to hold constant debt levels over time. That is, in the steady state both the aggregate and the individual levels of debt are zero in this case.

Implied Business Cycles

As in the previous subsections, we use the functional forms $F(k,h) = k^{\alpha}h^{1-\alpha}$ and $\Phi(x) = \frac{\phi}{2}x^2$. And we calibrate the model as follows: $\delta = 0.1$. $\alpha = 0.32$, $\rho = 0.42$, $\omega = 1.4550$, $\phi = 0.028$, $r = 0.04$, $\bar{x} = 0.51$, $\theta = 0.9933$, and

Table 4.5: Perpetual Youth Model: Implied Second Moments

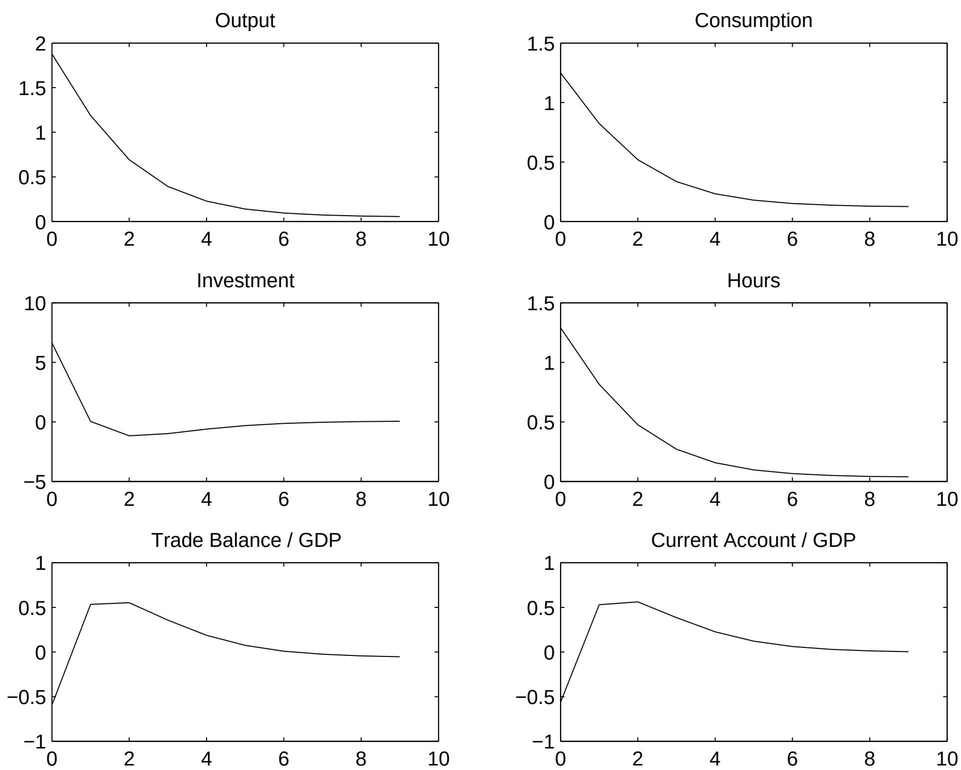
Variable	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
Output	3.2	0.63	1
Consumption	2.8	0.8	0.89
Investment	8.9	0.065	0.69
Hours	2.2	0.63	1
Trade-Balance-To-Output Ratio	1.6	0.46	-0.099
Current-Account-To-Output Ratio	1.4	0.3	0.059

Note. Standard deviations are measured in percentage points.

$\beta = 0.9596$. Note that the calibration makes agents relatively impatient, in the sense that $\beta(1+r) < 1$. The reason for imposing tis parameterization is that it allows for a positive aggregate level of external debt in the steady state, and a positive steady-state trade-balance-to-output ratio of about 2 percent, as in the calibration of the models discussed earlier in this section.

Table 4.5 displays unconditional second moments implied by the model and figure 4.4 depicts impulse responses to a one-percent increase in total factor productivity. The model dynamics are quite similar to those obtained under the alternative ways of inducing stationarity discussed in this section. In particular, the model generates the same ranking of volatilities and quite similar patterns of autocorrelations and correlations with output. In addition, as in the other models discussed in this section, in response to a positive innovation in total factor productivity, the perpetual-youth model implies expansions in output, consumption, investment, and hours and a deterioration in the trade balance and the current account.

Figure 4.4: Perpetual Youth Model: Impulse Response to a One-Percent Productivity Shock



4.4 Appendix A: Log-Linearization

The log-linear version of the system (4.39)-(4.6) and (4.8)-(4.14) is given by

$$s_{tb}\widehat{d}_t = s_{tb}\frac{r}{1+r}\widehat{r}_{t-1} + s_{tb}(1+r)\widehat{d}_{t-1} - r[\widehat{y}_t - s_c\widehat{c}_t - s_i\widehat{i}_t]$$

$$\widehat{y}_t = \widehat{A}_t + \alpha\widehat{k}_t + (1-\alpha)\widehat{h}_t$$

$$\widehat{k}_{t+1} = (1-\delta)\widehat{k}_t + \delta\widehat{i}_t$$

$$\widehat{\lambda}_t = \frac{r}{1+r}\widehat{r}_t + \epsilon_{\beta c}\widehat{c}_t + \epsilon_{\beta h}\widehat{h}_t + E_t\widehat{\lambda}_{t+1}$$

$$\widehat{\lambda}_t = \frac{(1-\beta)\epsilon_c}{(1-\beta)\epsilon_c - \beta\epsilon_{\beta c}}[\epsilon_{cc}\widehat{c}_t + \epsilon_{ch}\widehat{h}_t] - \frac{\beta\epsilon_{\beta c}}{(1-\beta)\epsilon_c - \beta\epsilon_{\beta c}}[\widehat{\eta}_t + \epsilon_{\beta cc}\widehat{c}_t + \epsilon_{\beta ch}\widehat{h}_t]$$

$$\widehat{\eta}_t = (1-\beta)[\epsilon_c E_t\widehat{c}_{t+1} + \epsilon_h E_t\widehat{h}_{t+1}] + \beta[E_t\widehat{\eta}_{t+1} + \epsilon_{\beta c}\widehat{c}_t + \epsilon_{\beta h}\widehat{h}_t]$$

$$\frac{(1-\beta)\epsilon_h}{(1-\beta)\epsilon_h + \beta\epsilon_{\beta h}}[\epsilon_{hc}\widehat{c}_t + \epsilon_{hh}\widehat{h}_t] + \frac{\beta\epsilon_{\beta h}}{(1-\beta)\epsilon_h + \beta\epsilon_{\beta h}}[\widehat{\eta}_t + \epsilon_{\beta hc}\widehat{c}_t + \epsilon_{\beta hh}\widehat{h}_t] = \widehat{\lambda}_t + \widehat{A}_t + \alpha\widehat{k}_t - \alpha\widehat{h}_t$$

$$\begin{aligned}\widehat{\lambda}_t + \phi k\widehat{k}_{t+1} - \phi k\widehat{k}_t \quad = \quad & \epsilon_{\beta c}\widehat{c}_t + \epsilon_{\beta h}\widehat{h}_t + E_t\widehat{\lambda}_{t+1} + \beta(\beta^{-1} + \delta - 1)[E_t\widehat{A}_{t+1} \\ & +(1-\alpha)E_t\widehat{h}_{t+1} - (1-\alpha)\widehat{k}_{t+1} + \beta\phi k E_t\widehat{k}_{t+2} - \beta\phi k\widehat{k}_{t+1}\end{aligned}$$

$$\widehat{r}_t = 0$$

$$\hat{A}_t = \rho\hat{A}_{t-1} + \epsilon_t,$$

where $\epsilon_{\beta c} \equiv c\beta_c/\beta$, $\epsilon_{\beta h} \equiv h\beta_h/\beta$, $\epsilon_{\beta cc} \equiv c\beta_{cc}/\beta_c$, $\epsilon_{\beta ch} \equiv h\beta_{ch}/\beta_c$, $\epsilon_c \equiv cU_c/U$, $\epsilon_{cc} = cU_{cc}/U_c$, $\epsilon_{ch} = hU_{ch}/U_c$, $s_{tb} \equiv tb/y$, $s_c \equiv c/y$, $s_i = i/y$. In the log-linearization we are using the particular forms assumed for the production function and the capital adjustment cost function.

4.5 Appendix B: Solving Dynamic General Equilibrium Models

The equilibrium conditions of the simple real business cycle model we studied in the previous chapter takes the form of a nonlinear stochastic vector difference equation. Reduced forms of this sort are common in Macroeconomics. A problem that one must face is that, in general, it is impossible to solve such systems. But fortunately one can obtain good approximations to the true solution in relatively easy ways. In the previous chapter, we introduced one particular strategy, consisting in linearizing the equilibrium conditions around the nonstochastic steady state. Here we explain in detail how to solve the resulting system of linear stochastic difference equations. In addition, we show how to use the solution to compute second moments and impulse response functions.

The equilibrium conditions of a wide variety of dynamic stochastic general equilibrium models can be written in the form of a nonlinear stochastic vector difference equation

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0, \tag{4.56}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t. The vector x_t denotes predetermined (or state) variables and the vector y_t denotes nonpredetermined (or control) variables. The initial value of the state vector x_0 is an initial condition for the economy. (Beyond the initial condition, the complete set of equilibrium conditions also includes a terminal condition, like a no-Ponzi game constraint. We omit such a constraint here because we focus on approximating stationary solutions.) The state vector x_t can be partitioned as $x_t = [x_t^1;\ x_t^2]'$. The vector x_t^1 consists of endogenous predetermined state variables and the vector x_t^2 of exogenous state variables. Specifically, we assume that x_t^2 follows the exogenous stochastic process given by

$$x_{t+1}^2 = \tilde{h}(x_t^2, \sigma) + \tilde{\eta}\sigma\epsilon_{t+1},$$

where both the vector x_t^2 and the innovation ϵ_t are of order $n_\epsilon \times 1$.[4] The vector ϵ_t is assumed to have a bounded support and to be independently and identically distributed, with mean zero and variance/covariance matrix I. The eigenvalues of the Jacobian of the function $\tilde{h}$ with respect to its first argument evaluated at the non-stochastic steady state are assumed to lie within the unit circle.

The solution to models belonging to the class given in equation (4.56) is of the form:

$$y_t = \hat{g}(x_t) \tag{4.57}$$

and

$$x_{t+1} = \hat{h}(x_t) + \eta\sigma\epsilon_{t+1}. \tag{4.58}$$

The vector x_t of predetermined variables is of size $n_x \times 1$ and the vector y_t of nonpredetermined variables is of size $n_y \times 1$. We define $n = n_x + n_y$. The function f then maps $R^{n_y} \times R^{n_y} \times R^{n_x} \times R^{n_x}$ into R^n.

[4]It is straightforward to accommodate the case in which the size of the innovations vector ϵ_t is different from that of x_t^2.

The matrix η is of order $n_x \times n_\epsilon$ and is given by

$$\eta = \begin{bmatrix} \emptyset \\ \tilde{\eta} \end{bmatrix}.$$

The shape of the functions $\hat{h}$ and $\hat{g}$ will in general depend on the amount of uncertainty in the economy. The key idea of perturbation methods is to interpret the solution to the model as a function of the state vector x_t and of the parameter σ scaling the amount of uncertainty in the economy, that is,

$$y_t = g(x_t, \sigma) \tag{4.59}$$

and

$$x_{t+1} = h(x_t, \sigma) + \eta\sigma\epsilon_{t+1}, \tag{4.60}$$

where the function g maps $R^{n_x} \times R^+$ into R^{n_y} and the function h maps $R^{n_x} \times R^+$ into R^{n_x}.

Given this interpretation, a perturbation methods finds a *local* approximation of the functions g and h. By a local approximation, we mean an approximation that is valid in the neighborhood of a particular point $(\bar{x}, \bar{\sigma})$.

Taking a Taylor series approximation of the functions g and h around the point $(x, \sigma) = (\bar{x}, \bar{\sigma})$ we have (for the moment to keep the notation simple, let's assume that nx=ny=1)

$$\begin{aligned} g(x,\sigma) &= g(\bar{x},\bar{\sigma}) + g_x(\bar{x},\bar{\sigma})(x-\bar{x}) + g_\sigma(\bar{x},\bar{\sigma})(\sigma-\bar{\sigma}) \\ &+\frac{1}{2}g_{xx}(\bar{x},\bar{\sigma})(x-\bar{x})^2 + g_{x\sigma}(\bar{x},\bar{\sigma})(x-\bar{x})(\sigma-\bar{\sigma}) \\ &+\frac{1}{2}g_{\sigma\sigma}(\bar{x},\bar{\sigma})(\sigma-\bar{\sigma})^2 + \ldots \\ h(x,\sigma) &= h(\bar{x},\bar{\sigma}) + h_x(\bar{x},\bar{\sigma})(x-\bar{x}) + h_\sigma(\bar{x},\bar{\sigma})(\sigma-\bar{\sigma}) \\ &+\frac{1}{2}h_{xx}(\bar{x},\bar{\sigma})(x-\bar{x})^2 \\ &+h_{x\sigma}(\bar{x},\bar{\sigma})(x-\bar{x})(\sigma-\bar{\sigma}) \\ &+\frac{1}{2}h_{\sigma\sigma}(\bar{x},\bar{\sigma})(\sigma-\bar{\sigma})^2 + \ldots, \end{aligned}$$

The unknowns of an n^{th} order expansion are the n-th order derivatives of the functions g and h evaluated at the point $(\bar{x},\bar{\sigma})$.

To identify these derivatives, substitute the proposed solution given by equations (4.59) and (4.60) into equation (4.56), and define

$$\begin{aligned} F(x,\sigma) &\equiv E_t f(g(h(x,\sigma)+\eta\sigma\epsilon',\sigma), g(x,\sigma), h(x,\sigma)+\eta\sigma\epsilon', x) \\ &= 0. \end{aligned} \tag{4.61}$$

Here we are dropping time subscripts. We use a prime to indicate variables dated in period $t+1$.

Because $F(x,\sigma)$ must be equal to zero for any possible values of x and σ, it must be the case that the derivatives of any order of F must also be equal to zero. Formally,

$$F_{x^k\sigma^j}(x,\sigma) = 0 \quad \forall x,\sigma,j,k, \tag{4.62}$$

where $F_{x^k\sigma^j}(x,\sigma)$ denotes the derivative of F with respect to x taken k times and with respect to σ taken j times.

As will become clear below, a particularly convenient point to approximate the functions g and h around is the non-stochastic steady state, $x_t = \bar{x}$ and $\sigma = 0$. We define the non-stochastic steady state as vectors $(\bar{x}, \bar{y})$ such that

$$f(\bar{y}, \bar{y}, \bar{x}, \bar{x}) = 0.$$

It is clear that $\bar{y} = g(\bar{x}, 0)$ and $\bar{x} = h(\bar{x}, 0)$. To see this, note that if $\sigma = 0$, then $E_t f = f$. The reason why the steady state is a particularly convenient point is that in most cases it is possible to solve for the steady state. With the steady state values in hand, one can then find the derivatives of the function F.

We are looking for approximations to g and h around the point $(x, \sigma) = (\bar{x}, 0)$ of the form

$$g(x,\sigma) = g(\bar{x},0) + g_x(\bar{x},0)(x-\bar{x}) + g_\sigma(\bar{x},0)\sigma$$

$$h(x,\sigma) = h(\bar{x},0) + h_x(\bar{x},0)(x-\bar{x}) + h_\sigma(\bar{x},0)\sigma$$

As explained earlier,

$$g(\bar{x},0) = \bar{y}$$

and

$$h(\bar{x},0) = \bar{x}.$$

The remaining unknown coefficients of the first-order approximation to g

and h are identified by using the fact that, by equation (4.62), it must be the case that:

$$F_\sigma(\bar{x}, 0) = 0.$$

and

$$F_x(\bar{x}, 0) = 0$$

To find those derivatives let's repeat equation (4.61)

$$\begin{aligned} F(x, \sigma) &\equiv E_t f(g(h(x, \sigma) + \eta\sigma\epsilon', \sigma), g(x, \sigma), h(x, \sigma) + \eta\sigma\epsilon', x) \\ &= 0. \end{aligned}$$

Taking derivative with respect to the scalar σ we find:

$$\begin{aligned} F_\sigma(\bar{x}, 0) &= E_t f_{y'}[g_x(h_\sigma + \eta\epsilon') + g_\sigma] + f_y g_\sigma + f_{x'}(h_\sigma + \eta\epsilon') \\ &= f_{y'}[g_x h_\sigma + g_\sigma] + f_y g_\sigma + f_{x'} h_\sigma \end{aligned}$$

This is a system of n equations. Then imposing

$$F_\sigma(\bar{x}, 0) = 0.$$

one can identify g_σ and h_σ:

$$\begin{bmatrix} f_{y'} g_x + f_{x'} & f_{y'} + f_y \end{bmatrix} \begin{bmatrix} h_\sigma \\ g_\sigma \end{bmatrix} = 0$$

This equation is linear and homogeneous in g_σ and h_σ. Thus, if a unique solution exists, we have that

$$h_\sigma = 0.$$

and

$$g_\sigma = 0.$$

These two expressions represent an important theoretical result. They show that in general, up to first order, one need not correct the constant term of the approximation to the policy function for the size of the variance of the shocks.

This result implies that in a first-order approximation the expected values of x_t and y_t are equal to their non-stochastic steady-state values $\bar{x}$ and $\bar{y}$. In this sense, we can say that in a first-order approximation the certainty equivalence principle holds, that is, the policy function is independent of the variance-covariance matrix of ϵ_t. This is an important limitation of first-order perturbation techniques. Because in many economic applications we are interested in finding the effect of uncertainty on the economy. For example, up to first-order the mean of the rate of return of all all assets must be same. Thus, first-order approximation techniques cannot be used to study risk premia. Another important question that can in general not be addressed with first-order perturbation techniques is how uncertainty affects welfare. This question is at the heart of the recent literature on optimal fiscal and monetary stabilization policy. Because in a first-order accurate solution the unconditional expectation of a variable is equal to the non-stochastic steady state, any two policies that give rise to the same steady state yield, up to first-order the same level of welfare.

To find g_x and h_x differentiate (4.61) with respect to x to obtain the following system

$$F_x(\bar{x}, 0) \quad = \quad f_{y'} g_x h_x + f_y g_x + f_{x'} h_x + f_x$$

Note that the derivatives of f evaluated at $(y', y, x', x) = (\bar{y}, \bar{y}, \bar{x}, \bar{x})$ are known. The above expression represents a system of $n \times n_x$ quadratic equations in the $n \times n_x$ unknowns given by the elements of g_x and h_x. Imposing

$$F_x(\bar{x}, 0) = 0$$

the above expression can be written as:

$$[f_{x'} \quad f_{y'}] \begin{bmatrix} I \\ g_x \end{bmatrix} h_x = -[f_x \quad f_y] \begin{bmatrix} I \\ g_x \end{bmatrix}$$

Let $A = [f_{x'} \quad f_{y'}]$ and $B = -[f_x \quad f_y]$. Note that both A and B are known.

Let $\hat{x}_t \equiv x_t - \bar{x}$, then postmultiplying the above system equation **??**) by $\hat{x}_t$ we obtain:

$$A \begin{bmatrix} I \\ g_x \end{bmatrix} h_x \hat{x}_t = B \begin{bmatrix} I \\ g_x \end{bmatrix} \hat{x}_t$$

Consider for the moment, a perfect foresight equilibrium. In this case, $h_x \hat{x}_t = \hat{x}_{t+1}$.

$$A \begin{bmatrix} I \\ g_x \end{bmatrix} \hat{x}_{t+1} = B \begin{bmatrix} I \\ g_x \end{bmatrix} \hat{x}_t$$

We are interested in solutions in which

$$\lim_{t\to\infty} |\hat{x}_t| < \infty$$

We will use this limiting conditions to find the matrix g_x. In particular, we will use the Schur decomposition method.

To solve the above system, we use the generalized Schur decomposition of the matrices A and B.[5] The generalized Schur decomposition of A and B is given by upper triangular matrices a and b and orthonormal matrices q and z satisfying:[6]

$$qAz = a$$

and

$$qBz = b.$$

Let

$$s_t \equiv z'[I; g_x]\hat{x}_t.$$

Then we have that

$$as_{t+1} = bs_t$$

Now partition a, b, z, and s_t as

$$a = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}, b = \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix}; z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}; s_t = \begin{bmatrix} s_t^1 \\ s_t^2 \end{bmatrix},$$

[5] More formal descriptions of the method can be found in Sims (1996) and Klein (2000).

[6] Recall that a matrix a is said to be upper triangular if elements $a_{ij} = 0$ for $i > j$. A matrix z is orthonormal if $z'z = zz' = I$.

where a_{22} and b_{22} are of order $n_y \times n_y$, z_{12} is of order $n_x \times n_y$, and s_t^2 is of order $n_y \times 1$. Then we have that

$$a_{22} s_{t+1}^2 = b_{22} s_t^2,$$

or

$$b_{22}^{-1} a_{22} s_{t+1}^2 = s_t^2.$$

Assume, without loss of generality, that the ratios $\text{abs}(a_{ii}/b_{ii})$ are decreasing in i. Suppose further that the number of ratios less than unity is exactly equal to the number of control variables, n_y, and that the number of ratios greater than one is equal to the number of state variables, n_x. By construction, the eigenvalues of $b_{22}^{-1} a_{22}$ are all less than unity in modulus.[7] Thus, the requirement $\lim_{j\to\infty} |s_{t+j}^2| < \infty$ is satisfied only if $s_t^2 = 0$. In turn, by the definition of s_t^2, this restriction implies that

$$(z_{12}' + z_{22}' g_x)\hat{x}_t = 0.$$

Because this condition has to hold for any value of the state vector, $\hat{x}_t$, it follows that it must be the case that

$$z_{12}' + z_{22}' g_x = 0.$$

[7] Here we are applying a number of properties of upper triangular matrices. Namely, (a) The inverse of a nonsingular upper triangular matrix is upper triangular. (b) the product of two upper triangular matrices is upper triangular. (c) The eigenvalues of an upper triangular matrix are the elements of its main diagonal.

Solving this expression for g_x yields

$$g_x = -{z'_{22}}^{-1} z'_{12}.$$

The fact that $s^2_t = 0$ also implies that

$$a_{11} s^1_{t+1} = b_{11} s^1_t,$$

or

$$s^1_{t+1} = {a_{11}}^{-1} b_{11} s^1_t$$

Now

$$s^1_t = (z'_{11} + z'_{21} gx)\hat{x}_t.$$

Replacing g_x, we have

$$s^1_t = [z'_{11} - z'_{21} {z'_{22}}^{-1} z'_{12}]\hat{x}_t.$$

Combining this expression with the equation describing the evolution of s_t shown two lines above, we get

$$\hat{x}_{t+1} = [z'_{11} - z'_{21} {z'_{22}}^{-1} z'_{12}]^{-1} {a_{11}}^{-1} b_{11} [z'_{11} - z'_{21} {z'_{22}}^{-1} z'_{12}]\hat{x}_t;$$

so that

$$h_x = [z'_{11} - z'_{21} {z'_{22}}^{-1} z'_{12}]^{-1} {a_{11}}^{-1} b_{11} [z'_{11} - z'_{21} {z'_{22}}^{-1} z'_{12}].$$

We can simplify this expression for h_x by using the following restrictions:

$$I = z'z = \begin{bmatrix} z'_{11}z_{11} + z'_{21}z_{21} & z'_{11}z_{12} + z'_{21}z_{22} \\ z'_{12}z_{11} + z'_{22}z_{21} & z'_{12}z_{12} + z'_{22}z_{22} \end{bmatrix}$$

to write:[8]

$$h_x = z_{11}a_{11}^{-1}b_{11}z_{11}^{-1}.$$

4.6 Local Existence and Uniqueness of Equilibrium

In the above discussion, we assumed that the number of eigenvalues of D with modulus less than unity is exactly equal to the number of control variables, n_y, and that the number of eigenvalues of D with modulus greater than one is equal to the number of state variables, n_x. In this case there is a unique local equilibrium. But not for every economy this is the case. Let's first consider the case that the number of eigenvalues of D with modulus greater than unity is equal to $m < ny$, which is less than the number of control variables. Then the requirement that we wish to study equilibria in which $lim_{j\to\infty}E_t|\hat{x}_{t+j}| < \infty$ will only yield m restrictions, rather than n_y restrictions. It follows that one can choose arbitrary initial values for $n_y - m$ elements of y_0 and the resulting first order solution will still be

[8]To obtain this simple expression for h_x, use element $(2,1)$ of $z'z$ to get $z'_{12}z_{11} = -z'_{22}z_{21}$. Premultiply by ${z'_{22}}^{-1}$ and post multiply by z_{11}^{-1} to get ${z'_{22}}^{-1}z'_{12} = -z_{21}{z_{11}}^{-1}$. Use this expression to eliminate ${z'_{22}}^{-1}z'_{12}$ From the square bracket in the expression for h_x. Then this square bracket becomes $[z'_{11} + z'_{21}z_{21}{z_{11}}^{-1}]$. Now use element $(1,1)$ of $z'z$ to write $z'_{21}z_{21} = I - z'_{11}z_{11}$. Using this equation to eliminate $z'_{21}z_{21}$ from the expression in square brackets, we get $[z'_{11} + (I - z'_{11}z_{11}){z_{11}}^{-1}]$, which is simply z_{11}^{-1}.

expected to converge back to the steady state. In this case the equilibrium is indeterminate.

