SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 — Prof. Kasa
Macroeconomic Theory — Fall 2006

FINAL EXAM - December 6

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (5 points each).

1. Ricardian Equivalence doesn't hold when taxes are distortionary.
2. There is slower convergence to the balanced growth steady state in the Cass-Koopmans model than in the Solow model.
3. If asset markets are efficient, then changes in stock prices are unpredictable.
4. Empirical evidence suggests that per capita income levels are converging across countries.
5. In the Diamond growth model, the competitive equilibrium is not Pareto efficient when the steady state interest rate exceeds the growth rate of output.

   The following questions are short answer. Be sure to explain and interpret your answer. Clarity and conciseness will be rewarded.

6. (15 points). Briefly compare and contrast the Ramsey and Mirrlees approaches to dynamic optimal taxation. How do their assumptions differ? How do their conclusions differ? Why?
7. (25 points). Suppose a large number of identical households have the following preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t})$$

   where

$$U(c_{1t}, c_{2t}) = \frac{(c_{1t}^{\alpha}, c_{2t}^{1-\alpha})^{1-\theta}}{1-\theta} \qquad 0 < \alpha < 1, \qquad \theta > 0$$

   There are two 'trees', one provides type-1 fruit (i.e., c_1), and the other provides type-2 fruit (i.e., c_2). Ownership shares in the trees are traded in competitive markets. Initially, each household owns one share of each tree. Let type-1 consumption be the numeraire.

   Assume the flow of 'dividends' from the type-1 tree follow the process

$$d_{1t+1}^{\alpha(1-\theta)} = d_{1t}^{\alpha(1-\theta)} \epsilon_{t+1}$$

   where $\epsilon \sim N(1, \sigma)$. For simplicity, assume dividends from the second tree are constant,

$$d_{2t} = \bar{d}_2 \quad \forall t$$

(a) Define a competitive equilibrium for this economy.

(b) What is the relative price of type-2 consumption (i.e., in units of type-1 goods)?

(c) Derive equilibrium share prices of both trees (in terms of the numberaire). Is the share price of the 'safe' tree (i.e., tree-2) necessarily less volatile than the share price of the 'risky' tree (i.e., tree-1)? Explain.

(d) Derive the price of a one-period ahead risk-free claim to type-1 consumption. Derive the price of a one-period ahead risk-free claim to type-2 consumption. Which is higher? Which is more volatile? Explain.

8. (25 points). Consider a simple model of 'divided government'. In particular, suppose there are now *two* planners. The first, a 'fiscal authority', picks taxes and spending (denoted by τ_t and g_t), and makes transfers to the representative household (with transfers denoted by v_t). The second planner is the 'efficiency authority', i.e., it takes taxes and transfers as given, and then decides on an optimal savings/consumption plan. That is, the second planner solves

$$\max_{\{c_t\},\{x_t\},\{k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{aligned} c_t + x_t + g_t &\leq (1-\tau_t)f(k_t) + v_t \\ k_{t+1} &\leq (1-\delta)k_t + x_t \\ k_0 &> 0 \quad \textit{given} \end{aligned}$$

Assume that the functions u and f have the 'usual' properties (i.e., Inada-type conditions that guarantee an interior solution).

(a) Assume that $0 < \tau_t = \bar{\tau} < 1$ (i.e., constant tax rate). Write down the second planner's Lagrangian, with λ_t denoting the multiplier on the resource constraint. Derive the first-order conditions that characterize the optimal paths of c_t, k_t, and λ_t. Now suppose that the first planner sets $v_t = \bar{\tau}f(k_t)$ and $g_t = \bar{g}$. (Remember, the second planner takes this as given). Prove that for $\bar{g}$ sufficiently small, there exists a unique interior steady state. Illustrate this steady state on a graph with (c, k) on the axes.

(b) Suppose the economy is at the steady state you just derived. Suddenly and unexpectedly the first planner announces that taxes will be permanently cut to $\tau' < \bar{\tau}$. Describe the new steady state. Use a graph to illustrate the economy's dynamic adjustment to the new steady state.

(c) Again suppose we are at a steady state, and now assume the first planner announces that capital taxes will be permanently eliminated T periods from now. Illustrate the economy's adjustment path. Do this using a graph with time on the horizontal axis and c_t and k_t on the vertical axis.

(d) How would your answers differ with a 'unified' government?

9. (25 points). Consider a simple discrete-time Real Business Cycle model where the representative agent has log utility and the capital stock completely depreciates after each period. Also assume the production function is Cobb-Douglas and population growth is zero. Thus, we can write the representative agent's objective function as:

$$\max_{C_t, L_t} E_t \sum_{j=0}^{\infty} \beta^j [\log C_{t+j} + b \log(1 - L_{t+j})]$$

and the economy-wide resource constraint as:

$$C_t + K_{t+1} = Y_t$$

where $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$ and the technology shock follows the autoregressive process, $\log Z_t = \rho \log Z_{t-1} + \varepsilon_t$, where ε_t is i.i.d.

(a) Write down the Bellman equation that characterizes the planner's problem.

(b) Solve for the optimal consumption and labor supply policy functions in terms of the unknown value function.

(c) Guess a log-linear functional form for the value function and solve for the relevant unknown coefficients. (Hint: To get the policy functions you don't actually have to solve for all of the unknown coefficients).

(d) Substitute the value function coefficients into the policy functions and interpret the resulting decision rules for consumption and labor supply. Are they consistent with observed business cycle facts?

10. (25 points). Consider an economy with aggregate production function

$$Y_t = A K_t^{1-\alpha} L_t^{\alpha}$$

All markets are competitive, labor supply is normalized to 1, capital fully depreciates after one period of use, and the government imposes a linear tax on capital income at rate τ, and uses the proceeds to fund government consumption (which either yields no utility, or enters additively into preferences).

Consider the following two preference/demographic specifications:

(a) All agents are infinitely-lived, with preferences

$$\sum_{t=0}^{\infty} \beta^t \ln c_t$$

(b) There are overlapping generations of 2-period lived agents, who can only work during the first period of their lives, and must finance second period consumption out of capital income earned from their first-period savings. The preferences of generation-t are given by

$$\ln c_t^y + \beta \ln c_t^o$$

(a) Characterize the equilibria of these two economies, and show that in the first economy taxation reduces output, while in the second, it does not.

(b) Explain intuitively why the response of the two economies differs.