On the other hand, if the number of eigenvalues of D with modulus greater than unity is greater than the number of control variables, n_y, then no local equilibrium exists. Let again m denote the number of eigenvalues of D greater than unity in modulus and assume that $m > n_y$. Then in order to ensure that $\lim E_t|\hat{x}_{t+j}| < \infty$ we must set m elements of $[x_0 y_0]$ equal to zero. This implies that $m - n_y$ elements of x_0 must be functions of the remaining $n_x - (m - n_y)$ elements. But this can never be the case, because x_0 is a vector of predetermined or exogenous variables and therefore its elements can take arbitrary values. In this case, we say no local equilibrium exists.

4.7 Second Moments

Start with the equilibrium law of motion of the deviation of the state vector with respect to its steady-state value, which is given by

$$\hat{x}_{t+1} = h_x \hat{x}_t + \sigma \eta \epsilon_{t+1}, \tag{4.63}$$

Covariance Matrix of x_t

Let

$$\Sigma_x \equiv E \hat{x}_t \hat{x}_t'$$

denote the unconditional variance/covariance matrix of $\hat{x}_t$ and let

$$\Sigma_\epsilon \equiv \sigma^2 \eta \eta'.$$

Then we have that

$$\Sigma_x = h_x \Sigma_x h_x' + \Sigma_\epsilon.$$

We will describe two numerical methods to compute Σ_x.

Method 1

One way to obtain Σ_x is to make use of the following useful result. Let A, B, and C be matrices whose dimensions are such that the product ABC exists. Then

$$\text{vec}(ABC) = (C' \otimes A) \cdot \text{vec}(B),$$

where the vec operator transforms a matrix into a vector by stacking its columns, and the symbol $\otimes$ denotes the Kronecker product. Thus if the vec operator is applied to both sides of

$$\Sigma_x = h_x \Sigma_x h_x' + \Sigma_\epsilon,$$

the result is

$$\begin{aligned} \text{vec}(\Sigma_x) &= \text{vec}(h_x \Sigma_x h_x') + \text{vec}(\Sigma_\epsilon) \\ &= \mathcal{F}\ \text{vec}(\Sigma_x) + \text{vec}(\Sigma_\epsilon), \end{aligned}$$

where

$$\mathcal{F} = h_x \otimes h_x.$$

Solving the above expression for $\text{vec}(\Sigma_x)$ we obtain

$$\text{vec}(\Sigma_x) = (I - \mathcal{F})^{-1}\text{vec}(\Sigma_\epsilon)$$

provided that the inverse of $(I-\mathcal{F})$ exists. The eigenvalues of $\mathcal{F}$ are products of the eigenvalues of the matrix h_x. Because all eigenvalues of the matrix h_x have by construction modulus less than one, it follows that all eigenvalues of $\mathcal{F}$ are less than one in modulus. This implies that $(I - \mathcal{F})$ is nonsingular and we can indeed solve for Σ_x. One possible drawback of this method is that one has to invert a matrix that has dimension $n_x^2 \times n_x^2$.

Method 2

The following iterative procedure, called doubling algorithm, may be faster than the one described above in cases in which the number of state variables (n_x) is large.

$$\Sigma_{x,t+1} = h_{x,t}\Sigma_{x,t}h'_{x,t} + \Sigma_{\epsilon,t}$$

$$h_{x,t+1} = h_{x,t}h_{x,t}$$

$$\Sigma_{\epsilon,t+1} = h_{x,t}\Sigma_{\epsilon,t}h'_{x,t} + \Sigma_{\epsilon,t}$$

$$\Sigma_{x,0} = I$$

$$h_{x,0} = h_x$$

$$\Sigma_{\epsilon,0} = \Sigma_\epsilon$$

Other second moments

Once the covariance matrix of the state vector, x_t has been computed, it is easy to find other second moments of interest. Consider for instance the covariance matrix $E\hat{x}_t\hat{x}'_{t-j}$ for $j > 0$. Let $\mu_t = \sigma\eta\epsilon_t$.

$$\begin{aligned} E\hat{x}_t\hat{x}'_{t-j} &= E[h_x^j\hat{x}_{t-j} + \sum_{k=0}^{j-1} h_x^k\mu_{t-k}]\hat{x}'_{t-j} \\ &= h_x^j E\hat{x}_{t-j}\hat{x}'_{t-j} \\ &= h_x^j\Sigma_x \end{aligned}$$

Similarly, consider the variance covariance matrix of linear combinations of the state vector x_t. For instance, the co-state, or control vector y_t is given by $y_t = \bar{y} + g_x(x_t - \bar{x})$, which we can write as: $\hat{y}_t = g_x\hat{x}_t$. Then

$$\begin{aligned} E\hat{y}_t\hat{y}'_t &= E g_x\hat{x}_t\hat{x}'_t g'_x \\ &= g_x[E\hat{x}_t\hat{x}'_t]g'_x \\ &= g_x\Sigma_x g'_x \end{aligned}$$

and, more generally,

$$\begin{aligned} E\hat{y}_t\hat{y}'_{t-j} &= g_x[E\hat{x}_t\hat{x}'_{t-j}]g'_x \\ &= g_x h_x^j\Sigma_x g'_x, \end{aligned}$$

for $j \geq 0$.

4.8 Impulse Response Functions

The impulse response to a variable, say z_t in period $t+j$ to an impulse in period t is defined as:

$$IR(z_{t+j}) \equiv E_t z_{t+j} - E_{t-1} z_{t+j}$$

The impulse response function traces the expected behavior of the system from period t on given information available in period t, relative to what was expected at time $t-1$. Using the law of motion $E_t \hat{x}_{t+1} = h_x \hat{x}_t$ for the state vector, letting x denote the innovation to the state vector in period 0, that is, $x = \eta\sigma\epsilon_0$, and applying the law of iterated expectations we get that the impulse response of the state vector in period t is given by

$$IR(\hat{x}_t) \equiv E_0 \hat{x}_t - E_{-1} \hat{x}_t = h_x^t [x_0 - E_{-1} x_0] = h_x^t [\eta\sigma\epsilon_0] = h_x^t x; \quad t \geq 0.$$

The response of the vector of controls $\hat{y}_t$ is given by

$$IR(\hat{y}_t) = g_x h_x^t x.$$

4.9 Matlab Code For Linear Perturbation Methods

Stephanie Schmitt-Grohé and I have written a suite of programs that are posted on the courses webpage: `www.columbia.edu/~mu2166/2nd_order.htm`. The program `gx_hx.m` computes the matrices g_x and h_x using the Schur decomposition method. The program `mom.m` computes second moments. The program `ir.m` computes impulse response functions.

4.10 Higher Order Approximations

In this chapter, we focused on a first-order approximation to the solution of a nonlinear system of stochastic difference equations of the form $E_t f(x_{t+1}, x_t) = 0.$. But higher order approximations are relatively easy to obtain. Indeed, there is a sense in which higher order approximations are simpler than the first order approximation. Namely, obtaining a higher-order approximation to the solution of the non-linear system is a sequential procedure. Specifically, the coefficients of the ith term of the jth-order approximation are given by the coefficients of the ith term of the ith order approximation, for $j > 1$ and $i < j$. So if the first-order approximation to the solution is available, then obtaining the second order approximation requires only to compute the coefficients of the quadratic terms, since the coefficients of the linear terms are those of the first order approximation. More importantly, obtaining the coefficients of the ith order terms of the approximate solution given all lower-order coefficients involves solving a *linear* system of equations.

Schmitt-Grohé and Uribe (2004) describe in detail how to obtain a second-order approximation to the solution of the nonlinear system and provide MATLAB code that implements the approximation.

4.11 Exercise

4.11.1 An RBC Small Open Economy with an internal debt-elastic interest-rate premium

Consider RBC open economy model with a debt elastic interest rate premium studied in Schmitt-Grohé and Uribe (SGU) (*JIE*, 2003, section 3, model 2). Modify the model by assuming that agents internalize the dependence of the interest rate premium on the level of debt. Specifically, suppose that the function $p(\cdot)$ depends upon the individual debt position, d_t, rather than on the aggregate per capita level of debt, $\tilde{d}_t$.

1. Derive the model's equilibrium conditions.

2. Use the same forms for the functions U, F, Φ, and p as in SGU. Calibrate the parameters γ, ω, α, ϕ, r, δ, ρ, σ_ϵ, β, $\bar{d}$, and ψ_2 using tables 1 and 2 in SGU. Calculate the model's nonstochastic steady state and compare it to that of model 2 in SGU.

3. Compute the unconditional standard deviation, serial correlation, and correlation with output of output, consumption, investment, hours, the trade balance-to-output ratio, and the current account-to-output ratio implied by the model. Compare these statistics to those associated with model 2 in SGU (reported in their table 3).

4. Compute the impulse response functions of output, consumption, investment, hours, the trade balance-to-output ratio, and the current account-to-output ratio implied by the model. Compare these im-

pulse responses to those associated with model 2 in SGU (shown in their figure 1).

Hint: You might find it convenient to use as a basis the matlab code associated with the SGU paper, located at `www.columbia.edu/~mu2166/closing.htm`

Chapter 5

The Terms of Trade

Three key stylized facts documented in chapter 1 are: (1) that emerging market economies are about twice as volatile as developed economies; (2) that private consumption spending is more volatile than output in emerging countries, but less volatile than output in developed countries; and (3) that the trade-balance-to-output ratio is significantly more countercyclical in emerging markets than it is in developed countries. Explaining this striking contrast between emerging and industrialized economies is at the top of the research agenda in small-open-economy macroeconomics. Broadly, the available theoretical explanations fall into two categories: One is that emerging market economies are subject to more volatile shocks than are developed countries. The second category of explanations argues that in emerging countries government policy tends to amplify business-cycle fluctuations whereas in developed countries public policy tends to mitigate aggregate instability. This and the following two chapters provide a progress report on the identification and quantification of exogenous sources of business cycles in small open economies. The present chapter concentrates on terms-of-trade shocks.

5.1 Defining the Terms of Trade

The terms of trade are defined as the relative price of exports in terms of imports. Letting P_t^x and P_t^m denote indices of world prices of exports and imports for a particular country, the terms of trade for that country are given by $tot_t \equiv P_t^x/P_t^m$.

Typically, emerging countries specialize in exports of a few primary commodities, such as metals, agricultural products, or oil. At the same time, emerging countries are normally small players in the world markets for the goods they export or import. It follows that for many small countries, the terms of trade can be regarded as an exogenous source of aggregate fluctuations. Because primary commodities display large fluctuations over time, the terms of trade have the potential to be an important source of business cycles in developing countries.

5.2 Empirical Regularities

Table 5.1 displays summary statistics relating the terms of trade to output, the components of aggregate demand, and the real exchange rate in the postwar era. In the table, the real exchange rate (rer) is defined as the relative price of consumption in terms of importable goods. Specifically, let P_t^c denote a domestic CPI index. Then the real exchange rate is given by P_t^c/P_t^m. A number of empirical regularities emerge from the table:

Table 5.1: The Terms of Trade and Business Cycles

Summary Statistic	Developed Countries	Developing Countries	Oil Exporting Countries
$\sigma(tot)$	4.70	10.0	18.0
$\rho(tot_t, tot_{t-1})$	0.47	0.40	0.50
$\sigma(tot)/\sigma(y)$	0.52	0.77	1.40
$\rho(tot, y)$	0.78	0.39	0.30
$\rho(tot, c)$	0.74	0.34	0.19
$\rho(tot, i)$	0.67	0.38	0.45
$\rho(tot, tb)$	0.24	0.28	0.33
$\rho(tot, rer)$	0.70	0.07	0.42

Source: Mendoza (1995), tables 1 and 3-6.

Note: tot, y, c, i, and tb denote, respectively, the terms of trade, output, consumption, investment, and the trade balance. The sample is 1955 to 1990 at annual frequency. The terms of trade are measured as the ratio of export to import unit values with 1900=100. All other variables are measured per capita at constant import prices. All variables are expressed in percent deviations from a HP trend constructed using a smoothing parameter of 100. The group of developed countries is formed by the US, UK, France, Germany, Italy, Canada, and Japan. The group of developing countries is formed by Argentina, Brazil, Chile, Mexico, Peru, Venezuela, Taiwan, India, Indonesia, Korea, Philippines, and Thailand. The group of oil-exporting countries is formed by Mexico, Venezuela, Saudi Arabia, Algeria, Cameroon, and Nigeria.

1. The terms of trade are twice as volatile in emerging countries as in developed countries, and they are almost twice as volatile in oil-exporting countries as in developing countries.

2. The terms of trade are half as volatile as output in developed countries, 75 percent as volatile as output in developing countries, and 150 percent as volatile as output in oil-exporting countries.

3. The terms of trade are procyclical. They are twice as procyclical in developed countries as in developing countries.

4. The terms of trade display positive but small serial correlation.

5. The correlation between the terms of trade and the trade balance is positive but small.

6. The terms of trade are positively correlated with the real exchange rate. This correlation is high for developed countries but almost nil for less developed countries.

The information provided in table 5.1 is mute on the importance of terms of trade shocks in explaining movements in aggregate activity. Later in this chapter, we attempt to answer this question by combining the empirical information contained in table 5.1 with the theoretical predictions of a fully specified dynamic general equilibrium model of the open economy.

5.2.1 TOT-TB Correlation: Two Early Explanations

The effects of terms-of-trade shocks on the trade balance is an old subject of investigation. More than half a century ago, Harberger (1950) and Laursen and Metzler (1950) formalized, within the context of a keynesian model, the conclusion that rising terms of trade should be associated with an improving trade balance. This conclusion became known as the Harberger-Laursen-Metzler (HLM) effect. This view remained more or less unchallenged until the early 1980s, when Obstfeld (1982) and Svensson and Razin (1983), using a dynamic optimizing model of the current account, concluded that the effect of terms of trade shocks on the trade balance depends crucially on the perceived persistence of the terms of trade. In their model a positive relation between terms of trade and the trade balance (i.e., the HLM effect) weakens as the terms of trade become more persistent and may even be overturned if the terms of trade are of a permanent nature. This view became known as the Obstfeld-Razin-Svensson (ORS) effect. Let us look at the HLM and ORS effects in some more detail.

The Harberger-Laursen-Metzler Effect

A simple way to obtain a positive relation between the terms of trade and the trade balance in the context of a Keynesian model is by starting with the national accounting identity

$$y_t = c_t + g_t + i_t + x_t - m_t,$$

where y_t denotes output, c_t denotes private consumption, g_t denotes public consumption, i_t denotes private investment, x_t denotes exports, and m_t denotes imports. Consider the following behavioral equations defining the dynamics of each component of aggregate demand. Public consumption and private investment are assumed to be independent of output. For simplicity, we will assume that these two varibles are constant over time and given by

$$g_t = \bar{g}$$

and

$$i_t = \bar{i},$$

respectively, where $\bar{g}$ and $\bar{i}$ are parameters. Consumption is assumed to be an increasing linear function of output

$$c_t = \bar{c} + \alpha y_t,$$

with $\alpha \in (0,1)$ and $\bar{c} > 0$ are parameters. Imports are assumed to be proportional to output,

$$m_t = \mu y_t,$$

with $\mu \in (0,1)$. In the jargon of the 1950s, the parameters α and μ are referred to as the marginal propensities to consume and import, respectively, whereas the term $\bar{c} + \bar{g} + \bar{i}$ is referred to as the autonomous component of domestic absorption. Output as well as all components of aggregate demand are expressed in terms of import goods. The quantity of goods exported in period t is denoted by q_t. Thus, the value of exports in terms of importables, x_t, is given by

$$x_t = tot_t q_t,$$

where tot_t denotes the terms of trade. The terms of trade are assumed to evolve exogenously, and the quantity of goods exported, q_t, is assumed to be constant and vigen by

$$q_t = \bar{q},$$

where $\bar{q}$ is a positive parameter. Using the behavioral equations to eliminate c_t, i_t, g_t, x_t, and m_t from the national income identity, and solving for output yields

$$y_t = \frac{\bar{c} + \bar{g} + \bar{i} + tot_t\bar{q}}{1 + \mu - \alpha}.$$

Letting $tb_t \equiv x_t - m_t$ denote the trade balance, we can write

$$tb_t = \frac{1 - \alpha}{1 + \mu - \alpha} tot_t\bar{q} - \frac{\mu(\bar{c} + \bar{g} + \bar{i})}{1 + \mu - \alpha}.$$

Clearly, this theory implies that an improvement in the terms of trade (an increase in tot_t) gives rise to an expansion in the trade surplus. This positive relation between the terms of trade and the trade balance is stronger the larger is the volume of exports, $\bar{q}$, the smaller is the marginal propensity to import, μ, and the smaller is the marginal propensity to consume α. The reason why μ increases the TOT multiplier is that a higher value of μ weakens the endogenous expansion in aggregate demand to an exogenous increase in exports, as a larger fraction of income is used to buy foreign goods. Similarly, a larger value of α reduces the TOT multiplier because it exacerbates the endogenous response of aggregate demand to a TOT shock through private consumption.

It is worth noting that in the context of this model, the sign of the effect of a TOT shock on the trade balance is independent of whether the terms of trade shocks are permanent or temporary in nature. This is the main contrast with the Obstfeld-Razin-Svensson effect.

The Obstfeld-Razin-Svensson Effect

The ORS effect is cast within the dynamic optimizing theoretical framework that differs fundamentally from the reduced-form Keynesian model we used to derive the HLM effect. Consider the small, open, endowment economy studied in chapter 2. This is an economy inhabited by an infinitely lived representative household with preferences described by the intertemporal utility function given in (2.5). Suppose that the good the household consumes is different from the good it is endowed with. The household, therefore exports the totality of its endowment and imports the totality of its consumption. Let tot_t denote the relative world price of exported goods in terms of imported goods, or the terms of trade. Assume for simplicity that the endowment of exportable goods is constant and normalized to unity, $y_t = 1$ for all t. The resource constraint is then given by

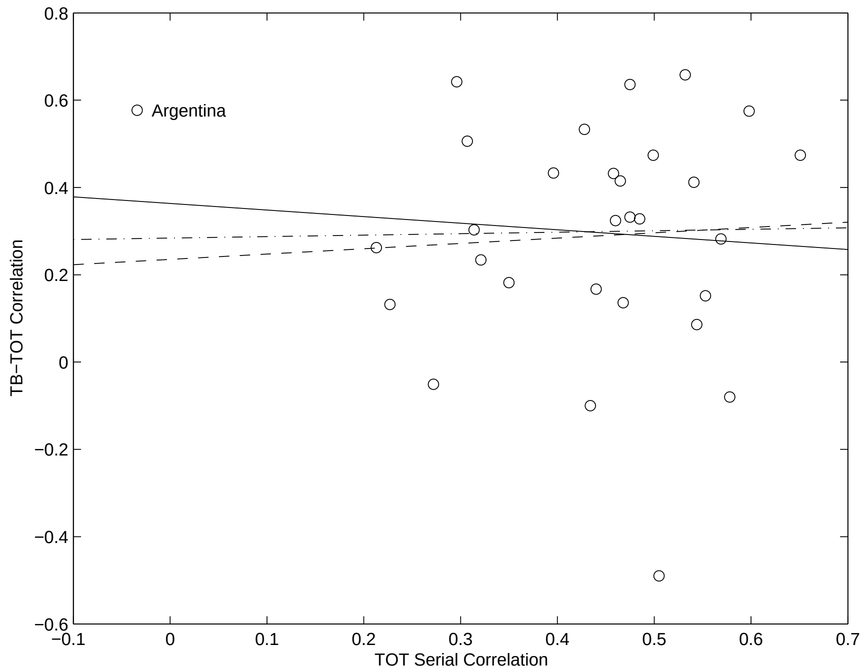
$$d_t = (1 + r)d_{t-1} + c_t - tot_t.$$

The borrowing constraint given in (2.3) prevents the household from engaging in Ponzi games. The economy is small in world product markets, so it takes the evolution of tot_t as exogenous. The model is therefore identical to the stochastic-endowment economy studied in chapter 2, with tot_t taking the place of y_t. We can then use the results derived in chapter 2 to draw the following conclusion: if the terms of trade are stationary then an increase in the terms of trade produces an improvement in the current account. Agents save in order to ensure higher future consumption. When terms of trade are nonstationary, an improvement in the terms of trade induces a trade balance deficit. In this case, the value of income is expected to grow over time, so agents can afford assuming higher current debts without sacrificing future expenditures.

This conclusion can be extended to a model with endogenous labor supply and capital accumulation. A simple way to do this is to modify the RBC model of chapter 4 by assuming again that households do not consume the good they produce. In this case, the productivity shock A_t can be interpreted as a terms-of-trade shock. An increase in the terms of trade produces an improvement in the trade balance if the terms of trade shock is transitory, but as the serial correlation of the terms of trade shock increases, an improvement in the terms of trade can lead to a deterioration in the current account driven by investment expenditures.

Is the ORS effect borne out in the data? If so, we should observe that countries experiencing more persistent terms-of-trade shocks should display lower correlations between the terms of trade and the trade balance than countries facing less persistent terms of trade shocks. Figure 5.1 plots the serial correlation of the terms of trade against the correlation of the trade balance with the terms of trade for 30 countries, including the G7 countries and 23 selected developing countries from Latin America, Africa, East Asia, and the Middle East. The 30 observations were taken from Mendoza (1995), table 1. The cloud of points, shown with circles, displays no pattern. The

Figure 5.1: TOT Persistence and TB-TOT Correlations



Source: Mendoza (1995), table 1.

Note: Each point corresponds to one country. The TOT serial correlation and the TB-TOT correlation are computed over the period 1955-1990. The sample includes the G-7 countries (United States, United Kingdom, France, Germany, Italy, Canada, and Japan), 6 countries from Latin America, (Argentina, Brazil, Chile, Mexico, Peru, Venezuela), 3 countries from the Middle East (Israel, Saudi Arabia, and Egypt), 6 countries from Asia (Taiwan, India, Indonesia, Korea, Philippines, and Thailand), and 8 countries from Africa (Algeria, Cameroon, Zaire, Kenya, Morocco, Nigeria, Sudan, and Tunisia). The solid line is the OLS fit and is given by $\text{corr}(TB, TOT) = 0.35 - 0.14\rho(TOT)$. The dashed line is the OLS fit after eliminating Argentina from the sample and is given by $\text{corr}(TB, TOT) = 0.23 + 0.12\rho(TOT)$. The dashed-dotted line is the OLS fit after eliminating the G7 countries, Saudi Arabia, and Argentina from the sample and is given by $\text{corr}(TB, TOT) = 0.28 + 0.03\rho(TOT)$.

OLS fit of the 30 points, shown with a solid line, displays a small negative slope of -0.14. The sign of the slope is indeed in line with the ORS effect: As the terms of trade shocks become more persistent, they should be expected to induce a smaller response in the trade balance. It is apparent in the graph, however, that the negative slope in the OLS regression is driven by a single observation, Argentina, the only country in the sample with a negative serial correlation of the terms of trade. Eliminating Argentina from the sample one obtains a positive OLS slope of 0.12.[1] The corresponding fitted relationship is shown with a broken line on figure 5.1.

A number of countries in figure 5.1 are likely to be large players in the world markets for the goods and services they import and/or export. Countries in this group would include all of the G7 nations, the largest economies in the world, and Saudi Arabia, a major oil exporter. For these countries, the terms of trade are not likely to be exogenous. Eliminating these 8 countries (as well as the outlier Argentina) from the sample, gives us a better idea of what the relation between the TB-TOT correlation and the TOT persistence looks for small emerging countries that take their terms of trade exogenously. The fitted line using this reduced sample has a negligible slope equal to 0.03 and is shown with a dash-dotted line in figure 5.1. We conclude that the observed relationship between the TB-TOT correlation and the persistence of TOT is close to nil.

Does this conclusion suggest that the empirical evidence presented here is against the Obstfeld-Razin-Svensson effect? Not necessarily. The ORS

[1]Indeed, Argentina is the only country whose elimination from the sample results in a positive slope.

effect requires isolating the effect of TOT shocks on the trade balance. The raw data is in principle driven by a multitude of shocks, of which the terms of trade is just one. Moreover, some of these shocks may directly affect both the trade balance and the terms of trade. Not controlling for these shocks may result in erroneously attributing part of their effect on the trade balance to the terms of trade. A case in point is given by world-interest-rate shocks. High world interest rates may be associated with depressed economic activity in developed and emerging economies alike. In turn, low levels of economic activity in the developed world are likely to be associated with a weak demand for primary commodities, and, as a result, with deteriorated terms of trade for the emerging countries producing those commodities. At the same time, high world interest rates are associated with contractions in aggregate demand and improvements in the trade balance in emerging countries. Under this scenario, the terms of trade and the trade balance are moving at the same time, but attributing all of the movement in the trade balance to changes in the terms of trade would be clearly misleading. As another example, suppose that domestic technology shocks are correlated with technology shocks in another country or set of countries. Suppose further that this other country or set of countries generates a substantial fraction of the demand for exports or the supply of imports of the country in question. In this case, judging the empirical validity of the ORS effect only on the grounds of raw correlations would be misplaced.

An important step in the process of isolating terms-of-trade shocks—or any kind of shock, for that matter—is identification. Data analysis based purely on statistical methods will in general not result in a successful identification of technology shocks. Economic theory must be used be at center stage in the identification process. The following exercise, which follows Mendoza (1995), represents an early step in the task of identifying the effects of terms-of-trade shocks on economic activity in emerging economies.

5.3 Terms-of-Trade Shocks in an RBC Model

Consider expanding the real-business-cycle model of chapter 4 to allow for terms-of-trade shocks. In doing this, we follow the work of Mendoza (1995). The household block of the model is identical to that of the standard RBC model studied in chapter 4. The main difference with the model of chapter 4 is that the model studied here features three sectors: a sector producing importable goods, a sector producing exportable goods, and a sector producing nontradable goods. An importable good is either an imported good or a good that is produced domestically but is highly substitutable with a good that is imported. Similarly, an exportable good is either an exported good or a good that is sold domestically but is highly substitutable with a good that is exported. A nontradable good is a good that is neither exportable nor importable.

5.3.1 Households

This block of the model is identical to that of the RBC model studied in chapter 4. The economy is populated by a large number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t), \tag{5.1}$$

where c_t denotes consumption, h_t denotes labor effort, and U is a period utility function taking the form

$$U(c, h) = \frac{\left[c\,(1-h)^{\omega}\right]^{1-\gamma}}{1-\gamma}.$$

The variable θ_t/θ_{t-1} is a time-varying discount factor and is assumed to evolve according to the following familiar law of motion:

$$\theta_{t+1} = \theta_t \beta(c_t, h_t), \tag{5.2}$$

where the function β is assumed to take the form

$$\beta(c, h) = [1 + c(1-h)^{\omega}]^{-\beta}.$$

We established in chapter 4 that the endogeneity of the discount factor serves the purpose of rendering the deterministic steady state independent of the country's initial net foreign asset position.

Households offer labor services for a wage w_t and own the stock of capital, k_t, which they rent at the rate u_t. The stock of capital evolves according to the following law of motion:

$$k_{t+1} = (1-\delta)k_t + i_t - \Phi(k_{t+1} - k_t), \tag{5.3}$$

where i_t denotes gross investment, which is assumed to be an importable good. The parameter $\delta \in [0, 1]$ denotes the capital depreciation rate. The function Φ introduces capital adjustment costs, and is assumed to satisfy $\Phi(0) = \Phi'(0) = 0$ and $\Phi'' > 0$. Under these assumptions, the steady-state level of capital is not affected by the presence of adjustment costs. As discussed in chapter 4, capital adjustment costs help curb the volatility of investment in small open economy models like the one studied here.

Households are assumed to be able to borrow or lend freely in international financial markets by buying or issuing risk-free bonds denominated in units of importable goods and paying the constant interest rate r^*. Letting d_t denote the debt position assumed by the household in period t and p_t^c denote the price of the consumption good, the period budget constraint of the household can be written as

$$d_t = (1 + r^*)d_{t-1} + p_t^c c_t + i_t - w_t h_t - u_t k_t. \tag{5.4}$$

The relative prices p_t^c, w_t, and u_t are expressed in terms of importable goods, which serve the role of numeraire. Households are subject to a no-Ponzi-game constraint of the form

$$\lim_{j\to\infty} \frac{E_t d_{t+j}}{(1 + r^*)^j} \leq 0. \tag{5.5}$$

The household seeks to maximize the utility function (5.1) subject to (5.2)-(5.5). Letting $\theta_t \eta_t$ and $\theta_t \lambda_t$ denote the Lagrange multipliers on (5.2) and (5.4), the first-order conditions of the household's maximization problem are (5.2), (5.4), (5.5) holding with equality, and

$$U_c(c_t, h_t) - \eta_t \beta_c(c_t, h_t) = \lambda_t p_t^c \tag{5.6}$$

$$-U_h(c_t, h_t) + \eta_t \beta_h(c_t, h_t) = \lambda_t w_t \tag{5.7}$$

$$\lambda_t = \beta(c_t, h_t)(1 + r_t) E_t \lambda_{t+1} \tag{5.8}$$

$$\lambda_t[1 + \Phi'(k_{t+1} - k_t)] = \beta(c_t, h_t) E_t \lambda_{t+1} \left[u_{t+1} + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{5.9}$$

$$\eta_t = -E_t U(c_{t+1}, h_{t+1}) + E_t \eta_{t+1} \beta(c_{t+1}, h_{t+1}) \tag{5.10}$$

5.3.2 Production of Consumption Goods

The consumption good, c_t, is produced by domestic firms. These firms operate a CES production function that takes tradable consumption goods, c_t^T, and nontradable consumption goods, c_t^N, as inputs. Formally,

$$c_t = [\chi (c_t^T)^{-\mu} + (1 - \chi)(c_t^N)^{-\mu}]^{-1/\mu}, \tag{5.11}$$

with $\mu > -1$. Firms operate in perfectly competitive product and input markets. They choose output and inputs to maximize profits, which are given by

$$p_t^c c_t - p_t^T c_t^T - p_t^N c_t^N,$$

where p_t^T and p_t^N denote, respectively, the relative prices of tradable and nontradable consumption goods in terms of importable goods. The first-order conditions associated with this profit-maximization problem are (5.11) and

$$\frac{c_t^N}{c_t^T} = \left(\frac{1-\chi}{\chi}\right)^{\frac{1}{1+\mu}} \left(\frac{p_t^T}{p_t^N}\right)^{\frac{1}{1+\mu}}, \tag{5.12}$$

$$\frac{c_t}{c_t^T} = \left(\frac{1}{\chi}\right)^{\frac{1}{1+\mu}} \left(\frac{p_t^T}{p_t^c}\right)^{\frac{1}{1+\mu}}. \tag{5.13}$$

It is clear from the first of these efficiency conditions that the elasticity of substitution between tradable and nontradable goods is given by $1/(1+\mu)$. From the second optimality condition, one observes that if the elasticity of substitution between tradables and nontradables is less than unity (or $\mu > 0$), then the share of tradables in total consumption, given by $p_t^T c_t^T/(p_t^c c_t)$, increases as the relative price of tradables in terms of consumption, p_t^T/p_t^c, increases.

5.3.3 Production of Tradable Consumption Goods

Tradable consumption goods, denoted c_t^T, are produced using importable consumption goods, c_t^M, and exportable consumption goods, c_t^X, via a Cobb-Douglas production function. Formally,

$$c_t^T = (c_t^X)^\alpha (c_t^M)^{1-\alpha}, \tag{5.14}$$

where $\alpha \in (0,1)$ is a parameter. Firms are competitive and aim at maximizing profits, which are given by

$$p_t^T c_t^T - p_t^X c_t^X - c_t^M,$$

where p_t^X denotes the relative price of exportable goods in terms of importable goods, or the terms of trade. Note that because the importable good plays the role of numeraire, we have that the relative price of importables in terms of the numeraire is always unity ($p_t^M = 1$). The optimality conditions associated with this problem are (5.14) and:

$$\frac{p_t^X c_t^X}{p_t^T c_t^T} = \alpha \tag{5.15}$$

$$\frac{c_t^M}{p_t^T c_t^T} = 1 - \alpha. \tag{5.16}$$

These optimality conditions state that the shares of consumption of exportables and importables in total consumption expenditure in tradable goods are constant and equal to α and $1-\alpha$, respectively. This implication is a consequence of the assumption of Cobb-Douglas technology in the production of tradable consumption.

5.3.4 Production of Importable, Exportable, and Nontradable Goods

Exportable and importable goods are produced with capital as the only input, whereas nontradable goods are produced using labor services only. Formally, the three production technologies are given by

$$y_t^X = A_t^X (k_t^X)^{\alpha_X}, \tag{5.17}$$

$$y_t^M = A_t^M (k_t^M)^{\alpha_M}, \tag{5.18}$$

and

$$y_t^N = A_t^N (h_t^N)^{\alpha_N}, \tag{5.19}$$

where y_t^X denotes output of exportable goods, y_t^M denotes output of importable goods, and y_t^N denotes output of nontradable goods. The factors A_t^i denote exogenous and stochastic technology shocks in sectors $i = X, M, N$. The variable k_t^i denotes the capital stock in sector $i = X, M$, and the variable h_t^N denotes labor services employed in the nontradable sector. Firms demand input quantities to maximize profits, which are given by

$$p_t^X y_t^X + y_t^M + p_t^N y_t^N - w_t h_t^N - u_t(k_t^X + k_t^M).$$

The optimality conditions associated with this problem are

$$\frac{u_t k_t^X}{p_t^X y_t^X} = \alpha_X, \tag{5.20}$$

$$\frac{u_t k_t^M}{y_t^M} = \alpha_M, \tag{5.21}$$

and

$$\frac{w_t h_t^N}{p_t^N y_t^N} = \alpha_N. \tag{5.22}$$

According to these expressions, and as a consequence of the assumption of Cobb-Douglas technologies, input shares are constant.

5.3.5 Market Clearing

In equilibrium, the markets for capital, labor, and nontradables must clear. That is,

$$k_t = k_t^X + k_t^M, \tag{5.23}$$

$$h_t = h_t^N, \tag{5.24}$$

and

$$c_t^N = y_t^N. \tag{5.25}$$

Also, in equilibrium the evolution of the net foreign debt position of the economy is given by

$$d_t = (1 + r^*)d_{t-1} - p_t^X (y_t^X - c_t^X) - y_t^M + c_t^M + i_t. \tag{5.26}$$

5.3.6 Driving Forces

There are four sources of uncertainty in this economy: One productivity shock in each of the three sectors (importable, exportable, and nontradable), and the terms of trade. We assume that all shocks follow autoregressive processes of order one. Mendoza (1995) imposes four restrictions on the joint distribution of the exogenous shocks: (1) all four shocks share the same persistence. (2) The sectorial productivity shocks are assumed to be perfectly correlated. (3) The technology shocks affecting the production of importables and exportables are assumed to be identical. (4) Innovations to productivity shocks and terms-of-trade shocks are allowed to be correlated.

These assumptions give rise to the following laws of motion:

$$\ln p_t^X = \rho \ln p_{t-1}^X + \epsilon_t^p; \quad \epsilon_t^p \sim \mathcal{N}(0, \sigma_{\epsilon^p}^2).$$

$$\ln A_t^X = \rho \ln A_{t-1}^X + \epsilon_t^T.$$

$$\ln A_t^M = \rho \ln A_{t-1}^M + \epsilon_t^T.$$

$$\ln A_t^N = \rho \ln A_{t-1}^N + \epsilon_t^N.$$

$$\epsilon_t^T = \psi_T \epsilon_t^p + \nu_t^T; \quad \nu_t^T \sim \mathcal{N}(0, \sigma_{\nu^T}^2),\ E(\epsilon_t^p, \nu_t^T) = 0.$$

$$\epsilon_t^N = \psi_N \epsilon_t^T.$$

We are now ready to define a competitive equilibrium.

5.3.7 Competitive Equilibrium

A stationary competitive equilibrium is a set of stationary processes $\{c_t$, c_t^T, c_t^X, c_t^M, c_t^N, h_t, h_t^N, y_t^X, y_t^M, y_t^N, k_t, k_t^X, k_t^M, i_t, d_t, p_t^c, p_t^N, p_t^T, w_t, u_t, η_t, $\lambda_t\}_{t=0}^{\infty}$ satisfying equations (5.3) and (5.6)-(5.26), given the initial conditions k_0 and d_{-1} and the exogenous processes $\{A_t^X$, A_t^M, A_t^N, $p_t^X\}_{t=0}^{\infty}$.

5.3.8 Calibration

Mendoza (1995) presents two calibrations of the model, one matching key macroeconomic relations in developed countries, and the other matching key macroeconomic relations in developing countries.

In calibrating the driving forces of the developed-country version of the model, the parameters ρ and σ_{ϵ^p} are set to match the average serial correlation and standard deviation of the terms of trade for the group of G7 countries. Using the information presented in table 1 of Mendoza (1995) yields

$$\rho = 0.473,$$

and

$$\sigma_{\epsilon^p} = 0.047\sqrt{1-\rho^2}.$$

Using estimates of productivity shocks in five industrialized countries by Stockman and Tesar (1995), Mendoza (1995) sets the volatility of productivity shocks in the importable and exportable sectors at 0.019 and the volatility of the productivity shock in the nontraded sector at 0.014. This implies that

$$\sqrt{\psi_T^2\sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2} = 0.019\sqrt{1-\rho^2},$$

and

$$\psi_N\sqrt{\psi_T^2\sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2} = 0.014\sqrt{1-\rho^2}.$$

Based on correlations between Solow residuals and terms of trade in five developed countries, Mendoza (1995) sets the correlation between the productivity shock in the exportable sector and the terms of trade at 0.165. This implies that

$$\frac{\psi_T\sigma_{\epsilon^p}}{\sqrt{\psi_T^2\sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2}} = 0.165.$$

Table 5.2 displays the parameter values implied by the above restrictions. This completes the calibration of the parameters defining exogenous driving forces in the developed-country model.

Table 5.2: Calibration

Parameter	Developed Country	Developing Country
σ_{ϵ^p}	0.041	0.011
σ_{ν^T}	0.017	0.032
ρ	0.47	0.41
ψ_T	0.067	-0.156
ψ_N	0.74	0.74
r^*	0.04	0.04
α_X	0.49	0.57
α_M	0.27	0.70
α_N	0.56	0.34
δ	0.1	0.1
ϕ	0.028	0.028
γ	1.5	2.61
μ	0.35	-0.22
α	0.3	0.15
ω	2.08	0.79
β	0.009	0.009

In calibrating the driving forces of the developing-country model, the parameter ρ and σ_p are picked to match the average serial correlation and standard deviation of the terms of trade for the group of developing countries reported in table 1 of Mendoza (1995). This implies:

$$\rho = 0.414,$$

and

$$\sigma_{\epsilon^p} = 0.12\sqrt{1-\rho^2}.$$

Mendoza (1995) assumes that the standard deviation of productivity shocks in the traded sector are larger than in the nontraded sector by the same proportion as in developed countries. This means that the parameter ψ_N takes the value 0.74 as in the developed-country model. Mendoza sets the standard deviation of productivity shocks in the traded sectors at 0.04 and their correlation with the terms of trade at -0.46 to match the observed average standard deviation of GDP and the correlation of GDP with TOT in developing countries. This yields the restrictions:

$$\sqrt{\psi_T^2\sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2} = 0.04\sqrt{1-\rho^2},$$

and

$$\frac{\psi_T\sigma_{\epsilon^p}}{\sqrt{\psi_T^2\sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2}} = -0.46.$$

The implied parameter values are shown in table 5.2. This completes the calibration of the exogenous driving forces for the developing-country version

Table 5.3: Data and Model Predictions

Variable	$\frac{\sigma_x}{\sigma_{TOT}}$		$\rho_{x_t,x_{t-1}}$		$\rho_{x,GDP}$		$\rho_{x,TOT}$	
	G7	DCs	G7	DCs	G7	DCs	G7	DCs
TOT								
Data	1	1	.47	.41	.78	.25	1	1
Model	1	1	.47	.41	.78	.32	1	1
TB								
Data	1.62	1.60	.33	.38	.18	-.17	.34	.32
Model	3.5	.86	.37	.69	-.11	-.45	.19	.08
GDP								
Data	1.69	1.30	.49	.49	1	1	.78	.25
Model	.86	.47	.68	.82	1	1	.78	.32
C								
Data	1.59	1.27	.44	.42	.96	.89	.74	.18
Model	1.01	1.32	.85	.99	.81	.75	.39	.03
I								
Data	1.90	1.62	.51	.49	.84	.72	.66	.26
Model	2.0	.94	.14	.11	.67	.39	.70	.28
RER								
Data	1.44	1.23	.38	.43	.58	.52	.70	.12
Model	.57	.60	.79	.95	.80	.71	.57	.25

Source: Mendoza (1995).

of the model. Table 5.2 also displays the values assigned to the remaining parameters of the model.[2]

5.3.9 Model Performance

Table 5.3 presents a number of data summary statistics from developed (G7) and developing countries (DCs) and their theoretical counterparts. The following list highlights a number of empirical regularities and comments on the model's ability to capture them.

[2]See Mendoza, 1995 for more details.

1. In the data, the terms of trade are procyclical, although much less so in developing countries than in G7 countries. The model captures this fact relatively well.

2. The observed terms of trade are somewhat persistent. This fact is matched by construction; recall that the parameter ρ is set to pin down the serial correlation of the terms of trade in developing and G7 countries.

3. The terms of trade are less volatile than GDP. The model fails to capture this fact.

4. The terms of trade are positively correlated with the trade balance. The model captures this empirical regularity, but underestimates the TB-TOT correlation, particularly for developing countries.

5. The trade balance is countercyclical in DCs but procyclical in G7 countries. In the model, the trade balance is countercyclical in both, developed and developing countries. The failure of the model to capture the procyclicality of the trade balance in developed countries should be taken with caution, for other authors estimate negative TB-GDP correlations for developed countries. The model appears to overestimate the countercyclicality of the trade balance.

6. In the data, the real exchange rate (RER) is measured as the ratio of the domestic CPI to an exchange-rate-adjusted, trade-weighted average of foreign CPIs. In the model, the RER is defined as the relative price of consumption in terms of importables and denoted by p^c_t. In the

data, the RER is procyclical. The model captures this fact, although it overestimates somewhat the RER-GDP correlation.

7. The RER is somewhat persistent (with a serial correlation of less than 0.45 for both developing and G7 countries). In the model, the RER is highly persistent, with an autocorrelation above 0.75 for both types of country.

5.3.10 How Important Are the Terms of Trade?

To assess the contribution of the terms of trade to explaining business cycles in developed and developing countries, one can run the counterfactual experiment of computing equilibrium dynamics after shutting off all sources of uncertainty other than the terms of trade themselves. In the context of the model of this section, one must set all productivity shocks at their deterministic steady-state values. This is accomplished by setting $\sigma_{\nu^T} = \psi_T = 0$.

Mendoza (1995) finds that when the volatility of all productivity shocks is set equal to zero in the developed-country version of the model, the volatility of output deviations from trend measured at import prices falls from 4.1 percent to 3.6 percent. Therefore, in the model the terms of trade explain about 88 percent of the volatility of output. When output is measured in terms of domestic prices, the terms of trade explain about 66 percent of output movements.

When the same experiment is performed in the context of the developing-country version of the model, shutting off the variance of the productivity shocks results in an increase in the volatility of output. The reason for this increase in output volatility is that in the benchmark calibration the terms of trade are negatively correlated with productivity shocks—note in table 5.2 that $\psi_T < 0$ for developing countries. Taking this result literally would lead to the illogical conclusion that terms of trade explain more than 100 percent of output fluctuations in the developing-country model. What is wrong?

One difficulty with the way we have measured the contribution of the terms of trade is that it is not based on a variance decomposition of output. A more satisfactory way to assess the importance of terms-of-trade and productivity shocks would be to define the terms of trade shock as ϵ_t^p and the productivity shock as ν_t^T. One justification for this classification is that ϵ_t^p affects both the terms of trade and sectoral total factor productivities, while ν_t^T affects sectoral total factor productivities but not the terms of trade. Under this definition of shocks, the model with only terms of trade shocks results when σ_{ν^T} is set equal to zero. Note that the parameter ψ_T must not be set to zero. An advantage of this approach is that, because the variance of output can be decomposed into a nonnegative fraction explained by ϵ_t^p and a nonnegative part explained by ν_t^T, the contribution of the terms of trade will always be a nonnegative number no larger than 100 percent.

Exercise 5.1 *Using the model presented in this section, compute a variance decomposition of output. What fraction of output is explained by terms-of-trade shocks in the developed- and developing-country versions of the model?*

Chapter 6

Interest-Rate Shocks

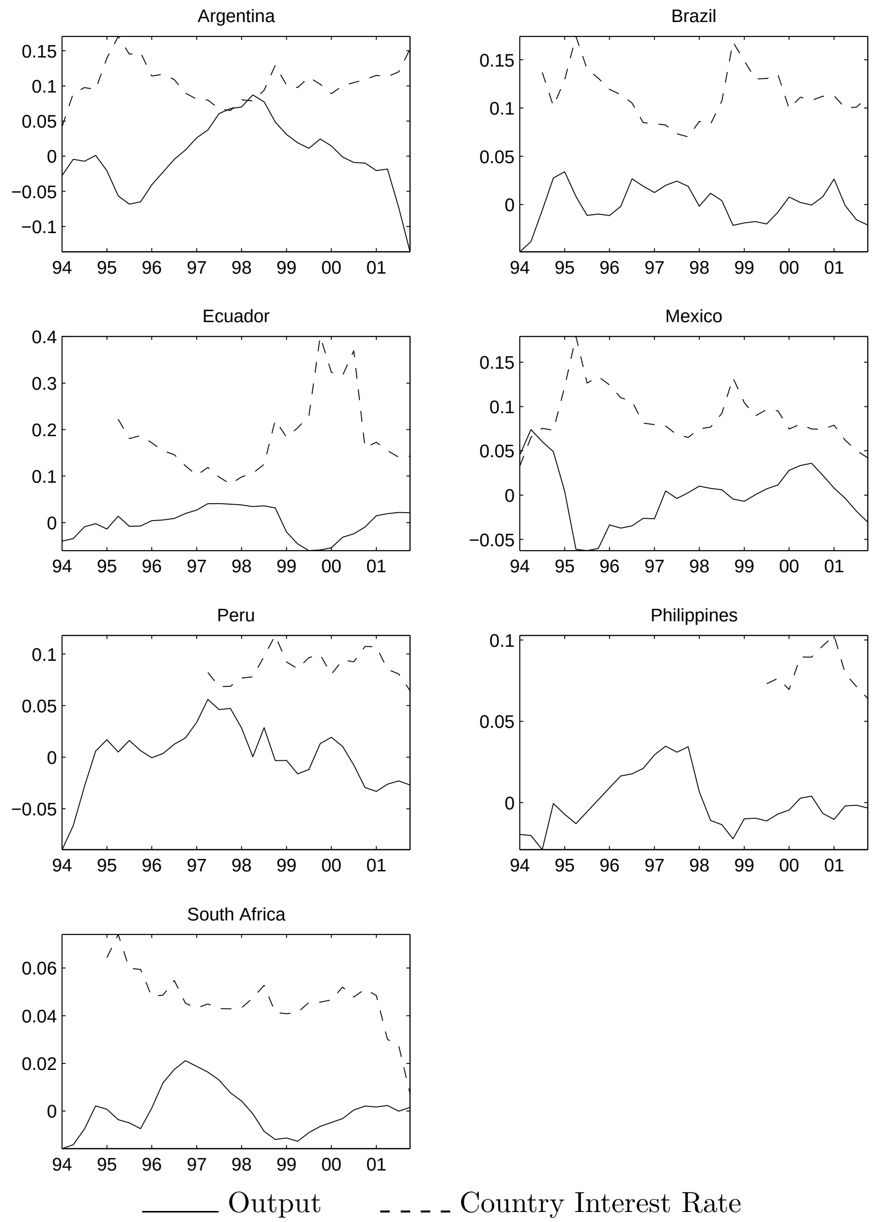
Business cycles in emerging market economies are correlated with the interest rate that these countries face in international financial markets. This observation is illustrated in figure 6.1, which depicts detrended output and the country interest rate for seven developing economies between 1994 and 2001. Periods of low interest rates are typically associated with economic expansions and times of high interest rates are often characterized by depressed levels of aggregate activity.[1]

Data like those shown in figure 6.1 have motivated researches to ask what fraction of observed business cycle fluctuations in emerging markets is due to movements in country interest rate. This question is complicated by the fact that the country interest rate is unlikely to be completely exogenous to the country's domestic conditions.[2] To clarify ideas, let R_t denote the gross

[1]The estimated correlations (p-values) are: Argentina -0.67 (0.00), Brazil -0.51 (0.00), Ecuador -0.80 (0.00), Mexico -0.58 (0.00), Peru -0.37 (0.12), the Philippines -0.02 (0.95), South Africa -0.07 (0.71).

[2]There is a large literature arguing that domestic variables affect the interest rate at which emerging markets borrow externally. See, for example, Edwards (1984), Cline

Figure 6.1: Country Interest Rates and Output in Seven Emerging Countries



Note: Output is seasonally adjusted and detrended using a log-linear trend. Country interest rates are real yields on dollar-denominated bonds of emerging countries issued in international financial markets. Data source: output, IFS; interest rates, EMBI+.

Source: Uribe and Yue (2006).

interest rate at which the country borrows in international markets, or the country interest rate. This interest rate can be expressed as $R_t = R_t^{us} S_t$. Here, R^{us} denotes the world interest rate, or the interest rate at which developed countries, like the U.S., borrow and lend from one another, and S_t denotes the gross country interest-rate spread, or country interest-rate premium. Because the interest-rate premium is country specific, in the data we find an Argentine spread, a Colombian spread, etc. If the country in question is a small player in international financial markets, as many emerging economies are, it is reasonable to assume that the world interest rate R_t^{us}, is completely exogenous to the emerging country's domestic conditions. We can't say the same, however, about the country spread S_t. An increase in output, for instance, may induce foreign lenders to lower spreads on believes that the country's ability to repay its debts has improved.

Interpreting the country interest rate as an exogenous variable when in reality it has an endogenous component is likely to result in an overstatement of the importance of interest rates in explaining business cycles. To see why, consider the following example. Suppose that the interest rate R_t is purely endogenous. Thus, its contribution to generating business cycles is nil. Assume, furthermore, that R_t is countercyclical, i.e., foreign lenders reduce the country spread in response to expansions in aggregate activity. The researcher, however, wrongly assumes that the interest rate is purely exogenous. Suppose now that a domestic productivity shock induces an expansion in output. In response to this output increase, the interest rate falls. The researcher, who believes R_t is exogenous, erroneously attributes

(1995), and Cline and Barnes (1997).

part of the increase in output to the decline in R_t. The right conclusion, of course, is that all of the movement in output is due to the productivity shock.

It follows that in order to quantify the macroeconomic effects of interest rate shocks, the first step is to identify the exogenous components of country spreads and world interest rate shocks. Necessarily, the identification process must combine statistical methods and economic theory. The particular combination adopted in this chapter draws heavily from Uribe and Yue (2006).

6.1 An Empirical Model

Our empirical model takes the form of a first-order VAR system:

$$A \begin{bmatrix} \hat{y}_t \\ \hat{\imath}_t \\ tby_t \\ \hat{R}^{us}_t \\ \hat{R}_t \end{bmatrix} = B \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\imath}_{t-1} \\ tby_{t-1} \\ \hat{R}^{us}_{t-1} \\ \hat{R}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon^y_t \\ \epsilon^i_t \\ \epsilon^{tby}_t \\ \epsilon^{rus}_t \\ \epsilon^r_t \end{bmatrix} \tag{6.1}$$

where y_t denotes real gross domestic output, i_t denotes real gross domestic investment, tby_t denotes the trade balance to output ratio, R^{us}_t denotes the gross real US interest rate, and R_t denotes the gross real (emerging) country interest rate. A hat on y_t and i_t denotes log deviations from a log-linear trend. A hat on R^{us}_t and R_t denotes simply the log. We measure R^{us}_t as the 3-month gross Treasury bill rate divided by the average gross US inflation over the past four quarters.[3] We measure R_t as the sum of J. P. Morgan's EMBI+ stripped spread and the US real interest rate. Output, investment, and the trade balance are seasonally adjusted.

To identify the shocks in the empirical model, Uribe and Yue (2006) impose the restriction that the matrix A be lower triangular with unit diagonal elements. Because R_t^{us} and R_t appear at the bottom of the system, this identification strategy presupposes that innovations in world interest rates (ϵ_t^{rus}) and innovations in country interest rates (ϵ_t^r) percolate into domestic real variables with a one-period lag. At the same time, the identification scheme implies that real domestic shocks (ϵ_t^y, ϵ_t^i, and ϵ_t^{tby}) affect financial markets contemporaneously. This identification strategy is a natural one, for, conceivably, decisions such as employment and spending on durable consumption goods and investment goods take time to plan and implement. Also, it seems reasonable to assume that financial markets are able to react quickly to news about the state of the business cycle.[4]

An additional restriction imposed on the VAR system, is that the world interest rate R_t^{us} follows a simple univariate $AR(1)$ process (i.e., $A_{4i} = B_{4i} = 0$, for all $i \neq 4$). Uribe and Yue (2006) adopt this restriction primarily because it is reasonable to assume that disturbances in a particular (small) emerging country will not affect the real interest rate of a large country like the United States.

The country-interest-rate shock, ϵ_t^r, can equivalently be interpreted as a

[3]Using a more forward looking measure of inflation expectations to compute the US real interest rate does not significantly alter our main results.

[4]Uribe and Yue (2006), discuss an alternative identification strategy consisting in placing financial variables 'above' real variables first in the VAR system.

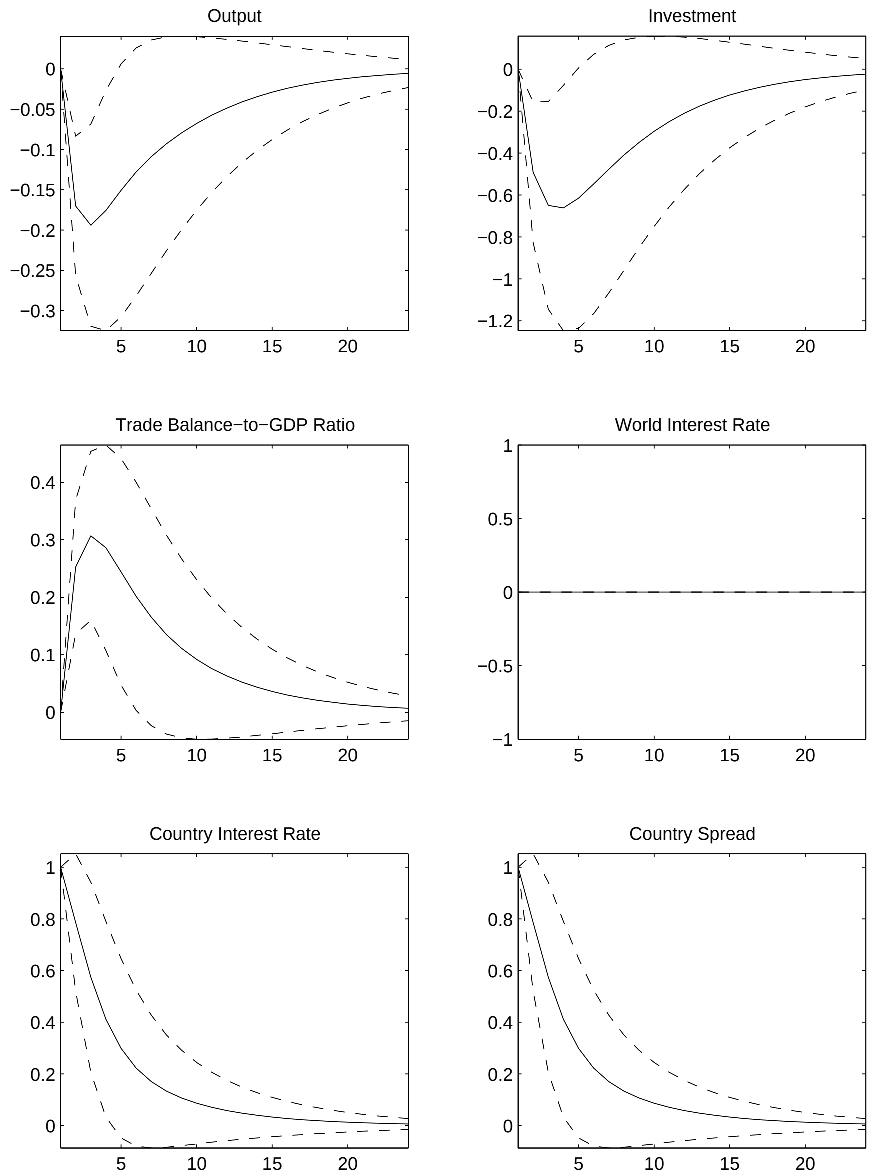
country spread shock. To see this, consider substituting in equation (6.1) the country interest rate $\hat{R}_t$ using the definition of country spread, $\hat{S}_t \equiv \hat{R}_t - \hat{R}^{us}_t$. Clearly, because R^{us}_t appears as a regressor in the bottom equation of the VAR system, the estimated residual of the newly defined bottom equation, call it ϵ^s_t, is identical to ϵ^r_t. Moreover, it is obvious that the impulse response functions of $\hat{y}_t$, $\hat{\imath}_t$, and tby_t associated with ϵ^s_t are identical to those associated with ϵ^r_t. Therefore, throughout the paper we indistinctly refer to ϵ^r_t as a country interest rate shock or as a country spread shock.

After estimating the VAR system (6.1), Uribe and Yue use it to address a number of questions central to disentangle the effects of country-spread shocks and world-interest-rate shocks on aggregate activity in emerging markets: First, how do US-interest-rate shocks and country-spread shocks affect real domestic variables such as output, investment, and the trade balance? Second, how do country spreads respond to innovations in US interest rates? Third, how and by how much do country spreads move in response to innovations in emerging-country fundamentals? Fourth, how important are US-interest-rate shocks and country-spread shocks in explaining movements in aggregate activity in emerging countries? Fifth, how important are US-interest-rate shocks and country-spread shocks in accounting for movements in country spreads? We answer these questions with the help of impulse response functions and variance decompositions.

6.2 Impulse Response Functions

Figure 6.2 displays with solid lines the impulse response function implied

Figure 6.2: Impulse Response To Country-Spread Shock



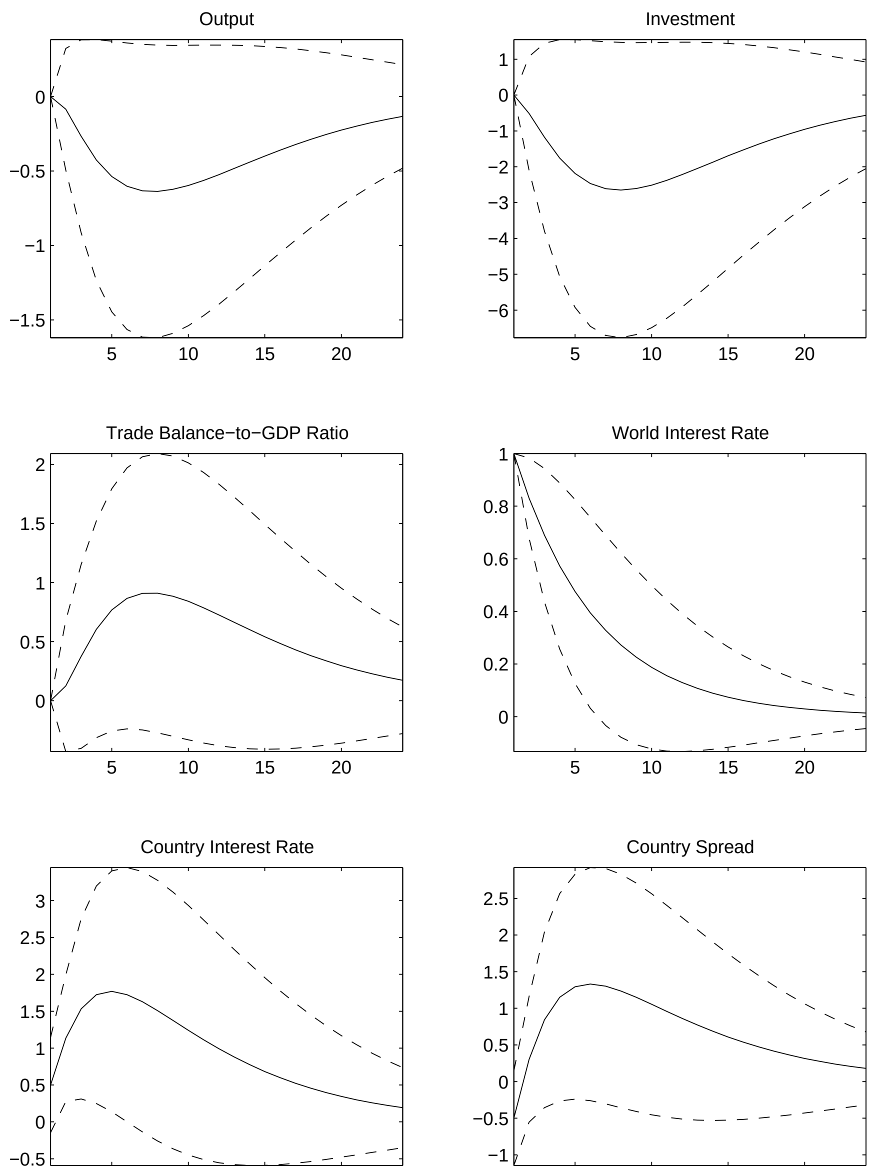
Notes: (1) Solid lines depict point estimates of impulse responses, and broken lines depict two-standard-deviation error bands. (2) The responses of Output and Investment are expressed in percent deviations from their respective log-linear trends. The responses of the Trade Balance-to-GDP ratio, the country interest rate, the US interest rate, and the country spread are expressed in percentage points. The two-standard-error bands are computed using the delta method.

by the VAR system (6.1) to a unit innovation in the country spread shock, ϵ_t^r. Broken lines depict two-standard-deviation bands.[5] In response to an unanticipated country-spread shock, the country spread itself increases and then quickly falls toward its steady-state level. The half life of the country spread response is about one year. Output, investment, and the trade balance-to-output ratio respond as one would expect. They are unchanged in the period of impact, because of our maintained assumption that external financial shocks take one quarter to affect production and absorption. In the two periods following the country-spread shock, output and investment fall, and subsequently recover gradually until they reach their pre-shock level. The adverse spread shock produces a larger contraction in aggregate domestic absorption than in aggregate output. This is reflected in the fact that the trade balance improves in the two periods following the shock.

Figure 6.3 displays the response of the variables included in the VAR system (6.1) to a one percentage point increase in the US interest rate shock, ϵ_t^{rus}. The effects of US interest-rate shocks on domestic variables and country spreads are measured with significant uncertainty, as indicated by the width of the 2-standard-deviation error bands. The point estimates of the impulse response functions of output, investment, and the trade balance, however, are qualitatively similar to those associated with an innovation in the country spread. That is, aggregate activity and gross domestic investment contract, while net exports improve. However, the quantitative effects of an innovation in the US interest rate are much more pronounced than those caused by a country-spread disturbance of equal magnitude. For

[5] These bands are computed using the delta method.

Figure 6.3: Impulse Response To A US-Interest-Rate Shock



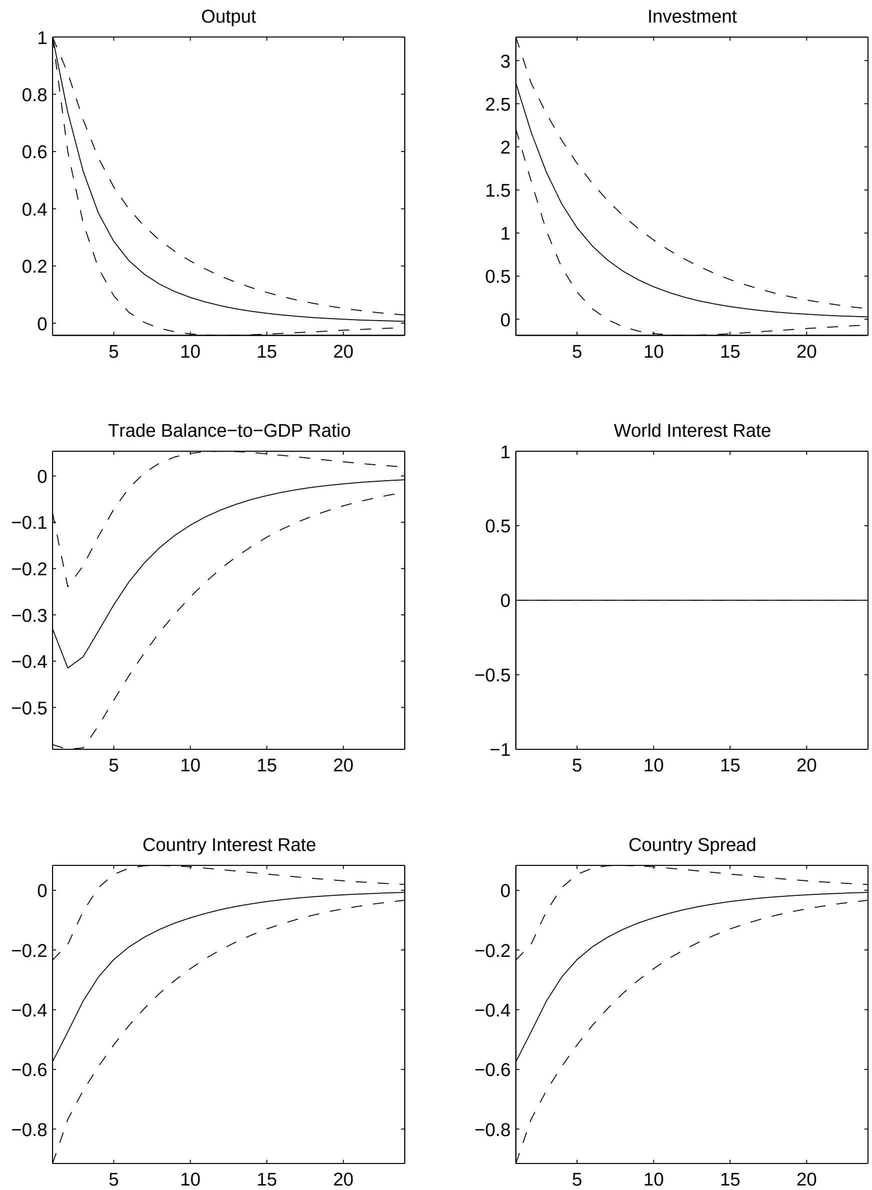
Notes: (1) Solid lines depict point estimates of impulse responses, and broken lines depict two-standard-deviation error bands. (2) The responses of Output and Investment are expressed in percent deviations from their respective log-linear trends. The responses of the Trade Balance-to-GDP ratio, the country interest rate, and the US interest rate are expressed in percentage points.

instance, the trough in the output response is twice as large under a US-interest-rate shock than under a country-spread shock.

It is remarkable that the impulse response function of the country spread to a US-interest-rate shock displays a delayed overshooting. In effect, in the period of impact the country interest rate increases but by less than the jump in the US interest rate. As a result, the country spread initially falls. However, the country spread recovers quickly and after a couple of quarters it is more than one percentage point above its pre-shock level. Thus, country spreads increase significantly in response to innovations in the US interest rate but with a short delay. The negative impact effect is in line with the findings of Eichengreen and Mody (1998) and Kamin and Kleist (1999). We note, however, that because the models estimated by these authors are static in nature, by construction, they are unable to capture the rich dynamic relation linking these two variables. The overshooting of country spreads is responsible for the much larger response of domestic variables to an innovation in the US interest rate than to an innovation in the country spread of equal magnitude.

We now ask how innovations in output, ϵ_t^y, impinge upon the variables of our empirical model. The model is vague about the precise nature of output shocks. They can reflect variations in total factor productivity, the terms-of-trade, etc. Figure 6.4 depicts the impulse response function to a one-percent increase in the output shock. The response of output, investment, and the trade balance is very much in line with the impulse response to a positive productivity shock implied by the small open economy RBC model (see figure 4.1). The response of investment is about three times as large as that

Figure 6.4: Impulse Response To An Output Shock



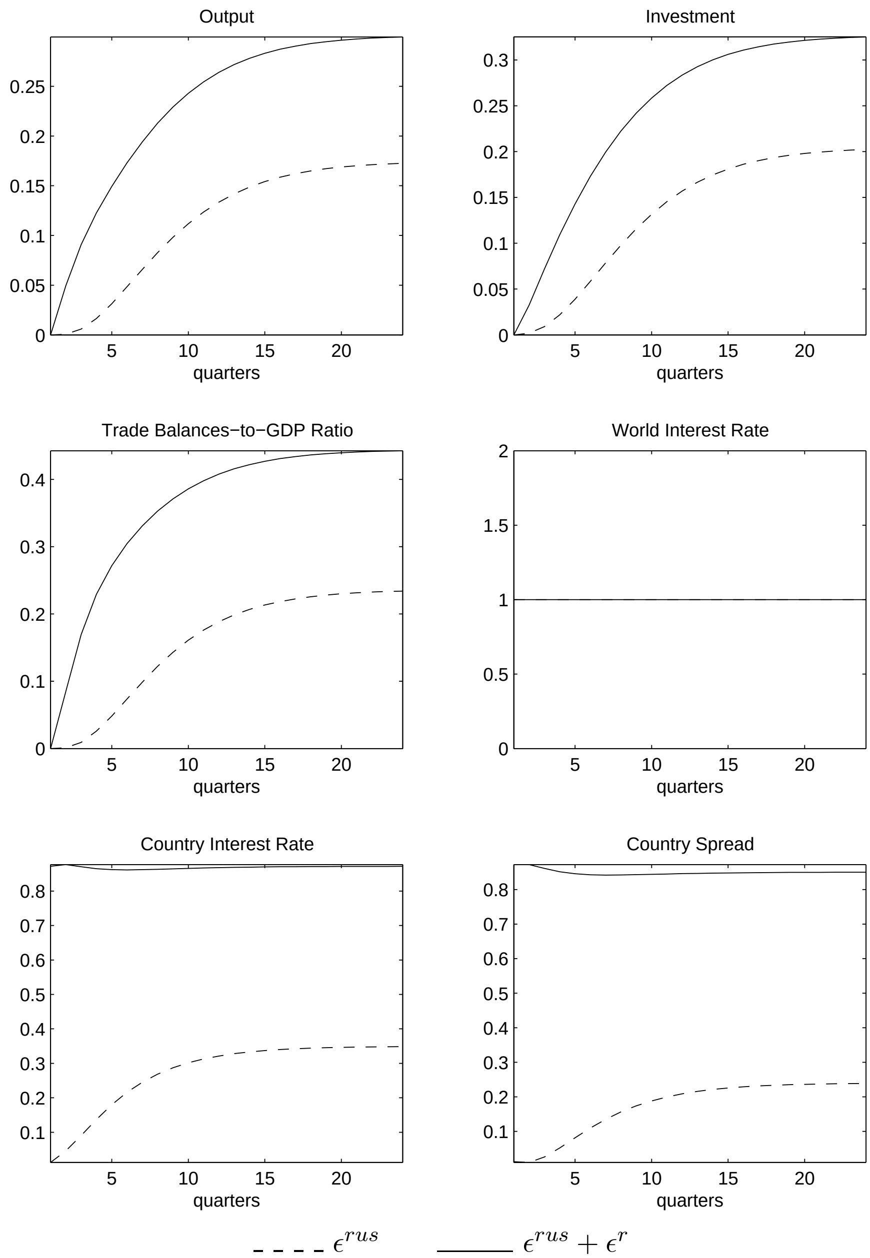
Notes: (1) Solid lines depict point estimates of impulse response functions, and broken lines depict two-standard-deviation error bands. (2) The responses of Output and Investment are expressed in percent deviations from their respective log-linear trends. The responses of the Trade Balance-to-GDP ratio, the country interest rate, and the US interest rate are expressed in percentage points.

of output. At the same time, the trade balance deteriorates significantly by about 0.4 percent and after two quarters starts to improve, converging gradually to its steady-state level. More interestingly, the increase in output produces a significant reduction in the country spread of about 0.6 percent. The half life of the country spread response is about five quarters. The countercyclical behavior of the country spread in response to output shocks suggests that country interest rates behave in ways that exacerbates the business-cycle effects of output shocks.

6.3 Variance Decompositions

Figure 6.5 displays the variance decomposition of the variables contained in the VAR system (6.1) at different horizons. Solid lines show the fraction of the variance of the forecasting error explained jointly by US-interest-rate shocks and country-spread shocks (ϵ_t^{rus} and ϵ_t^r). Broken lines depict the fraction of the variance of the forecasting error explained by US-interest-rate shocks (ϵ_t^{rus}). Because ϵ_t^{rus} and ϵ_t^r are orthogonal disturbances, the vertical difference between the solid line and the broken line represents the variance of the forecasting error explained by country-spread shocks at dif-

Figure 6.5: Variance Decomposition at Different Horizons



Note: Solid lines depict the fraction of the variance of the k-quarter-ahead forecasting error explained jointly by ϵ_t^{rus} and ϵ_t^{r} at different horizons. Broken lines depict the fraction of the variance of the forecasting error explained by ϵ_t^{rus} at different horizons.

ferent horizons.[6,7] Note that as the forecasting horizon approaches infinity, the decomposition of the variance of the forecasting error coincides with the decomposition of the unconditional variance of the series in question.

For the purpose of the present discussion, we associate business-cycle fluctuations with the variance of the forecasting error at a horizon of about five years. Researchers typically define business cycles as movements in time series of frequencies ranging from 6 quarters to 32 quarters (Stock and Watson, 1999). Our choice of horizon falls in the middle of this window.

According to our estimate of the VAR system given in equation (6.1), innovations in the US interest rate, ϵ_t^{rus}, explain about 20 percent of movements in aggregate activity in emerging countries at business cycle frequency. At the same time, country-spread shocks, ϵ_t^r, account for about 12 percent of

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[6]These forecasting errors are computed as follows. Let $x_t \equiv [\hat{y}_t\ \hat{\imath}_t\ tby_t\ \hat{R}_t^{us}\ \hat{R}_t]$ be the vector of variables included in the VAR system and $\epsilon_t \equiv [\epsilon_t^y\ \epsilon_t^i\ \epsilon_t^{tby}\ \epsilon_t^{rus}\ \epsilon_t^r]$ the vector of disturbances of the VAR system. Then, one can write the MA(∞) representation of x_t as $x_t = \sum_{j=0}^{\infty} C_j \epsilon_{t-j}$, where $C_j \equiv (A^{-1}B)^j A^{-1}$. The error in forecasting x_{t+h} at time t for $h > 0$, that is, $x_{t+h} - E_t x_{t+h}$, is given by $\sum_{j=0}^{h} C_j \epsilon_{t+h-j}$. The variance/covariance matrix of this h-step-ahead forecasting error is given by $\Sigma^{x,h} \equiv \sum_{j=0}^{h} C_j \Sigma_\epsilon C_j'$, where Σ_ϵ is the (diagonal) variance/covariance matrix of ϵ_t. Thus, the variance of the h-step-ahead forecasting error of x_t is simply the vector containing the diagonal elements of $\Sigma^{x,h}$. In turn, the variance of the error of the h-step-ahead forecasting error of x_t due to a particular shock, say ϵ_t^{rus}, is given by the diagonal elements of the matrix $\Sigma^{x,\epsilon^{rus},h} \equiv \sum_{j=0}^{h} (C_j \Lambda_4) \Sigma_\epsilon (C_j \Lambda_4)'$, where Λ_4 is a 5×5 matrix with all elements equal to zero except element (4,4), which takes the value one. Then, the broken lines in figure 6.5 are given by the element-by-element ratio of the diagonal elements of $\Sigma^{x,\epsilon^{rus},h}$ to the diagonal elements of the matrix $\Sigma^{x,h}$ for different values of h. The difference between the solid lines and the broken lines (i.e., the fraction of the variance of the forecasting error due to ϵ_t^r) is computed in a similar fashion but using the matrix Λ_5.

[7]We observe that the estimates of ϵ_t^y, ϵ_t^i, ϵ_t^{tby}, and ϵ_t^r (i.e., the sample residuals of the first, second, third, and fifth equations of the VAR system) are orthogonal to each other. But because $\hat{y}_t$, $\hat{\imath}_t$, and tby_t are excluded from the R_t^{us} equation, we have that the estimates of ϵ_t^{rus} will in general not be orthogonal to the estimates of ϵ_t^y, ϵ_t^i, or ϵ_t^{tby}. However, under our maintained specification assumption that the US real interest rate does not systematically respond to the state of the business cycle in emerging countries, this lack of orthogonality should disappear as the sample size increases.

aggregate fluctuations in these countries. Thus, around one third of business cycles in emerging economies is explained by disturbances in external financial variables. These disturbances play an even stronger role in explaining movements in international transactions. In effect, US-interest-rate shocks and country-spread shocks are responsible for about 43 percent of movements in the trade balance-to-output ratio in the countries included in our panel.

Variations in country spreads are largely explained by innovations in US interest rates and innovations in country-spreads themselves. Jointly, these two sources of uncertainty account for about 85 percent of fluctuations in country spreads. Most of this fraction, about 60 percentage points, is attributed to country-spread shocks. This last result concurs with Eichengreen and Mody (1998), who interpret this finding as suggesting that arbitrary revisions in investors sentiments play a significant role in explaining the behavior of country spreads.

The impulse response functions shown in figure 6.4 establish empirically that country spreads respond significantly and systematically to domestic macroeconomic variables. At the same time, the variance decomposition performed in this section indicates that domestic variables are responsible for about 15 percent of the variance of country spreads at business-cycle frequency. A natural question raised by these findings is whether the feedback from endogenous domestic variables to country spreads exacerbates domestic volatility. Here we make a first step at answering this question. Specifically, we modify the $\hat{R}_t$ equation of the VAR system by setting to zero the coefficients on $\hat{y}_{t-i}$, $\hat{i}_{t-i}$, and tby_{t-i} for $i = 0, 1$. We then compute

Table 6.1: Aggregate Volatility With and Without Feedback of Spreads from Domestic Variables Model

Variable	Feedback	No Feedback
	Std. Dev.	Std. Dev.
$\hat{y}$	3.6450	3.0674
$\hat{\imath}$	14.1060	11.9260
tby	4.3846	3.5198
R	6.4955	4.7696

the implied volatility of $\hat{y}_t$, $\hat{i}_t$, tby_t and $\hat{R}_t$ in the modified VAR system at business-cycle frequency (20 quarters). We compare these volatilities to those emerging from the original VAR model. Table 6.1 shows that the presence of feedback from domestic variables to country spreads significantly increases domestic volatility. In particular, when we shut off the endogenous feedback, the volatility of output falls by 16 percent and the volatility of investment and the trade balance-to-GDP ratio fall by about 20 percent. The effect of feedback on the cyclical behavior of the country spread itself is even stronger. In effect, when feedback is negated, the volatility of the country interest rate falls by about one third.

Of course, this counterfactual exercise is subject to Lucas' (1976) celebrated critique. For one should not expect that in response to changes in the coefficients defining the spread process all other coefficients of the VAR system will remain unaltered. As such, the results of table 6.1 serve solely as a way to motivate a more adequate approach to the question they aim to address. This more satisfactory approach necessarily involves the use of a theoretical model economy where private decisions change in response to alterations in the country-spread process. We follow this route next.

6.4 A Theoretical Model

The process of identifying country-spread shocks and US-interest-rate shocks involves a number of restrictions on the matrices defining the VAR system (6.1). To assess the plausibility of these restrictions, it is necessary to use the predictions of some theory of the business cycle as a metric. If the estimated shocks imply similar business cycle fluctuations in the empirical as in theoretical models, we conclude that according to the proposed theory, the identified shocks are plausible.

Accordingly, we will assess the plausibility of our estimated shocks in four steps: First, we develop a standard model of the business cycle in small open economies. Second, we estimate the deep structural parameters of the model. Third, we feed into the model the estimated version of the fourth and fifth equations of the VAR system (6.1), describing the stochastic laws of motion of the US interest rate and the country spread. Finally, we compare estimated impulse responses (i.e., those shown in figures 6.2 and 6.3) with those implied by the proposed theoretical framework.

The basis of the theoretical model presented here is the standard neoclassical growth model of the small open economy (e.g., Mendoza, 1991). We depart from the canonical version of the small-open-economy RBC model along four dimensions. First, as in the empirical model, we assume that in each period, production and absorption decisions are made prior to the realization of that period's world-interest-rate shock and country-spread shock. Thus, innovations in the world interest rate or the country spread are assumed to have allocative effects with a one-period lag. Second, preferences are assumed to feature external habit formation, or catching up with the Joneses as in Abel (1990). This feature improves the predictions of the standard model by preventing an excessive contraction in private non-business absorption in response to external financial shocks. Habit formation has been shown to help explain asset prices and business fluctuations in both developed economies (e.g., Boldrin, Christiano, and Fisher, 2001) and emerging countries (e.g., Uribe, 2002). Third, firms are assumed to be subject to a working-capital constraint. This constraint introduces a direct supply side effect of changes in the cost of borrowing in international financial markets, and allows the model to predict a more realistic response of domestic output to external financial shocks. Fourth, the process of capital accumulation is assumed to be subject to gestation lags and convex adjustment costs. In combination, these two frictions prevent excessive investment volatility, induce persistence, and allow for the observed nonmonotonic (hump-shaped) response of investment in response to a variety of shocks (see Uribe, 1997).

6.4.1 Households

Consider a small open economy populated by a large number of infinitely lived households with preferences described by the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \mu \tilde{c}_{t-1}, h_t), \tag{6.2}$$

where c_t denotes consumption in period t, $\tilde{c}_t$ denotes the cross-sectional average level of consumption in period t, and h_t denotes the fraction of time devoted to work in period t. Households take as given the process for $\tilde{c}_t$. The single-period utility index U is assumed to be increasing in its first argument, decreasing in its second argument, concave, and smooth. The parameter $\beta \in (0, 1)$ denotes a subjective discount factor, and the parameter μ measures the intensity of external habit formation.

Households have access to two types of asset, physical capital and an internationally traded bond. The capital stock is assumed to be owned entirely by domestic residents. Households have three sources of income: wages, capital rents, and interest income bond holdings. Each period, households allocate their wealth to purchases of consumption goods, purchases of investment goods, and purchases of financial assets. The household's period-by-period budget constraint is given by

$$d_t = R_{t-1}d_{t-1} + \Psi(d_t) + c_t + i_t - w_t h_t - u_t k_t, \tag{6.3}$$

where d_t denotes the household's debt position in period t, R_t denotes the gross interest rate faced by domestic residents in financial markets, w_t denotes the wage rate, u_t denotes the rental rate of capital, k_t denotes the stock of physical capital, and i_t denotes gross domestic investment. We assume that households face costs of adjusting their foreign asset position. We introduce these adjustment costs with the sole purpose of eliminating the familiar unit root built in the dynamics of standard formulations of the small open economy model. The debt-adjustment cost function $\Psi(\cdot)$ is assumed to be convex and to satisfy $\Psi(\bar{d}) = \Psi'(\bar{d}) = 0$, for some $\bar{d} > 0$. Earlier in chapter 4, we compared a number of standard alternative ways to induce stationarity in the small open economy framework, including the one used here, and conclude that they all produce virtually identical implications for business fluctuations.[8]

The process of capital accumulation displays adjustment costs in the form of gestation lags and convex costs as in Uribe (1997). Producing one unit of capital good requires investing 1/4 units of goods for four consecutive periods. Let s_{it} denote the number of investment projects started in $t-i$ for $i = 0, 1, 2, 3$. Then investment in period t is given by

$$i_t = \frac{1}{4} \sum_{i=0}^{3} s_{it}. \tag{6.4}$$

In turn, the evolution of s_{it} is given by

$$s_{i+1t+1} = s_{it}. \tag{6.5}$$

The stock of capital obeys the following law of motion:

$$k_{t+1} = (1-\delta)k_t + k_t \Phi\left(\frac{s_{3t}}{k_t}\right), \tag{6.6}$$

where $\delta \in (0,1)$ denotes the rate of depreciation of physical capital. The

[8]The debt adjustment cost can be decentralized as follows. Suppose that financial transactions between domestic and foreign residents require financial intermediation by domestic institutions (banks). Suppose there is a continuum of banks of measure one that behave competitively. They capture funds from foreign investors at the country rate R_t and lend to domestic agents at the rate R_t^d. In addition, banks face operational costs, $\Psi(d_t)$, that are increasing and convex in the volume of intermediation, d_t. The problem of domestic banks is then to choose the volume d_t so as to maximize profits, which are given by $R_t^d[d_t - \Psi(d_t)] - R_t d_t$, taking as given R_t^d and R_t. It follows from the first-order condition associated with this problem that the interest rate charged to domestic residents is given by $R_t^d = \frac{R_t}{1-\Psi'(d_t)}$, which is precisely the shadow interest rate faced by domestic agents in the centralized problem (see the Euler condition (6.10) below). Bank profits are assumed to be distributed to domestic households in a lump-sum fashion. This digression will be of use later in the paper when we analyze the firm's problem.

process of capital accumulation is assumed to be subject to adjustment costs, as defined by the function Φ, which is assumed to be strictly increasing, concave, and to satisfy $\Phi(\delta) = \delta$ and $\Phi'(\delta) = 1$. These last two assumptions ensure the absence of adjustment costs in the steady state and that the steady-state level of investment is independent of Φ. The introduction of capital adjustment costs is commonplace in models of the small open economy. As discussed in chapters 3 and 4, adjustment costs are a convenient and plausible way to avoid excessive investment volatility in response to changes in the interest rate faced by the country in international markets.

Households choose contingent plans $\{c_{t+1}, h_{t+1}, s_{0,t+1}, d_{t+1}\}_{t=0}^{\infty}$ so as to maximize the utility function (6.2) subject to the budget constraint (6.3), the laws of motion of total investment, investment projects, and the capital stock given by equations (6.4)-(6.6), and a borrowing constraint of the form

$$\lim_{j\to\infty} E_t \frac{d_{t+j+1}}{\prod_{s=0}^{j} R_{t+s}} \leq 0 \tag{6.7}$$

that prevents the possibility of Ponzi schemes. The household takes as given the processes $\{\tilde{c}_{t-1}, R_t, w_t, u_t\}_{t=0}^{\infty}$ as well as c_0, h_0, k_0, $R_{-1}d_{-1}$, and s_{it} for $i = 0, 1, 2, 3$. The Lagrangian associated with the household's optimization problem can be written as:

$$\begin{aligned}\mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t - \mu\tilde{c}_{t-1}, h_t) + \lambda_t \left[d_t - R_{t-1}d_{t-1} - \Psi(d_t) + w_t h_t + u_t k_t - \frac{1}{4}\sum_{i=0}^{3} s_{it} - c_t \right] \right.\\ &+ \left. \lambda_t q_t \left[(1-\delta)k_t + k_t \Phi\left(\frac{s_{3t}}{k_t}\right) - k_{t+1} \right] + \lambda_t \sum_{i=0}^{2} \nu_{it}(s_{it} - s_{i+1t+1}) \right\},\end{aligned}$$

where λ_t, $\lambda_t \nu_{it}$, and $\lambda_t q_t$ are the Lagrange multipliers associated with constraints (6.3), (6.5), and (6.6), respectively. The optimality conditions associated with the household's problem are (6.3), (6.4)-(6.7) all holding with equality and

$$E_t\lambda_{t+1} = U_c(c_{t+1} - \mu\tilde{c}_t, h_{t+1}) \tag{6.8}$$

$$E_t[w_{t+1}\lambda_{t+1}] = -U_h(c_{t+1} - \mu\tilde{c}_t, h_{t+1}) \tag{6.9}$$

$$\lambda_t\left[1 - \Psi'(d_t)\right] = \beta R_t E_t\lambda_{t+1} \tag{6.10}$$

$$E_t\lambda_{t+1}\nu_{0t+1} = \frac{1}{4}E_t\lambda_{t+1} \tag{6.11}$$

$$\beta E_t\lambda_{t+1}\nu_{1t+1} = \frac{\beta}{4}E_t\lambda_{t+1} + \lambda_t\nu_{0t} \tag{6.12}$$

$$\beta E_t\lambda_{t+1}\nu_{2t+1} = \frac{\beta}{4}E_t\lambda_{t+1} + \lambda_t\nu_{1t} \tag{6.13}$$

$$\beta E_t\left[\lambda_{t+1}q_{t+1}\Phi'\left(\frac{s_{3t+1}}{k_{t+1}}\right)\right] = \frac{\beta}{4}E_t\lambda_{t+1} + \lambda_t\nu_{2t} \tag{6.14}$$

$$\lambda_t q_t = \beta E_t\left\{\lambda_{t+1}q_{t+1}\left[1 - \delta + \Phi\left(\frac{s_{3t+1}}{k_{t+1}}\right) - \frac{s_{3t+1}}{k_{t+1}}\Phi'\left(\frac{s_{3t+1}}{k_{t+1}}\right)\right] + \lambda_{t+1}u_{t+1}\right\}. \tag{6.15}$$

It is important to recall that, because of our assumed information structure, the variables c_{t+1}, h_{t+1}, and s_{0t+1} all reside in the information set of period t. Equation (6.8) states that in period t households choose consumption and leisure for period $t+1$ in such as way as to equate the marginal utility of consumption in period $t+1$ to the expected marginal utility of wealth in that period, $E_t\lambda_{t+1}$. Note that in general the marginal utility of wealth will differ from the marginal utility of consumption ($\lambda_t \neq U_c(c_t - \mu\tilde{c}_{t-1}, h_t)$), because current consumption cannot react to unanticipated changes in wealth. Equation (6.9) defines the household's labor supply schedule, by equating the marginal disutility of effort in period $t+1$ to the expected utility value of the wage rate in that period. Equation (6.10) is an asset pricing relation equating the intertemporal marginal rate of substitution in consumption to the rate of return on financial assets. Note that, because of the presence of frictions to adjust bond holdings, the relevant rate of return on this type of asset is not simply the market rate R_t but rather the shadow rate of return $R_t/[1-\Psi'(d_t)]$. Intuitively, when the household's debt position is, say, above its steady-state level $\bar{d}$, we have that $\Psi'(d_t) > 0$ so that the shadow rate of return is higher than the market rate of return, providing further incentives for households to save, thereby reducing their debt positions. Equations (6.11)-(6.13) show how to price investment projects at different stages of completion. The price of an investment project in its ith quarter of gestation equals the price of a project in the i-1 quarter of gestation plus 1/4 units of goods. Equation (6.14) links the cost of producing a unit of capital to the shadow price of installed capital, or Tobin's Q, q_t. Finally, equation (6.15) is a pricing condition for physical capital. It equates the revenue from selling one unit of capital today, q_t, to the discounted value of renting the unit of capital for one period and then selling it, $u_{t+1}+q_{t+1}$, net of depreciation and adjustment costs.

6.4.2 Firms

Output is produced by means of a production function that takes labor services and physical capital as inputs,

$$y_t = F(k_t, h_t), \tag{6.16}$$

where the function F is assumed to be homogeneous of degree one, increasing in both arguments, and concave. Firms hire labor and capital services from perfectly competitive markets. The production process is subject to a working-capital constraint that requires firms to hold non-interest-bearing assets to finance a fraction of the wage bill each period. Formally, the working-capital constraint takes the form

$$\kappa_t \geq \eta w_t h_t; \quad \eta \geq 0,$$

where κ_t denotes the amount of working capital held by the representative firm in period t.

The debt position of the firm, denoted by d_t^f, evolves according to the following expression

$$d_t^f = R_{t-1}^d d_{t-1}^f - F(k_t, h_t) + w_t h_t + u_t k_t + \pi_t - \kappa_{t-1} + \kappa_t,$$

where π_t denotes distributed profits in period t, and $R_t^d \equiv \frac{R_t}{1-\Psi'(d_t)}$ is the interest rate faced by nonfinancial domestic agents, as shown in footnote 8. The interest rate R_t^d will in general differ from the country interest rate R_t—the interest rate that domestic banks face in international financial markets—because of the presence of debt-adjustment costs.

Define the firm's total net liabilities at the end of period t as $a_t = R_t^d d_t^f - \kappa_t$. Then, we can rewrite the above expression as

$$\frac{a_t}{R_t} = a_{t-1} - F(k_t, h_t) + w_t h_t + u_t k_t + \pi_t + \left(\frac{R_t^d - 1}{R_t^d}\right) \kappa_t.$$

We will limit attention to the case in which the interest rate is positive at all times. This implies that the working-capital constraint will always bind, for otherwise the firm would incur in unnecessary financial costs, which would be suboptimal. So we can use the working-capital constraint holding with equality to eliminate κ_t from the above expression to get

$$\frac{a_t}{R_t^d} = a_{t-1} - F(k_t, h_t) + w_t h_t \left[1 + \eta\left(\frac{R_t^d - 1}{R_t^d}\right)\right] + u_t k_t + \pi_t. \qquad (6.17)$$

It is clear from this expression that the assumed working-capital constraint increases the unit labor cost by a fraction $\eta(R_t^d - 1)/R_t^d$, which is increasing in the interest rate R_t^d.

The firm's objective is to maximize the present discounted value of the stream of profits distributed to its owners, the domestic residents. That is,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \pi_t.$$

We use the household's marginal utility of wealth as the stochastic discount factor because households own domestic firms. Using constraint (6.17) to eliminate π_t from the firm's objective function the firm's problem can be stated as choosing processes for a_t, h_t, and k_t so as to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \frac{a_t}{R_t^d} - a_{t-1} + F(k_t, h_t) - w_t h_t \left[1 + \eta\left(\frac{R_t^d - 1}{R_t^d}\right)\right] - u_t k_t \right\},$$

subject to a no-Ponzi-game borrowing constraint of the form

$$\lim_{j\to\infty} E_t \frac{a_{t+j}}{\prod_{s=0}^{j} R_{t+s}^d} \le 0.$$

The first-order conditions associated with this problem are (6.10), (6.17), the no-Ponzi-game constraint holding with equality, and

$$F_h(k_t, h_t) = w_t \left[1 + \eta \left(\frac{R_t^d - 1}{R_t^d}\right)\right] \tag{6.18}$$

$$F_k(k_t, h_t) = u_t. \tag{6.19}$$

It is clear from the first of these two efficiency conditions that the working-capital constraint distorts the labor market by introducing a wedge between the marginal product of labor and the real wage rate. This distortion is larger the larger the opportunity cost of holding working capital, $(R_t^d - 1)/R_t^d$, or the higher the intensity of the working capital constraint, η.[9] We also observe that any process a_t satisfying equation (6.17) and the firm's no-Ponzi-game constraint is optimal. We assume that firms start out with no liabilities. Then, an optimal plan consists in holding no liabilities at all times ($a_t = 0$ for all $t \geq 0$), with distributed profits given by

$$\pi_t = F(k_t, h_t) - w_t h_t \left[1 + \eta \left(\frac{R_t^d - 1}{R_t^d}\right)\right] - u_t k_t$$

In this case, d_t represents the country's net debt position, as well as the amount of debt intermediated by local banks. We also note that the above three equations together with the assumption that the production technology is homogeneous of degree one imply that profits are zero at all times ($\pi_t = 0 \; \forall \; t$).

[9]The precise form taken by this wedge depends on the particular timing assumed in modeling the use of working capital. Here we adopt the shopping-time timing. Alternative assumptions give rise to different specifications of the wedge. For instance, under a cash-in-advance timing the wedge takes the form $1 + \eta(R_t^d - 1)$.

6.4.3 Driving Forces

One advantage of our method to assess the plausibility of the identified US-interest-rate shocks and country-spread shocks is that one need not feed into the model shocks other than those whose effects one is interested in studying. This is because we empirically identified not only the distribution of the two shocks we wish to study, but also their contribution to business cycles in emerging economies. In formal terms, we produced empirical estimates of the coefficients associated with ϵ_t^r and ϵ_t^{rus} in the MA(∞) representation of the endogenous variables of interest (output, investment, etc.). So using the economic model, we can generate the corresponding theoretical MA(∞) representation and compare it to its empirical counterpart. It turns out that up to first order, one only needs to know the laws of motion of R_t and R_t^{us} to construct the coefficients of the theoretical MA(∞) representation. We therefore close our model by introducing the law of motion of the country interest rate R_t. This process is the estimate of the bottom equation of the VAR system (6.1) and is given by

$$\begin{aligned} \hat{R}_t &= 0.63\hat{R}_{t-1} + 0.50\hat{R}_t^{us} + 0.35\hat{R}_{t-1}^{us} - 0.79\hat{y}_t + 0.61\hat{y}_{t-1} + 0.11\hat{\imath}_t - 0.10\hat{\imath}_{t-1} \\ &+ 0.29tby_t - 0.19tby_{t-1} + \epsilon_t^r, \end{aligned} \tag{6.20}$$

where ϵ^r is an i.i.d. disturbance with mean zero and standard deviation 0.031. As indicated earlier, the variable tby_t stands for the trade balance-to-GDP ratio and is given by:[10]

$$tby_t = \frac{y_t - c_t - i_t - \Psi(d_t)}{y_t}. \tag{6.21}$$

Because the process for the country interest rate defined by equation (6.20) involves the world interest rate R_t^{us}, which is assumed to be an exogenous random variable, we must also include this variable's law of motion as part of the set of equations defining the equilibrium behavior of the theoretical model. Accordingly, we estimate R_t^{us} as follows an AR(1) process and obtain

$$\hat{R}_t^{us} = 0.83\hat{R}_{t-1}^{us} + \epsilon_t^{rus}, \tag{6.22}$$

where ϵ_t^{rus} is an i.i.d. innovation with mean zero and standard deviation 0.007.

6.4.4 Equilibrium, Functional Forms, and Parameter Values

In equilibrium all households consume identical quantities. Thus, individual consumption equals average consumption across households, or

$$c_t = \tilde{c}_t; \quad t \geq -1. \tag{6.23}$$

An equilibrium is a set of processes c_{t+1}, $\tilde{c}_{t+1}$, h_{t+1}, d_t, i_t, k_{t+1}, s_{it+1} for $i = 0, 1, 2, 3$, R_t, R_t^d, w_t, u_t, y_t, tby_t, λ_t, q_t, and ν_{it} for $i = 0, 1, 2$ satisfying

[10] In an economy like the one described by our theoretical model, where the debt-adjustment cost $\Psi(d_t)$ are incurred by households, the national income and product accounts would measure private consumption as $c_t + \Psi(d_t)$ and not simply as c_t. However, because of our maintained assumption that $\Psi'(\bar{d}) = 0$, it follows that both measures of private consumption are identical up to first order.

conditions (6.3)-(6.16), (6.18)-(6.21), and (6.23), all holding with equality, given c_0, c_{-1}, y_{-1}, i_{-1}, i_0, h_0, the processes for the exogenous innovations ϵ_t^{rus} and ϵ_t^r, and equation (6.22) describing the evolution of the world interest rate.

We adopt the following standard functional forms for preferences, technology, capital adjustment costs, and debt adjustment costs,

$$U(c-\mu\tilde{c},h) = \frac{\left[c-\mu\tilde{c}-\omega^{-1}h^{\omega}\right]^{1-\gamma}-1}{1-\gamma},$$

$$F(k,h) = k^{\alpha}h^{1-\alpha},$$

$$\Phi(x) = x - \frac{\phi}{2}(x-\delta)^2; \quad \phi > 0,$$

$$\Psi(d) = \frac{\psi}{2}(d-\bar{d})^2.$$

In calibrating the model, the time unit is meant to be one quarter. Following Mendoza (1991), we set $\gamma = 2$, $\omega = 1.455$, and $\alpha = .32$. We set the steady-state real interest rate faced by the small economy in international financial markets at 11 percent per year. This value is consistent with an average US interest rate of about 4 percent and an average country premium of 7 percent, both of which are in line with actual data. We set the depreciation rate at 10 percent per year, a standard value in business-cycle studies.

There remain four parameters to assign values to, ψ, ϕ, η, and μ. There is no readily available estimates for these parameters for emerging economies. We therefore proceed to estimate them. Our estimation procedure follows Christiano, Eichenbaum, and Evans (2001) and consists in choosing values for the four parameters so as to minimize the distance between the estimated impulse response functions shown in figure 6.2 and the corresponding impulse responses implied by the model.[11] In our exercise we consider the first 24 quarters of the impulse response functions of 4 variables (output, investment, the trade balance, and the country interest rate), to 2 shocks (the US-interest-rate shock and the country-spread shock). Thus, we are setting 4 parameter values to match 192 points. Specifically, let IR^e denote the 192×1 vector of estimated impulse response functions and $IR^m(\psi, \phi, \eta, \mu)$ the corresponding vector of impulse responses implied by the theoretical model, which is a function of the four parameters we seek to estimate. Then our estimate of (ψ, ϕ, η, μ) is given by

$$\text{argmax}_{\{\psi,\phi,\eta,\mu\}}[IR^e - IR^m(\psi, \phi, \eta, \mu)]'\Sigma_{IR^e}^{-1}[IR^e - IR^m(\psi, \phi, \eta, \mu)],$$

where Σ_{IR^e} is a 192×192 diagonal matrix containing the variance of the impulse response function along the diagonal. This matrix penalizes those elements of the estimated impulse response functions associated with large error intervals. The resulting parameter estimates are $\psi = 0.00042$, $\phi = 72.8$, $\eta = 1.2$, and $\mu = 0.2$. The implied debt adjustment costs are small. For example, a 10 percent increase in d_t over its steady-state value $\bar{d}$ maintained over one year has a resource cost of 4×10^{-6} percent of annual GDP. On the other hand, capital adjustment costs appear as more significant. For

[11] A key difference between the exercise presented here and that in Christiano et al. is that here the estimation procedure requires fitting impulse responses to multiple sources of uncertainty (i.e., country-interest-rate shocks and world-interest-rate shocks, whereas in Christiano et al. the set of estimated impulse responses used in the estimation procedure are originated by a single shock.

Table 6.2: Parameter Values

Symbol	Value	Description
β	0.973	Subjective discount factor
γ	2	Inverse of intertemporal elasticity of substitution
μ	0.204	Habit formation parameter
ω	1.455	$1/(\omega - 1)$ = Labor supply elasticity
α	0.32	capital elasticity of output
ϕ	72.8	Capital adjustment cost parameter
ψ	0.00042	Debt adjustment cost parameter
δ	0.025	Depreciation rate (quarterly)
η	1.2	Fraction of wage bill subject to working-capital constraint
R	2.77%	Steady-state real country interest rate (quarterly)

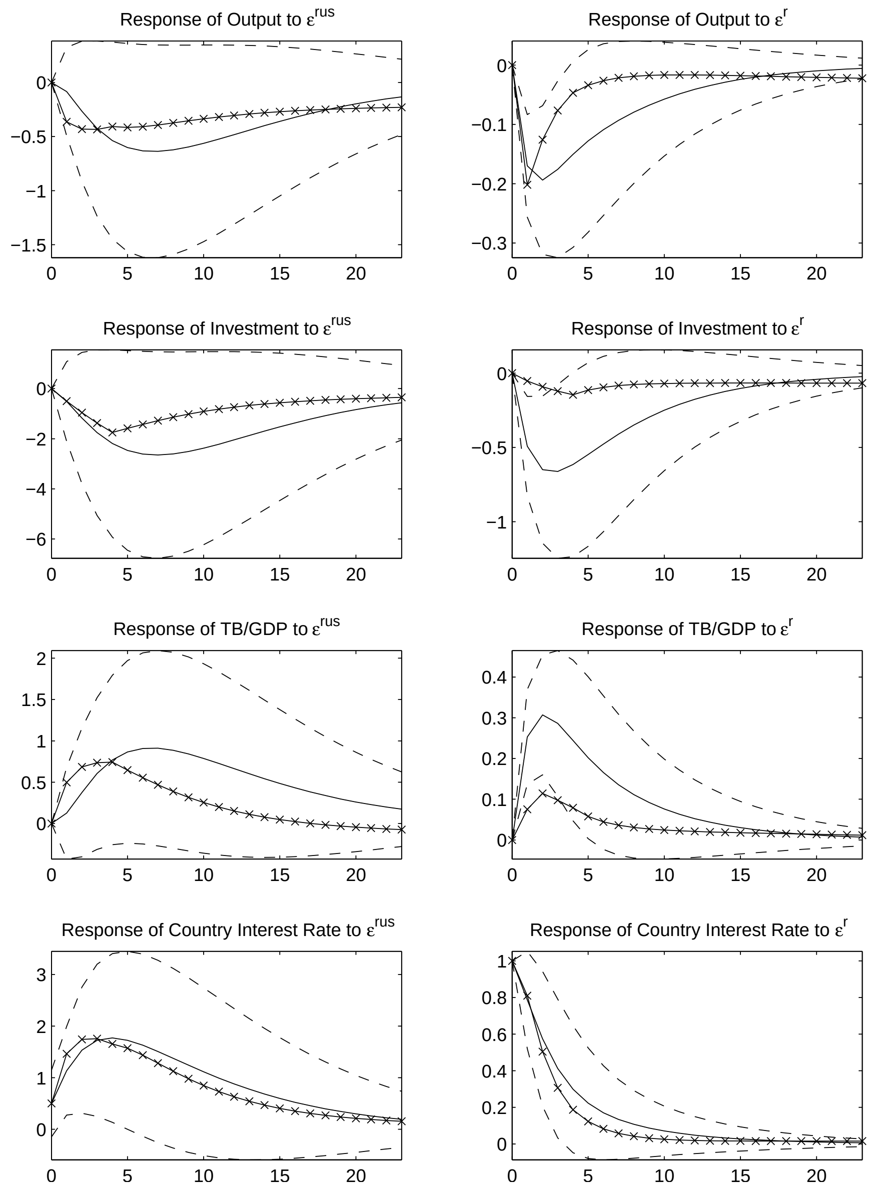
instance, starting in a steady-state situation, a 10 percent increase in investment for one year produces an increase in the capital stock of 0.88 percent. In the absence of capital adjustment costs, the capital stock increases by 0.96 percent. The estimated value of η implies that firms maintain a level of working capital equivalent to about 3.6 months of wage payments. Finally, the estimated degree of habit formation is modest compared to the values typically used to explain asset-price regularities in closed economies (e.g., Constantinides, 1990). Table 6.2 gathers all parameter values.

6.5 Theoretical and Estimated Impulse Responses

Figure 6.6 depicts impulse response functions of output, investment, the trade balance-to-GDP ratio, and the country interest rate.[12] The left col-

[12]The Matlab code used to produce theoretical impulse response functions is available on line at `http://www.columbia.edu/~mu2166/uribe_yue_jie/uribe_yue_jie.html`.

Figure 6.6: Theoretical and Estimated Impulse Response Functions



______ Estimated IR   -x-x- Model IR   _ _ _ _ 2-std error bands around Estimated IR

Note: The first column displays impulse responses to a US interest rate shock (ϵ^{rus}), and the second column displays impulse responses to a country-spread shock (ϵ^{r}).

umn shows impulse responses to a US-interest-rate shock (ϵ_t^{rus}), and the right column shows impulse responses to a country-spread shock (ϵ_t^r). Solid lines display empirical impulse response functions, and broken lines depict the associated two-standard-error bands. This information is reproduced from figures 6.2 and 6.3. Crossed lines depict theoretical impulse response functions.

The model replicates three key qualitative features of the estimated impulse response functions: First, output and investment contract in response to either a US-interest-rate shock or a country-spread shock. Second, the trade balance improves in response to either shock. Third, the country interest rate displays a hump-shaped response to an innovation in the US interest rate. Fourth, the country interest rate displays a monotonic response to a country-spread shock. We therefore conclude that the scheme used to identify the parameters of the VAR system (6.1) is indeed successful in isolating country-spread shocks and US-interest-rate shocks from the data.

6.6 The Endogeneity of Country Spreads

According to the estimated process for the country interest rate given in equation (6.20), the country spread $\hat{S}_t = \hat{R}_t - \hat{R}_t^{us}$ moves in response to four types of variable: its own lagged value S_{t-1} (the autoregressive component), the exogenous country-spread shock ϵ_t^r (the sentiment component), current and past US interest rates R_t^{us} and R_{t-1}^{us}), and current and past values of a set of domestic endogenous variables, $\hat{y}_t$, $\hat{y}_{t-1}$, $\hat{\imath}_t$, $\hat{\imath}_{t-1}$, $\hat{tby}_t$, $\hat{tby}_{t-1}$. A natural question is to what extent the endogeneity of country spreads contributes to exacerbating aggregate fluctuations in emerging countries.

We address this question by means of two counterfactual exercises. The first exercise aims at gauging the degree to which country spreads amplify the effects of world-interest-rate shocks. To this end, we calculate the volatility of endogenous macroeconomic variables due to US-interest-rate shocks in a world where the country spread does not directly depend on the US interest rate. Specifically, we assume that the process for the country interest rate is given by

$$\begin{aligned}\hat{R}_t &= 0.63\hat{R}_{t-1} + \hat{R}^{us}_t - 0.63\hat{R}^{us}_{t-1} - 0.79\hat{y}_t + 0.61\hat{y}_{t-1} + 0.11\hat{\imath}_t - 0.12\hat{\imath}_{t-1} \\ &+ 0.29 tby_t - 0.19 tby_{t-1} + \epsilon^r_t.\end{aligned} \quad (6.24)$$

This process differs from the one shown in equation (6.20) only in that the coefficient on the contemporaneous US interest rate is unity and the coefficient on the lagged US interest rate equals -0.63, which is the negative of the coefficient on the lagged country interest rate. This parametrization has two properties of interest. First, it implies that, given the past value of the country spread, $\hat{S}_{t-1} = \hat{R}_{t-1} - \hat{R}^{us}_{t-1}$, the current country spread, S_t, does not directly depend upon current or past values of the US interest rate. Second, the above specification of the country-interest-rate process preserves the dynamics of the model in response to country-spread shocks. The process for the US interest rate is assumed to be unchanged (i.e., given by equation (6.22)). We note that in conducting this and the next counterfactual exercises we do not reestimate the VAR system. The reason is that doing so would alter the estimated process of the country spread shock ϵ^r_t.

Table 6.3: Endogeneity of Country Spreads and Aggregate Instability

	Std. Dev. due to ϵ^{rus}			Std. Dev. due to ϵ^{r}		
Variable	Baseline Model	No R^{us}	No $\hat{y}_t$ $\hat{\imath}$, or tby	Baseline Model	No R^{us}	No $\hat{y}_t$ $\hat{\imath}$, or tby
$\hat{y}$	1.110	0.420	0.784	0.819	0.819	0.639
$\hat{\imath}$	2.245	0.866	1.580	1.547	1.547	1.175
tby	1.319	0.469	0.885	0.663	0.663	0.446
R	3.509	1.622	2.623	4.429	4.429	3.983
S	2.515	0.347	1.640	4.429	4.429	3.983

Note: The variable S denotes the country spread and is defined as $S = R/R^{us}$. A hat on a variable denotes log-deviation from its non-stochastic steady-state value.

This would amount to introducing two changes at the same time. Namely, changes in the endogenous and the sentiment components of the country spread process.

The precise question we wish to answer is: what process for $\hat{R}_t$ induces higher volatility in macroeconomic variables in response to US-interest-rate shocks, the one given in equation (6.20) or the one given in equation (6.24)? To answer this question, we feed the theoretical model first with equation (6.20) and then with equation (6.24) and in each case compute a variance decomposition of output and other endogenous variables of interest. The result is shown in table 6.3. We find that when the country spread is assumed not to respond directly to variations in the US interest rate (i.e., under the process for R_t given in equation (6.24)) the standard deviation of output and the trade balance-to-output ratio explained by US-interest-rate shocks is about two thirds smaller than in the baseline scenario (i.e., when R_t follows the process given in equation (6.20)). This indicates that the aggregate effects of US-interest-rate shocks are strongly amplified by the dependence of country spreads on US interest rates.

A second counterfactual experiment we wish to conduct aims to assess the macroeconomic consequences of the fact that country spreads move in response to changes in domestic variables, such as output and the external accounts. To this end, we use our theoretical model to compute the volatility of endogenous domestic variables in an environment where country spreads do not respond to domestic variables. Specifically, we replace the process for R_t given in equation (6.20) with the process

$$\hat{R}_t = 0.63\hat{R}_{t-1} + 0.50\hat{R}^{us}_t + 0.35\hat{R}^{us}_{t-1} + \epsilon^r_t. \tag{6.25}$$

Table 6.3 displays the outcome of this exercise. We find that the equilibrium volatility of output, investment, and the trade balance-to-output ratio explained jointly by US-interest-rate shocks and country-spread shocks (ϵ^{rus}_t and ϵ^r_t) falls by about one fourth when the feedback from endogenous domestic variables to country spreads is shut off.[13] We conclude that the fact that country spreads respond to the state of domestic business conditions significantly exacerbates aggregate instability in emerging countries.

[13]Ideally, this particular exercise should be conducted in an environment with a richer battery of shocks capable of explaining a larger fraction of observed business cycles than that accounted by ϵ^{rus}_t and ϵ^r_t alone.

Chapter 7

Sovereign Debt

Why do countries pay their international debts? This is a fundamental question in open-economy macroeconomics. A key distinction between international and domestic debts is that the latter are enforceable. Countries typically have in place domestic judicial systems capable of punishing defaulters. Thus, one reason why residents of a given country honor their debts with other residents of the same country is because creditors are protected by a government able and willing to apply force against delinquent debtors. At the international level the situation is quite different. For there is no such a thing as a supernational authority with the capacity to enforce financial contracts between residents of different countries. Defaulting on international financial contracts appears to have no legal consequences. If agents have no incentives to pay their international debts, then lenders should have no reason to lend internationally to begin with. Yet, we do observe a significant amount of borrowing and lending across nations. It follows that international borrowers must have reasons to repay their debts other than pure legal enforcement.

Two main reasons are typically offered for why countries honer their international debts: economic sanctions and reputation. Economic sanctions may take many forms, such as seizures of debtor country's assets located abroad, trade embargoes, import tariffs and quotas, etc.[1] Intuitively, the stronger is the ability of creditor countries to impose economic sanctions, the weaker the incentives for debtor countries to default.

A reputational motive to pay international debts arises when creditor countries have the ability to exclude from international financial markets countries with a reputation of being defaulters. Being isolated from international financial markets is costly, as it precludes use of the current account to smooth out consumption in response to aggregate domestic income shocks. As a result, countries may choose to repay their debts simply to preserve their reputation and thereby maintain access to international financing.

This chapter investigates whether the existing theories of sovereign debt are capable of explaining the observed levels of sovereign debt. Before plunging into theoretical models of country debt, however, we will present some stylized facts about international lending and default that will guide us in evaluating the existing theories.

[1] The use of force by one country or a group of countries to collect debt from another country was not uncommon until the beginning of the twentieth century. In 1902, an attempt by Great Britain, Germany, and Italy to collect the public debt of Venezuela by force prompted the Argentine jurist Luis-María Drago, who at the time was serving as minister of foreign affairs of Argentina, to articulate a doctrine stating that no public debt should be collected from a sovereign American state by armed force or through the occupation of American territory by a foreign power. The Drago doctrine was approved by the Hague Conference of 1907.

7.1 Empirical Regularities

Table 7.1 displays average debt-to-GNP ratios over the period 1970-2000 for a number of emerging countries that defaulted upon or restructured their external debt at least once between 1824 and 1999. The table also displays average debt-to-GNP ratios at the beginning of default or restructuring episodes. The data suggest that at the time of default debt-to-GNP ratios are significantly above average. In effect, for the countries considered in the sample, the debt-to-GNP ratio at the onset of a default or restructuring episode was on average 14 percentage points above normal times. The information provided in the table is silent, however, about whether the higher debt-to-GNP ratios observed at the brink of default episodes obey to a contraction in aggregate activity or to a faster-than-average accumulation of debt in periods immediately preceding default or both.

Table 7.1 also shows the country premium paid by the 9 emerging countries listed over a period starting on average in 1996 and ending in 2002. During this period, the interest rate at which these 9 countries borrowed in the international financial market was on average about 6 percentage points above the interest rate at which developed countries borrow from one another. There is evidence that country spreads are higher the higher the debt-to-GNP ratio. Akitoby and Stratmann (2006), estimate a semielasticity of the spread with respect to the debt-to-GNP ratio of 1.3 (see their table 3, column 4). That is, they estimate

$$\frac{\partial \log \text{country spread}}{\partial \text{debt-to-GNP ratio}} = 1.3.$$

Table 7.1: Debt-to-GNP Ratios and Country Premiums Among Defaulters

Country	Average Debt-to-GNP Ratio	Debt-to-GNP Ratio at Year of Default	Average Country Spread
Argentina	37.1	54.4	1756
Brazil	30.7	50.1	845
Chile	58.4	63.7	186
Colombia	33.6		649
Egypt	70.6	112.0	442
Mexico	38.2	46.7	593
Philippines	55.2	70.6	464
Turkey	31.5	21.0	663
Venezuela	41.3	46.3	1021
Average	44.1	58.1	638

Notes: The sample includes only emerging countries with at least one external-debt default or restructuring episode between 1824 and 1999. Debt-to-GNP ratios are averages over the period 1970-2000. Country spreads are measured by EMBI country spreads, produced by J.P. Morgan, and expressed in basis points, and are averages through 2002, with varying starting dates as follows: Argentina 1993; Brazil, Mexico, and Venezuela, 1992; Chile, Colombia, and Turkey, 1999; Egypt 2002; Philippines, 1997. Debt-to-GNP ratios at the beginning of a default episodes are averages over the following default dates in the interval 1970-2002: Argentina 1982 and 2001; Brazil 1983; Chile 1972 and 1983; Egypt 1984; Mexico 1982; Philippines 1983; Turkey 1978; Venezuela 1982 and 1995. Colombia did not register an external default or restructuring episode between 1970 and 2002.

Source: Own calculations based on Reinhart, Rogoff, and Savastano (2003), tables 3 and 6.

Thus, if, as documented in table 7.1, the debt-to-GNP ratio is 14 percentage points higher at the beginning of a debt default or restructuring episode than during normal times, and the average country spread during normal times is about 640 basis points, it follows that at the beginning of a default episode the country premium increases on average by $1.3 \times 640 \times 0.14 = 116$ basis points, or 1.16 percent. This increase in spreads might seem small for a country that is at the brink of default. We note, however, that this increase in the country premium is only the part of the total increase in country spreads that is attributable to changes in the debt-to-GNP ratio. Country spreads are known to respond systematically to other variables that may take different values during normal and default times. For instance, Akitoby and Stratmann (2006) and others have documented that spreads increase significantly with the rate of inflation and decrease significantly with the foreign reserve-to-GDP ratio. Because around default periods inflation tends to be high and foreign reserves tend to be low, these two factors contribute to higher country spreads in periods immediately preceding a default episode.

Table 7.2 displays empirical probabilities of default for 9 emerging countries over the period 1824-1999. On average, the probability of default is about 3 prevent per year. That is, countries defaulted on average once every 33 years. Table 7.2 also reports the average number of years countries are in state of default or restructuring after a default or restructuring episode. After a debt crisis, countries are in state of default for about 11 years on average. If one assumes that while in state of default countries have limited access to fresh funds from international markets, one would conclude that

Table 7.2: Probability of Default and Length of Default State 1824-1999

Country	Probability of Default per year	Years in State of Default per Default Episode
Argentina	0.023	11
Brazil	0.040	6
Chile	0.017	14
Colombia	0.040	10
Egypt	0.011	11
Mexico	0.046	6
Philippines	0.006	32
Turkey	0.034	5
Venezuela	0.051	7
Average	0.030	11

Note: The sample includes only emerging countries with at least one external-debt default or restructuring episode between 1824 and 1999. Therefore, the average probability is conditional on at least one default in the sample period.

Source: Own calculations based on Reinhart, Rogoff, and Savastano (2003), table 1.

default causes countries to be in financial autarky for about a decade. But the connection between being in state of default and being in financial autarky should not be taken too far. For being in state of default with a set of lenders, does not necessarily preclude the possibility of obtaining new loans from other lenders with which the borrower has no unpaid debts.

7.2 The Cost of Default

Default and debt restructuring episodes are typically accompanied by significant declines in aggregate activity. Sturzenegger (2003) finds that after controlling for a number of factors that explain economic growth, the cumulative output loss associated with default or debt restructuring episodes in the 1980s was of about 4 percent over four years.

Default episodes are also associated with disruptions in international trade. Rose (2005) investigates this issue empirically. The question of whether default disrupts international trade is of interest because if for some reason trade between two countries is significantly diminished as a result of one country defaulting on its financial debts with other cuntries, then maintaining access to international trade could represent a reason why countries tend to honor their international financial obligations. Rose estimates an equation of the form

$$T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m R_{ijt-m} + \epsilon_{ijt},$$

where T_{ijt} is a measure of average bilateral trade between countries i and

j in period t. Rose identifies default with dates in which a country enters a debt restructuring deal with the Paris Club. The Paris Club is an informal association of creditor-country finance ministers and central bankers that meets to negotiate bilateral debt rescheduling agreements with debtor-country governments. The regressor R_{ijt} is a proxy for default. It is a binary variable equal to unity if countries i and j renegotiated debt in period t in the context of the Paris Club and zero otherwise. The main focus of Rose's work is the estimation of the coefficients ϕ_{ijm}.

Rose's empirical model belongs to the family of gravity models. The variable X_{ijt} is a vector of regressors including (current and possibly lagged) characteristics of the country pair ij at time t such as output, population, distance, area, sharing of a common language, sharing of land borders, membership to the same free trade agreement, country pair-specific dummies, etc. The vector X_{ijt} also includes current and lagged values of a variable IMF_{ijt}, that takes the values 0, 1, or 2, respectively, if neither, one, or both countries i and j engaged in an IMF program at time t.

The data set used for the estimation of the model covers all bilateral trades between 217 countries between 1948 and 1997 at an annual frequency. The sample contains 283 Paris-Club debt-restructuring deals. Rose finds sensible estimates of the parameters pertaining to the gravity model. Specifically, countries that are more distant geographically trade less, whereas high-income country pairs trade more. Countries that share a common currency, a common language, a common border, or membership in a regional free trade agreement trade more. Landlocked countries and islands trade less, and most of the colonial effects are large and positive. The inception of IMF programs is associated with an accumulated contraction in trade of about 10 percent over three years.

Default, as measured by the debt restructuring variable R_{ijt} has a significant and negative effect on bilateral trade. Rose estimates the parameter ϕ_{ijm} to be on average about 0.07 and the lag length, M, to be about 15 years. This means that entering in a debt restructuring agreement with a member of the Paris Club leads to a decline in bilateral trade of about 7 percent per year for about 15 years. Thus, the cumulative effect of default on trade is about one year worth of trade in the long run. Based on this finding, Rose concludes that one reason why countries pay back their international financial obligations is fear of trade disruptions in the case of default.

Do the estimated values of ϕ_m really capture the effect of trade sanctions imposed by creditor countries to defaulting countries? Countries undergoing default or restructuring of their external financial obligations typically are subject to severe economic distress, which may be associated with a general decline in international trade that is not specific to creditor countries. If this is indeed the case, then the coefficients ϕ_m would be picking up the combined effects of trade sanctions and of general economic distress during default episodes. To disentangle these two effects, Martínez and Sandleris (2008) estimate the following variant of Rose's gravity model:

$$T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m R_{ijt-m} + \sum_{m=0}^{M} \gamma_m G_{ijt-m} + \epsilon_{ijt},$$

where G_{ijt-m} is a binary variable taking the value one if either country i

or country j is a debtor country renegotiating in the context of the Paris Club in period t, and zero otherwise. Notice that, unlike variable R_{ijt}, variable G_{ijt} is unity as long as one of the countries is a renegotiating debtor, regardless of whether or not the other country in the pair is the renegotiating creditor. This regressor is meant to capture the general effect of default on trade with all trading countries, not just with those with which the debtor country is renegotiating debt arrears. In this version of the gravity model, evidence of trade sanctions would require a point estimate for $\sum_{m=0}^{M} \phi_m$ that is negative and significant, and evidence of a general effect of default on trade would require a negative and significant estimate of $\sum_{m=0}^{M} \gamma_m$. Martínez and Sandleris estimate $\sum_{m=0}^{15} \gamma_m$ to be -0.41. Thtat is, when a country enters in default its international trade falls by about 40 percent over 15 years with all countries. More importantly, they obtain a point estimate of $\sum_{m=0}^{15} \gamma_m$ that is positive and equal to 0.01. The sign of the point estimates are robust to setting the number of lags, M, at 0, 5, or 10. This result would point at the absence of trade sanctions if creditor countries acted in isolation against defaulters. However, if creditors behave collectively by applying sanctions to defaulters whether or not they are directly affected (i.e., even if the creditor and the debtor are not renegotiating) then the γ_m coefficients would erroneously be capturing sanction effects.

Martínez and Sandleris control for collective-sanction effects by estimating two additional variants of the gravity model. One is of the form

$$T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m CRED_{ijt-m} + \sum_{m=0}^{M} \gamma_m G_{ijt-m} + \epsilon_{ijt},$$

where $CRED_{ijt}$ is a binary variable that takes the value 1 if one of the countries in the pair ij is a debtor renegotiating its debt and the other is a creditor, independently of whether or not it is renegotiating with the debtor country in the pair. Evidence of trade sanctions would require $\sum_{m=0}^{M} \phi_m$ to be negative and significant. The point estimate of $\sum_{m=0}^{M} \phi_m$ turns out to be sensitive to the lag length considered. At lag lengths of 0, 5, and 10 years the point estimate is positive and equal to 0.09, 0.19, and 0.01, respectively. But when the lag length is set at 15 years, the point estimate turns negative and equal to -0.19. The second variant of the gravity model considered aims at disentangling the individual and collective punishment effects. It takes the form:

$$T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m ACRED_{ijt-m} + \sum_{m=0}^{M} \xi_m NACRED_{ijt-m} + \sum_{m=0}^{M} \gamma_m NOTCRED_{ijt-m} + \epsilon_{ijt}.$$

Here, $ACRED_{itt}$, $NACRED_{itt}$, and $NOTCRED_{itt}$ are all binary variables taking the values 1 or 0. The variable $ACRED_{itt}$ takes the value 1 if one of the countries in the pair ij is a defaulter negotiating its debt in the context of the Paris Club in period t and the other country is the negotiating Paris Club member. The variable $NACRED_{itt}$ takes the value 1 if one of the countries in the pair ij is a defaulter negotiating its debt in the context of the Paris Club in period t and the other country is a nonnegotiating Paris Club member. The variable $NOTCRED_{itt}$ takes the value 1 if one of the countries in the pair ij is a defaulter negotiating its debt in the context of the Paris Club in period t and the other country is not a member of the Paris Club. In this variant of the model, evidence of collective trade sanctions would require both $\sum_{m=0}^{M}\phi_m$ and $\sum_{m=0}^{M}\xi_m$ to be negative and significant. The cumulative effect of default on trade between defaulters and nonaffected creditors, given by $\sum_{m=0}^{M}\phi_m$, is consistently negative and robust across lag lengths. Specifically, it takes the values -0.0246, -0.2314, -0.4675, and -0.5629, at lag lengths of 0, 5, 10, and 15 years, respectively. However, the cumulative effect of default on trade between defaulters and directly affected creditors, given by $\sum_{m=0}^{M}\phi_m$, is again sensitive to the specified lag length, taking positive values at short and medium lag lengths and turning negative at long lag lengths. Specifically, the point estimate is 0.0631, 0.0854, 0.0119, and -0.3916 at lag lengths of 0, 5, 10, and 15, respectively.

We interpret the work of Martínez and Sandleris as suggesting that the importance of trade sanctions as a cost of default depends crucially upon one's beliefs regarding the magnitude of the delay with which creditors are able or willing to punish defaulter debtors. if one believes that a reasonable period over which creditors apply trade sanctions to defaulting debtors is less than a decade, then the gravity model offers little evidence of trade sanctions to defaulters. Virtually all of the observed decline in the bilateral trade of debtors after a default episode can be attributed to economic distress and not to punishment inflicted by creditors. However, if one believes that creditors have good memory and are capable of castigating defaulting debtors many years (more than a decade) after a default episode, then the gravity model identifies a significant punishment component in the observed decline in bilateral trade following default episodes of about 50 percent of the trade volume cumulated over 15 years.

7.3 Default Incentives With State-Contingent Contracts

The focus of this section is to analyze the structure of international debt contracts when agents have accesss to state-contingent financial instruments but may lack commitment to honor debt obligations. The material in this section draws from the influential work of Grossman and Van Huyck (1988).[2]

Consider a one-period economy facing a stochastic endowment given by

$$y_s = y + \epsilon_s,$$

where $s = 1, \ldots, S$ denotes the state of nature, $y > 0$ is a constant, and ϵ_s is a random endowment shock with mean zero. There are $S > 1$ possible states of nature. Let us assume, without loss of generality, that $\epsilon_1 < \epsilon_2 < \cdots < \epsilon_S$.

Before the realization of the state of nature, the households can buy insurance from foreign lenders in the form of state-contingent debt contracts. Specifically, these debt contracts stipulate that the country must pay d_s units of goods to foreign lenders in state s. Foreign lenders are assumed to be risk neutral. to operate in a perfectly competitive market, and to face an opportunity cost of funds equal to zero. These assumptions imply that debt contracts must carry an expected payment of zero. Formally, letting π_s denote the probability of occurrence of state s, the zero-expected-profit

[2]A similar exposition appears in Obstfeld and Rogoff (1996).

condition can be written as

$$\sum_{s=1}^{S} \pi_s d_s = 0. \tag{7.1}$$

The representative household in the domestic economy seeks to maximize the utility function

$$\sum_{s=1}^{S} \pi_s u(c_s), \tag{7.2}$$

where c_s denotes consumption in state of nature s and $u(\cdot)$ denotes a strictly increasing and strictly concave utility index. In each state of nature $s = 1, \cdots, S$, the household's budget constraint is given by

$$c_s = y + \epsilon_s - d_s \tag{7.3}$$

We are now ready to characterize the form of the optimal external debt contract. We begin by considering the case in which households can commit to honor their promises.

7.3.1 Optimal Debt Contract With Commitment

The household's problem consists in choosing a state-continget debt contract d_s, $s = 1, \cdots, S$, to maximize the utility function (7.2) subject to the participation constraint (7.1) and to the budget constraint (7.3). The Lagrangian associated with this problem can be written as

$$\mathcal{L} = \sum_{s=1}^{S} \pi_s \left[u(y + \epsilon_s - d_s) + \lambda d_s \right],$$

where λ denotes the Lagrange multiplier associated with the participation constraint. Note that λ is not state contingent. The first-order conditions associated with the representative household's problem are (7.1), (7.3), and

$$u'(c_s) = \lambda.$$

Noting that the multiplier λ is independent of the state of nature, this expression states that consumption is constant across states of nature. That is, the optimal external debt contract achieves perfect consumption smoothing. In particular, under the optimal contract consumption equals the average endowment in all states, or

$$c_s = y,$$

for all s. The associated debt payments are exactly equal to the endowment shocks,

$$d_s = \epsilon_s.$$

Under the optimal contract, domestic risk-averse households transfer all of their income uncertainty to risk-neutral foreign lenders. In this way, the domestic households receive payments from the rest of the world when the endowment realization is low and must transfer resources to the rest of the world when the domestic endowment is high. Furthermore, payments to (from) the rest of the world are larger the farther is output above (below) its mean value. Indeed, we have

$$\frac{\mathrm{d}d_s}{\mathrm{d}\epsilon_s} = 1,$$

which means that transfer payments to the rest of the world move one to one with income innovations.

7.3.2 Optimal Debt Contract Without Commitment

In the economy under analysis, there are no negative consequences for not paying debt obligations. In addition, debtors have incentives not to pay. In effect, in any state of the world in which the contract stipulates a payment to foreign lenders (i.e., in states in which the endowment is above average), the debtor country would be better off consuming the resources it owes. After consuming this resources, the world simply ends, so debtors cannot be punished for having defulted.

The perfect-risk-sharing equilibrium we analyzed in the previous subsection was built on the basis that the sovereign can resist the temptation to default. What if this commitment to honoring debts was absent? Clearly, in our one-period world, the country would default in any state in which the contract stipulates a payment to the rest of the world. It then follows that any debt contract must include the additional incentive-compatibility constraint

$$d_s \leq 0, \tag{7.4}$$

for all s. The representative household's problem then consists in maximizing the utility function (7.2) subject to the participation constraint (7.1), to the budget constraint (7.3), and to the incentive-compatibility constraint (7.4).

Restrictions (7.1) and (7.4) together state that debt payments must be zero on average and never positive. The only debt contract that can satisfy these two requirements simultaneously is clearly

$$d_s = 0,$$

for all s. That is, a trivial contract stipulating no transfers of any sort in any state. It follows that under lack of commitment international risk sharing breaks down. No meaningful debt contract can be supported in equilibrium. As a result, the country is in complete financial autarky and must consume its endowment in every state

$$c_s = y + \epsilon_s,$$

for all s. This consumption profile has the same mean as the one that can be supported with commitment, namely, $y \equiv Ey_s$. However, the consumption plan under commitment is constant across states, whereas the one associated with autarky inherits the volatility of the endowment process. It follows immediately that risk-averse households (i.e., households with concave preferences) are worse off in the financially autarkic economy. Put differently, commitment is welfare increasing.

Because in the economy without commitment international transfers are constant (and equal to zero) across states, we have that

$$\frac{\mathrm{d}d_s}{\mathrm{d}\epsilon_s} = 0.$$

This result is in sharp contrast with what we obtained under full commitment. In that case, the derivative of debt payments with respect to the endowment is unity at all endowment levels.

Direct Sanctions

Suppose that foreign lenders (or their representative governments) could punish defaulting sovereigns by seizing national property (such as financial assets, exports, etc.). One would expect that this type of actions would deter borrowers from defaulting at least as long as debt obligations do not exceed the value of the seizure. What is the shape of the optimal debt contract that emerges in this type of environment?

We model direct sanctions by assuming that in the case of default lenders can seize $k > 0$ units of goods from the delinquent debtor. It follows that the borrower will honor all debts not exceeding k in value. Formally, this means that the incentive-compatibility constraint now takes the form

$$d_s \leq k. \tag{7.5}$$

The representative household's problem then consists in maximizing the utility function (7.2) subject to the participation constraint (7.1), to the budget constraint (7.3), and to the incentive-compatibility constraint (7.5). The Lagrangian associated with this problem can be written as

$$\mathcal{L} = \sum_{s=1}^{S} \pi_s \left[u(y + \epsilon_s - d_s) + \lambda d_s + \gamma_s (k - d_s) \right],$$

where λ denotes the Lagrange multiplier associated with the participation constraint and γ_s denotes the Lagrange multiplier associated with the incentive-compatibility constraint in state s (there are S such multipliers in total, one for each state of nature). The first-order conditions associated with the representative household's problem are (7.1), (7.3), (7.5), and

$$u'(c_s) = \lambda - \gamma_s, \tag{7.6}$$

$$\gamma_s \geq 0, \tag{7.7}$$

and the slackness condition

$$(k - d_s)\gamma_s = 0. \tag{7.8}$$

In states in which the incentive-compatibility constraint does not bind, i.e., when $d_s < k$, the slackness condition (7.8) states that the Lagrange multiplier γ_s must vanish. It then follows from optimality condition (7.6) that the marginal utility of consumption equals λ for all states of nature in which the incentive compatibility constraint is not binding. This means that consumption is constant across all states in which the incentive-compatibility constraint does not bind.

The budget constraint (7.3) implies that across states in which the incentive-compatibility constraint is not binding, payments to foreign lenders must differ from the endowment innovation ϵ_s by only a constant. Formally, we have that

$$d_s = \bar{d} + \epsilon_s,$$

for all s in which $d_s < k$, where $\bar{d}$ is a constant. Based on our analysis of the case with commitment, in which large payments to the rest of the world take place in state of nature featuring large endowments, it is natural to conjecture that the optimal contract will feature the incentive compatibility constraint binding at relatively high levels of income and not binding at relatively low levels of income. Let $\bar{\epsilon}$ denote the smallest endowment level at which the incentive compatibility constraint binds. Then, we have

$$d_s = \begin{cases} \bar{d} + \epsilon_s & \text{for } \epsilon_s < \bar{\epsilon} \\ k & \text{for } \epsilon_s \geq \bar{\epsilon} \end{cases} . \tag{7.9}$$

We will show shortly that the debt contract described by this expression is indeed continuous in the endowment. In particular, we will show that as $S \to \infty$, we have that

$$\bar{d} + \bar{\epsilon} = k. \tag{7.10}$$

We will also show that if this condition holds, then the constant $\bar{d}$ is indeed positive. This means that under the optimal debt contract without commitment but with direct sanctions the borrower enjoys less insurance than in the case of full commitment. This is because in relatively low-endowment states (i.e., states in which the incentive-compatibility constraint does not bind) the borrower must pay $\bar{d}+\epsilon_s$, which is a larger sum than the one that is stipulated for the same state in the optimal contract with full commitment, given simply by ϵ_s.

To see that if condition (7.10) holds then $\bar{d}$ is positive, write the participation constraint (7.1), which indicates that debt payments must be nil on average, as

$$\begin{aligned} 0 &= \sum_{\epsilon_s < \bar{\epsilon}} \pi_s(\bar{d} + \epsilon_s) + \sum_{\epsilon_s \geq \bar{\epsilon}} \pi_s k \\ &= \sum_{\epsilon_s < \bar{\epsilon}} \pi_s(\bar{d} + \epsilon_s) + \sum_{\epsilon_s \geq \bar{\epsilon}} \pi_s(\bar{\epsilon} + \bar{d}) \\ &= \bar{d} + \sum_{\epsilon_s < \bar{\epsilon}} \pi_s \epsilon_s + \sum_{\epsilon_s \geq \bar{\epsilon}} \pi_s \bar{\epsilon} \\ &= \bar{d} - \sum_{\epsilon_s \geq \bar{\epsilon}} \pi_s(\epsilon_s - \bar{\epsilon}), \end{aligned}$$

which implies that

$$\bar{d} = \sum_{\epsilon_s \geq \bar{\epsilon}} \pi_s(\epsilon_s - \bar{\epsilon}).$$

Clearly, as long as the incentive compatibility constraint binds, i.e., as long as $\bar{\epsilon} < \epsilon_S$, we have that the right-hand side of the above expression is positive, and therefore $\bar{d} > 0$.

In showing that $\bar{d}$ is positive, we made use of the conjecture that the debt contract is continuous in the endowment. To show that this conjecture indeed holds, it is convenient to assume that there is a continuum of states. in the interval $[\epsilon^L, \epsilon^H]$, with $\epsilon^L < \epsilon^H$, and density $\pi(\epsilon)$. The optimal contract sets $\bar{d}$ and $\bar{\epsilon}$ to maximize the utility function (7.2) subject to the participation constraint (7.1) and to the restriction that the debt contract take the form given in (7.9). Note that we are not imposing the continuity restriction (7.10). Rather, we wish to show that this condition emerges as part of the optimal contract. Given $\bar{d}$ and $\bar{\epsilon}$, the utility function is given by

$$\int_{\epsilon^L}^{\bar{\epsilon}} u(y - \bar{d})\pi(\epsilon)d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} u(y + \epsilon - k)\pi(\epsilon)d\epsilon.$$

Differentiating this expression with respect to $\bar{d}$ and $\bar{\epsilon}$ and setting the result equal to zero, yields

$$-u'(y-\bar{d})F(\bar{\epsilon})d\bar{d}+[u(y-\bar{d})-u(y+\bar{\epsilon}-k)]\pi(\bar{\epsilon})d\bar{\epsilon}=0, \qquad (7.11)$$

where $F(\bar{\epsilon}) \equiv \int_{\epsilon^L}^{\bar{\epsilon}} \pi(\epsilon)d\epsilon$ denotes the probability that ϵ is less than $\bar{\epsilon}$.

At the same time, differentiating the participation constraint, given by $\int_{\epsilon^L}^{\bar{\epsilon}}(\bar{d}+\epsilon)\pi(\epsilon)d\epsilon+[1-F(\bar{\epsilon})]k=0$, we obtain

$$[(y+\bar{\epsilon}-k)-(y-\bar{d})]\pi(\bar{\epsilon})d\epsilon+F(\bar{\epsilon})d\bar{d}=0 \qquad (7.12)$$

Combining equations (7.11) and (7.12) we obtain

$$-u'(y-\bar{d})[(y-\bar{d})-(y+\bar{\epsilon}-k)]+[u(y-\bar{d})-u(y+\bar{\epsilon}-k)]=0,$$

Clearly, this expression is satisfied if $\bar{d}+\bar{\epsilon}=k$. This result proves that that if the optimal contract problem has a local maximum, then the implied transfer payment is continuous in ϵ.

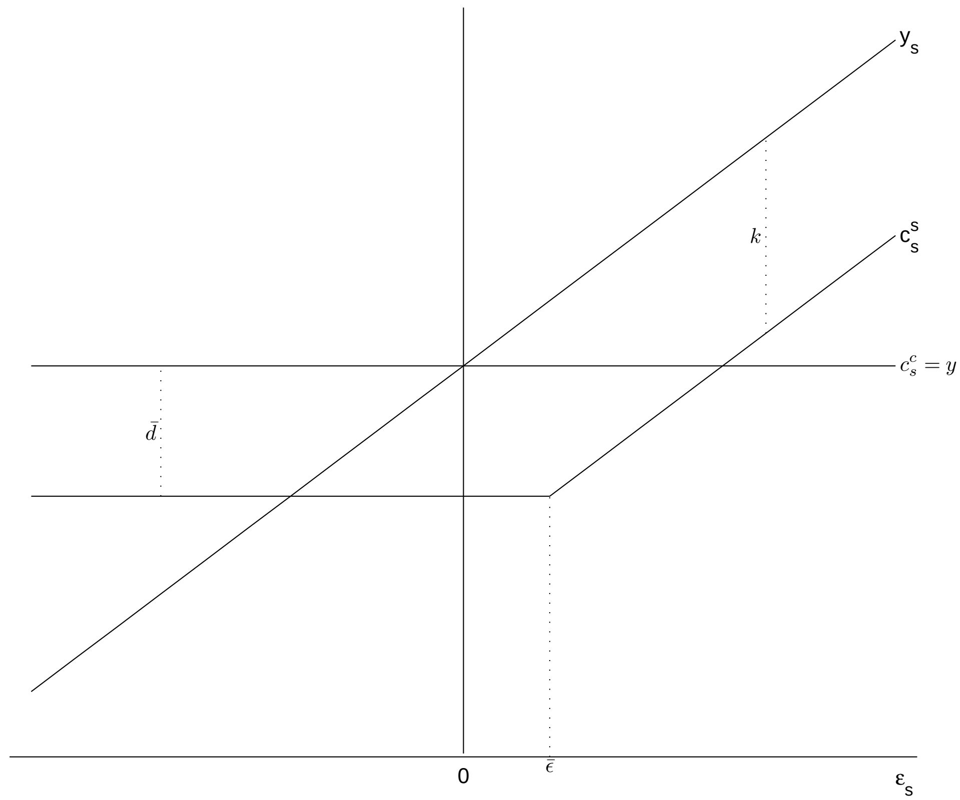
Our analysis shows that the case with no commitment and direct sanctions falls in between the case with full commitment and the case with no commitment and no direct sanctions. In particular, the derivative of the optimal payment with respect to the endowment, $dd_s/d\epsilon_s$ equals unity for $\epsilon_s<\bar{\epsilon}$ (as in the case with full commitment), and equals zero for ϵ_s larger than $\bar{\epsilon}$ (as in the case without commitment and no direct sanctions):

$$\frac{d\mathrm{d}_s}{d\epsilon_s} = \begin{cases} 1 & \epsilon_s < \bar{\epsilon} \\ 0 & \epsilon_s \geq \bar{\epsilon} \end{cases} .$$

Note also that if the sanction is sufficiently large—specifically, if $k > \epsilon_S$—then $\bar{\epsilon} > \epsilon_S$ and the optimal contract is identical to the one that results in the case of full commitment. By contrast, if the ability of crdditors to impose sanctions is sufficiently limited—specifically, if $k = 0$—then $\bar{\epsilon} = \epsilon_1$ and the optimal contract stipulates financial autarky as in the case with neither commitment nor direct sanctions. It follows, perhaps paradoxically, that the larger the ability of creditors to punish debtor countries in case of default, the higher the welfare of the debtor countries themselves.

Finally, it is of interest to compare the consumption profiles across states in the model with commitment and in the model with direct sanctions and no commitment. Figure 7.1 provides a graphical representation of this comparison. In the model with commitment, consumption is perfectly smooth across states and equal to the average endowment. As mentioned earlier, in this case the risk-averse debtor country transfers all of the risk to risk neutral lenders. In the absence of commitment, consumption smoothing is a direct function of the ability of the lender to punish debtors in the case of default. Consumption is flat in low-endowment states (from ϵ_1 to $\bar{\epsilon}$) and increasing in the endowment in high-endowment states (from $\bar{\epsilon}$ to ϵ_S). The reduced ability of the risk averse agent to transfer risk to risk neutral lenders is reflected in two features of the consumption profile. First, the profile is

Figure 7.1: Consumption Profiles Under Full Commitment and No Commitment With Direct Sanctions



Note: c_s^c and c_s^s denotes the levels of consumption under full commitment and direct sanctions, respectively, y_s denotes output, and ϵ_s denotes the endowment shock.

no longer flat across all states of nature. Second, the flat segment of the consumption profile is lower than the level of consumption achieved under full commitment.

Reputation

Suppose now that creditors do not have access to direct sanctions to punish debtors who choose to default. Instead, assume that creditors have the ability to exclude delinquent debtors from financial markets. Because financial autarky entails the cost of an elevated consumption volatility, financial exclusion has the potential to support international lending. Debtor countries pay their obligations to maintain their performing status.

Clearly, the model we have in mind can no longer be a one-period model like the one studied thus far. Time is at the center of any reputational model of debt. Accordingly, we will assume that the debtor country lives for ever and has preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \sum_{s=1}^{S} \pi_s u(c_s).$$

Consider designing a debt contract with the following three characteristics: First, it must be signed before period 0, and it stipulates precise payments in every date $t \geq 0$ and state $s \in \{1, 2, \ldots, S\}$. Second, payments are state contingent, but time independent. That is, the contract stipulates that in any state s, the country must pay d_s to foreign lenders independently of t. Third, for every state and date pair (s, t), the policy of never defaulting—i.e., the policy of honoring the debt contract in date/state (s, t)

and in every possible subsequent date/date— welfare dominates the policy of defaulting in (s,t). These restrictions imply that the following incentive compatibility constraint must hold in each state of nature: each state

$$u(y+\epsilon_s-d_s)+\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s-d_s)\geq u(y+\epsilon_s)+\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s). \tag{7.13}$$

The left-hand side of this expression is the welfare level associated with maintaining a good standing. The right-hand side represents the welfare level associated with financial autarky.

Because the problem is stationary, in the sense that the optimal contract is time independent, it suffices to maximize the period utility index, $\sum_{s=1}^{S}\pi_s u(y+\epsilon_s-d_s)$, subject to the above incentive-compatibility constraint and to the participation constraint (7.1). The Lagrangian associated with this problem is

$$\begin{aligned}\mathcal{L} &= \sum_{s=1}^{S}\pi_s u(y+\epsilon_s-d_s)\\ &+ \lambda\sum_{s=1}^{S}\pi_s d_s\\ &+ \sum_{s=1}^{S}\gamma_s\left[u(y+\epsilon_s-d_s)+\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s-d_s)-u(y+\epsilon_s)-\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s)\right]\end{aligned}$$

The first-order conditions associated with the problem of choosing the transfer schedule d_s are

$$u'(y+\epsilon_s-d_s)=\frac{\lambda}{1+\frac{\gamma_s}{\pi_s}+\frac{\beta}{1-\beta}\sum_{s=1}^{S}\gamma_s} \tag{7.14}$$

and the slackness condition

$$\gamma_s \left[u(y+\epsilon_s - d_s) + \frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s - d_s) - u(y+\epsilon_s) - \frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s)\right] = 0. \tag{7.15}$$

In states in which the incentive compatibility constraint (7.15) is not binding—low-endowment states in which the incentives to default are small, as payments to creditors are either negative or small—the slackness condition (7.15) stipulates that the Lagrange multiplier γ_s must vanish. In these states, the optimality condition (7.14) becomes

$$u'(y+\epsilon_s - d_s) = \frac{\lambda}{1+\frac{\beta}{1-\beta}\sum_{s=1}^{S}\gamma_s}$$

Because the right-hand side of this expression is independent of the state of nature s, we have that the marginal utility of consumption must be constant across these states. This, in turn, implies that consumption is constant across these states, and that transfers are of the form $d_s = \bar{d} + \epsilon_s$, where $\bar{d}$ is a constant. Note that over these states, consumption and payments to or from the rest of the world behave exactly as in the case with direct sanctions: domestic risk-averse agents transfer their endowment shock plus a constant to risk-neutral foreign lenders

What is the pattern of transfers in states in which the incentive compatibility constraint is binding? In these states, restriction (7.13) holds with equality:

$$u(y+\epsilon_s-d_s)+\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s-d_s) = u(y+\epsilon_s)+\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s). \tag{7.16}$$

How does the optimal transfer d_s vary across states in which the collateral constraint is binding? Does it increase as one moves from low- to high-endowment states,and by how much? To answer this question, consider two states, s and s' in which the incentive compatibility constraint binds and such that $\epsilon_{s'} > \epsilon_s$. Notice that in equation (7.16), showing the incentive compatibility constraint holding with equality, the terms $\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s)$ and $\frac{\beta}{1-\beta}\sum_{s=1}^{S}\pi_s u(y+\epsilon_s-d_s)$ are both unchanged as we move from $\epsilon_{s'}$ to $\epsilon_{s''}$. Only the first terms on the right- and left-hand sides of (7.16) change as we evalluate that expression at different states of nature. If ϵ_s and $\epsilon_{s'}$ are very close to each other, then we can approximate the effect on the optimal transfer of moving from one state to the other by differentiating (7.16) with respect to the current transfer and the current endowment, which yields

$$\frac{\mathrm{d}d_s}{\mathrm{d}\epsilon_s} = \frac{u'(y+\epsilon_s-d_s)-u'(y+\epsilon_s)}{u'(y+\epsilon_s-d_s)}.$$

Because the incentive compatibility constraint binds only when the risk-averse agent must make payments ($d_s > 0$)—there are no incentives to default when the risk-averse agent receives income from the risk-neutral lender—and because the utility index is strictly concave, it follows that $u'(y+\epsilon_s-d_s) > u'(y+\epsilon_s)$ in all states in which the incentive compatibility constraint binds. This implies that when the incentive compatibility constraint binds we have

$$0 < \frac{\mathrm{d}d_s}{\mathrm{d}\epsilon_s} < 1.$$

That is, payments to the foreign lender increase with the level of income, but less than one for one. It might seem counterintuitive that as the current endowment increases the payment to creditors that can be supported without default also increases. After all, the higher is the current level of endowment, the higher is the level of current consumption that can be achieved upon default. The intuition behind the direct relation between income and payments is that *given* a positive level of current payments, $d_s > 0$, a small increase in current endowment, ϵ_s, raises the current-period utility associated with not defaulting, $u(y + \epsilon - d_s)$, by more than it raises the utility associated with the alternative of defaulting, $u(y + \epsilon_s)$. (This is because the period utility function is assumed to be strictly concave.) It follows that in states in which $d_s > 0$, the higher is the current endowment, the higher is the level of payments to foreign lenders that can be supported without inducing default. This does not mean that default incentives are weaker the higher is the level of the endowment. Recall that the analysis in this paragraph is restricted to states in which the incentive compatibility constraint is binding. The incentive compatibility constraint tends to bind in relatively high-endowment states.

We close by noting that the positive slope of the payment schedule with respect to the endowment (when the incentive compatibility constraint is binding) presents a contrast with the pattern that emerges in the case of direct sanctions. In that case, when the incentive compatibility constraint binds payments equal the maximum punishment k, which implies that the slope of the payment schedule equals zero.

7.4 Default Incentives With Non-State-Contingent Contracts

In a world with complete financial markets, optimal risk-sharing arrangements stipulate positive payoffs in low-income states and negative payoffs in high-income states. In this way, the optimal financial contract facilitates a smooth level of consumption across states of nature. An implication of this result is that default incentives are stronger in high-income states (when the agent must make payments) and weaker in low-income states (when the agent receives transfers). In the real world, however, as documented earlier in this chapter, countries tend to default during economic contractions. One of our goals in this section is to explain this empirical regularity. To this end, we remove the assumption that financial markets are complete. Indeed, we focus on the polar case of a single non-state-contingent asset. In this environment, debts assumed in the current period impose financial obligation in the next period that are independent of whether income in that period is high or low. the debtor is no longer able to design debt contracts that pay a high interest rate in good states and a low interest rate in bad states. As a result, debtors facing high debt obligations and low endowments will have strong incentives to default. The pioneer model of Eaton and Gersowitz (1981), which we study in this section, represents the first formalization of this idea. Our version of the Eaton-Gersowitz model follows Arellano (2005).

Consider a small open economy populated by a large number of identical individuals. Preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t denotes consumption in period t, u is a period utility function assumed to be strictly increasing and strictly concave, and $\beta \in (0,1)$ is a parameter denoting the subjective discount factor. Each period $t \geq 0$, the representative household is endowed with y_t units of consumption goods. This endowment is assumed to be exogenous, stochastic, and i.i.d., with a distribution featuring a bounded support $Y \equiv [\underline{y}, \bar{y}]$.

At the beginning of each period, the household can be either in good financial standing or in bad financial standing If the household is in bad financial standing, then it is prevented from borrowing or lending in financial markets. As a result, the household is forced to consume its endowment. Formally, consumption of a household in bad financial standing is given by

$$c = y.$$

We drop the time subscript in expressions where all variables are dated in the current period. If the household is in good financial standing, it can choose to default on its debt obligations or to honor its debt. If it chooses to default, then it immediately acquires a bad financial status. If it chooses to honor its debt, then it maintains its good financial standing until the beginning of the next period. Households that are in good standing and choose not to default face the following budget constraint:

$$c + d = y + q(d')d',$$

where d denotes the household's debt due in the current period, d' denotes the debt acquired in the current period and due in the next period, and $q(d')$ denotes the market price of the household's debt. Note that the price of debt depends on the amount of debt acquired in the current period and due next period, d', but not on the level of debt acquired in the previous period and due in the current period, d. This is because the default decision in the next period depends on the amount of debt due then. Notice also that $q(\cdot)$ is independent of the current level of output. This is because of the assumed i.i.d. nature of the endowment, which implies that its current value conveys no information about future expected endowment levels. If instead we had assumed that y were serially correlated, then bond prices would depend on the level of current endowment.

We assume that 'bad financial standing' is an absorbent state. This means that once the household falls into bad standing, it remains in that status forever. The household enters in bad standing when it defaults on its financial obligations. The value function associated with bad financial standing is denoted $v^b(y)$ and is given by

$$v^b(y) = u(y) + \beta E v^b(y').$$

Here, y' denotes next period's endowment.

For an agent in good standing, the value function associated with continuing to participate in capital markets (i.e., not defaulting) is denoted by $v^c(d, y)$ and is given by

$$v^c(d, y) = \max_{d'} \left\{ u(y + q(d')d' - d) + \beta E v^g(d', y') \right\},$$

subject to

$$d' \leq \bar{d},$$

where $v^g(d, y)$ denotes the value function associated with being in good financial standing, and is given by

$$v^g(d, y) = \max\{v^b(y), v^c(d, y)\}.$$

The parameter $\bar{d} > 0$ is a debt limit that prevents agents from engaging in Ponzi games. In this economy, households choose to default when servicing the debt entails a cost in terms of forgone current consumption that is larger than the inconvenience of living in financial autarky forever. It is then reasonable to conjecture that default is more likely the larger the level of debt and the lower the current endowment. In what follows, we demonstrate that this intuition is in fact correct. We do so in steps.

The Default Set

The default set contains all endowment levels at which a household chooses to default given a particular level of debt. We denote the default set by

$D(d)$. Formally, the default set is defined by

$$D(d) = \{y \in Y : v^b(y) > v^c(d, y)\}.$$

Because it is never in the agent's interest to default when its asset position is nonnegative (or $d \leq 0$), it follows that $D(d)$ is empty for all $d \leq 0$.

The following proposition shows that at debt levels for which the default set is not empty, the economy must run trade surpluses.

Proposition 7.1 *If $D(d) \neq \emptyset$, then $q(d')d' - d < 0$ for all $d' \leq \bar{d}$.*

Proof: Suppose that $q\left(\widehat{d}\right)\widehat{d} - d \geq 0$ for some $\widehat{d} \leq \bar{d}$. Then,

$$\begin{aligned} v^c(d, y) &\equiv \max_{d' < \bar{d}} \left\{ u(y + q(d')d' - d) + \beta E v^g(d', y') \right\} \\ &\geq u(y + q\left(\widehat{d}\right)\widehat{d} - d) + \beta E v^g(\widehat{d}, y') \\ &\geq u(y) + \beta E v^b(y') \\ &\equiv v^b(y), \end{aligned}$$

for all $y \in Y$. The first inequality follows from the fact that $\widehat{d}$ is a feasible point of the constrained maximization problem that appears on the right-hand side of the first line of the above expression—i.e., $\bar{d}$ satisfies $\widehat{d} \leq \bar{d}$. The second inequality holds because, by assumption, $q(\widehat{d})\widehat{d} - d \geq 0$ and because, by definition, $v^g(\widehat{d}, y') \geq v^b(y')$. It follows that if $q(\widehat{d})\widehat{d} - d \geq 0$ for some $\widehat{d} \leq \bar{d}$, then $D(d) = \emptyset$. ■

This proposition states that if the household has a level of debt that puts it at risk of default, then if it is to continue to participate in the financial market, it must devote part of its current endowment to servicing the debt.

We now establish that in this economy households tend to default in bad times. Specifically, we show that if a household with a certain level of debt and income chooses to default then it will also choose to default at the same level of debt and a lower level of income. In other worlds, if the default set is not empty then it is indeed an interval with lower bound given by the lowest endowment level $\underline{y}$.

Proposition 7.2 *If $y_2 \in D(d)$ and $y_1 < y_2$, then $y_1 \in D(d)$.*

Proof: Suppose $D(d) \neq \emptyset$. Consider any $y \in Y$ such that $y \in D(d)$. Let $v^b_y(y) \equiv \partial v^b(y)/\partial y$ and $v^c_y(d,y) \equiv \partial v^c(d,y)/\partial y$. By the envelope theorem, $v^b_y(y) = u'(y)$ and $v^c_y(d,y) = u'(y + q(d')d' - d)$. By proposition 7.1, we have that $q(d')d' - d < 0$ for all $d' \leq \bar{d}$. This implies, by strict concavity of u, that $u'(y + q(d')d' - d) > u'(y)$. It follows that $v^b_y(y) - v^c_y(d,y) < 0$, for all $y \in D(d)$. That is, $v^b(y) - v^c(d,y)$ is a decreasing function of y for all $y \in D(d)$. This means that if $v^b(y_2) > v^c(d, y_2)$ for $y_2 \in D(d)$, then $v^b(y_1) > v^c(d, y_1)$ for $y_1 < y_2$. Equivalently, if $y_2 \in D(d)$, then $y_1 \in D(d)$ for any $y_1 < y_2$. ■

We have shown that the default set is an interval with a lower bound given by the lowest endowment $\underline{y}$. We now show that the default set $D(d)$ is a larger interval the larger the stock of debt. Put differently, the higher the debt, the larger the probability of default.

Proposition 7.3 *If $D(d) \neq \emptyset$, then $D(d)$ is an interval, $[\underline{y}, y^*(d)]$, where $y^*(d)$ is increasing in d if $y^*(d) < \bar{y}$.*

Proof: We already proved that the default set $D(d)$ is an interval. By definition, every $y \in D(d)$ satisfies $v^b(y) - v^c(d,y) > 0$. At the same time, we showed that $v_y^b(y) - v_y^c(d, y) < 0$ for all $y \in D(d)$. It follows that $y^*(d)$ is given either by $\bar{y}$ or (implicitly) by $v^b(y^*(d)) = v^c(d, y^*(d))$. Differentiating this expression yields

$$\frac{\mathrm{d}y^*(d)}{\mathrm{d}d}\frac{v_d^c(d, y^*(d))}{v_y^b(y^*(d)) - v_y^c(d, y^*(d))},$$

where $v_d^c(d, y) \equiv \partial v^c(d, y)/\partial d$. We have shown that $v_y^b(y^*(d)) - v_y^c(d, y^*(d)) < 0$. Using the definition of $v_d^c(d, y)$ and applying the envelope theorem, it follows that $v_d^c(d, y^*(d)) = -u'(y^*(d) + q(d')d' - d) < 0$. We then conclude that

$$\frac{\mathrm{d}y^*(d)}{\mathrm{d}d} > 0,$$

as stated in the proposition.■

Summarizing, we have obtained two important results: First, given the stock of debt, default is more likely the lower the level of output. Second, the larger the stock of debt, the higher the probability of default. These two results are in line with the stylized facts presented earlier in this chapter, indicating that at the time of default countries tend to display above-average debt-to-GNP ratios (see table 7.1).

Default Risk and the Country Premium

We now characterize the behavior of the country interest-rate premium in this economy. Let the world interest rate be constant and equal to $r^* > 0$. We assume that foreign lenders are risk neutral and perfectly competitive. It follows that the expected rate of return on the country's debt must equal r^*. If the country does not default, foreign lenders receive $1/q(d')$ units of goods per unit lent. If the country does default, foreign lenders receive nothing. Therefore, equating the expected rate of return on the domestic debt to the risk-free world interest rate, one obtains

$$1 + r^* = \frac{\text{Prob}\left\{y' \geq y^*(d')\right\}}{q(d')}.$$

The numerator on the right side of this expression is the probability that the country will not default next period. Letting $F(y)$ denote the cumulative density function of the endowment shock, we can write

$$q(d') = \frac{1 - F(y^*(d'))}{1 + r^*}.$$

Taking derivative with respect to next period's debt yields

$$\frac{dq(d')}{dd'} = \frac{-F'(y^*(d'))y^{*'}(d')}{1 + r^*} \leq 0.$$

The inequality follows because by definition $F' \geq 0$ and because, by proposition7.3, $y^{*'}(d') \geq 0$. It follows that the country spread, given by the difference between $1/q(d')$ and $1 + r^*$, is nondecreasing in the stock of debt. We summarize this result in the following proposition:

Proposition 7.4 *The country spread, given by* $1/q(d') - 1 - r^*$ *is nondecreasing in the stock of debt.*

7.5 Saving and the Breakdown of Reputational Lending

A key assumption of the reputational model of sovereign debt is that when a country defaults foreign lenders coordinate to exclude it from the possibility to borrow or lend in international financial markets. At a first glance, it might seem that what is important is that defaulters be precluded from borrowing in international financial markets. Bullow and Rogoff (1989) have shown, however, that the prohibiting defaulters to lend to foreign agents (or save in foreign assets) is crucial for the reputational model to work. If delinquent countries were not allowed to borrow but could run current account surpluses, no lending at all could be supported on reputational grounds alone.

To illustrate this insight in a simple setting, consider a deterministic economy. Suppose that a reputational equilibrium supports a path for external debt given by $\{d_t\}_{t=0}^{\infty}$, where d_t denotes the level of external debt assumed in period t and due in period $t+1$.[3] .Assume that default is punished with perpetual exclusion from borrowing in international financial markets, but that lending in these markets is allowed after default. This assumption and the fact that the economy operates under perfect foresight imply that any reputational equilibrium featuring positive debt at least one date must be characterized by no default. To see this, notice that if the country defaults at some date $T > 0$, then no foreign investor would want

[3]For an example of a deterministic model with sovereign debt supported by reputation, see Eaton and Fernández (1995).

to lend to this country in period $T-1$, since default would occur for sure one period later. Thus, $d_{T-1} \leq 0$. In turn, if the country is excluded from borrowing starting in period $T-1$, then it will have no incentives to honor any debts outstanding in that period. As a result, no foreign investor will be willing to lend to the country in period $T-2$. That is, $d_{T-2} \leq 0$. Continuing with this logic, we arrive at the conclusion that default in period T implies no debt at any time. That is, $d_t \leq 0$ for all $t \geq 0$.

It follows from this result that in an equilibrium with positive external debt the interest rate must equal the world interest rate $r^* > 0$, because the probability of default is nil. That is, the country premium is nil. The evolution of the equilibrium level of debt is then given by

$$d_t = (1 + r^*)d_{t-1} - tb_t,$$

for $t \geq 0$, where tb_t denotes the trade balance in period t. (In the economy studied in this section, $tb_{=}y_t - c_t$.) Let d_T be the maximum level of external debt in this equilibrium sequence. That is, $d_T \geq d_t$ for all $t \geq -1$. Does it pay for the country to honor this debt? The answer is no. The reason is that the country could default on this debt in $T+1$—and therefore be excluded from borrowing internationally forever thereafter—and still be able to run trade balances no larger than the ones that would have obtained in the absence of default. To see this, let $\widetilde{d}_t$ for $t > T$ denote the post-default path of external debt. Let the debt position acquired in the period of default be

$$\widetilde{d}_{T+1} = -tb_{T+1},$$

where tb_{T+1} is the trade balance prevailing in period $T+1$ under the original debt sequence $\{d_t\}$. We have that $-tb_{T+1} = d_{T+1} - (1+r^*)d_T$, which implies that

$$\widetilde{d}_{T+1} = d_{T+1} - (1+r^*)d_T. \tag{7.17}$$

Because by assumption $d_T \geq d_{T+1}$ and $r^* > 0$, we have that $\widetilde{d}_{T+1} < 0$. That is, the country can generate the no-default level of trade balance in period $T+1$ without having to borrow internationally. Let the external debt position in period $T+2$ be

$$\widetilde{d}_{T+2} = (1+r^*)\widetilde{d}_{T+1} - tb_{T+2},$$

where, again, tb_{T+2} is the trade balance prevailing in period $T+2$ under the original debt sequence $\{d_t\}$. Using (7.17) and the fact that $tb_{T+2} = (1+r^*)d_{T+1} - d_{T+2}$, we obtain

$$\widetilde{d}_{T+2} = d_{T+2} - (1+r^*)^2 d_T < 0.$$

The inequality follows because by assumption $d_{T+2} \leq d_T$ and $r^* > 0$. We have shown that the defaulting strategy can achieve the no-default level of trade balance in period $t+2$ without requiring any international borrowing. Continuing in this way, one obtains that the no-default sequence of trade balances, tb_t for $t \geq T+1$, can be supported by the debt path $\widetilde{d}_t$ satisfying

$$\widetilde{d}_t = d_t - (1+r^*)^{t-T} d_T,$$

which is strictly negative for all $t \geq T+1$. The fact that the entire post-default debt path is negative implies that the country could also implement a post default path of trade balances $\widetilde{tb}_t$ satisfying $\widetilde{tb}_t \leq tb_t$ for $t \geq T+1$ and $\widetilde{tb}_{t'} < tb_t$ for at least one $t' \geq T+1$ and still generate no positive debt at any date $t \geq T+1$. This new path for the trade balance would be strictly preferred to the no-default path because it would allow consumption to be strictly higher than under the no-default strategy in at least one period and to be at least as high as under the no-default strategy in all other periods (recall that $tb_t = y_t - c_t$). It follows that it pays for the country to default immediately after reaching the largest debt level d_T. But we showed that default in this perfect foresight economy implies zero debt at all times. Therefore, no external debt can be supported in equilibrium on reputational grounds (i.e., $d_T \leq 0$).

For simplicity, we derived the breakdown of the reputation model under saving using a model without uncertainty. But the result also holds in a stochastic environment (see Bullow and Rogoff, 1989).

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