SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 — Prof. Kasa
Macroeconomic Theory — Fall 2009

FINAL EXAM - December 9

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (5 points each).

1. According to the Lucas (1978) asset pricing model, stocks that are more highly correlated with consumption have higher average returns.
2. According to the Lucas (1988) growth model, an economy should recover more quickly from a plague that wipes out half the population, than from a war, which destroys half the physical capital stock (while keeping the stock of human capital relatively constant).
3. According to Romer's (1990) R& D growth model, countries that are larger and more populous will grow faster.

The following questions are short answer. Be sure to explain and interpret your answer. Clarity and conciseness will be rewarded.

4. (15 points). **Dynamic Responses to Alternative Tax Policies**. Consider a discrete-time Cass-Koopmans model with a government that must finance an exogenous stream of expenditures. Assume government purchases do not affect the production function or marginal utility of private consumption. Households maximize the utility function (note there is no uncertainty)

$$U_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

subject to the budget constraint

$$\sum_{t=0}^{\infty} \{q_t(1+\tau_{ct})c_t + q_t[k_{t+1} - (1-\delta)k_t]\} \leq \sum_{t=0}^{\infty} \{r_t(1-\tau_{kt})k_t + w_t n_t - q_t \tau_{ht}\}$$

where q_t is the time-0 pretax price of a unit of consumption (or investment) at time-t, τ_{ct} and τ_{kt}, are time-t taxes on consumption and capital income, and τ_{ht} denotes a lump-sum tax. Note that the household supplies labor inelastically.

(a) Derive the household's after-tax Euler equation.

(b) Write down the aggregate resource constraint. Use it along with the answer to part (a) to derive two difference equations that characterize the equilibrium time paths of c_t and k_t. (Note: You will need to use the firm's profit maximization conditions to substitute out the rental rate of capital, r_t, in terms of k_t.) Use a phase diagram to illustrate the dynamic evolution of the economy.

(c) Suppose the economy is in a steady state, and the government suddenly announces that government spending will permanently rise 10 periods from now. Compare and contrast how the economy responds to this announcement under three alternative financing schemes: (*i*) a permanent rise in lump-sum taxes beginning 10 periods from now, (*ii*) a permanent rise in consumption taxes beginning 10 periods from now, and (*iii*) a permanent rise in capital taxes beginning 10 periods from now. Focus on the responses of consumption, the capital stock, and the after-tax rate of return on capital. (Note: Do not attempt to calculate explicit time paths. Simply sketch the time paths of the responses.)

5. (20 points). **Disasters and the Equity Premium**. Consider a discrete-state version of the Lucas asset pricing model, with just two states. State 1 is 'normal times', and State 2 is a 'disaster'. In state 1 per capita consumption/dividends grow at a 3% annual rate. During a disaster per capita consumption falls 22% (i.e, $c_{t+1}/c_t = .78$). Suppose we know that average annual per capita consumption growth is 2% and the equity premium is 6%.

   (a) Using the available data, what must be the long-run average probability of being in the disaster state?

   (b) Suppose households have preferences

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma}$$

   where $\beta = .99$ and $\gamma = 5$. Use your answer to part (a) to calculate excess returns during normal times and during disasters. (Hint: Remember the 'excess return' is the difference between the stock market return and the risk-free rate. Let R_1^e be the market excess return during normal times and R_2^e be the market excess return during a disaster. Use the household's Euler equation (for excess returns) along with the constraint that the average equity premium is 6% to derive two (linear) equations in the unknows R_1^e and R_2^e). How much does the market crash during a disaster? How often do crashes occur (on average)?

   (c) What is the average risk-free rate in this economy?

6. (25 points). **Nonrenewable Resources and Growth**. Consider a standard Solow-style growth model with three inputs: Capital (K), Labor (L), and Energy (E). The flow of output is given by

$$Y(t) = A(t)K(t)^{\alpha}E(t)^{\gamma}L(t)^{1-\alpha-\gamma} \tag{1}$$

   where $g_A = \dot{A}/A$ is the exogenous rate of technological progress, and $n = \dot{L}/L$ is the exogenous rate of population (labor force) growth. Suppose there is a fixed stock of nonrenewable resources, R, and that energy use depletes this stock, so that

$$\dot{R} = -E$$

   For simplicity, suppose that a constant fraction, $0 < \eta < 1$, of the remaining stock is used at each point of time, so that $E = \eta R$.

   (a) Letting R_0 denote the initial stock of resources, derive an expression for the remaining stock at each moment of time. (Hint: Solve a simple linear differential equation). Use this to derive an expression for energy use at each moment of time.

(b) For simplicity, suppose that the saving rate is constant, so that $\dot{K} = sY - \delta K$. Prove that the economy converges to a balanced growth path, and that K/Y is constant along this path. Using equation (1) and the fact that K/Y is constant, calculate the growth rate of per capita output on the balanced growth path as a function of g_A, η, and n.

(c) Suppose that initially the economy uses energy at the rate $\eta = .04$, so that the half-life of the resource stock is $\ln(2)/.04 \approx 17$ years. Also suppose that capital's share of GDP is 20% ($\alpha = .20$), and that energy's share is 10% ($\gamma = .10$). Finally, suppose that as part of an international agreement the country must permanently cut its energy use to $\eta = .01$ (so now the half-life of the remaining stock is 69 years). Calculate the effects of this policy change on output. Be sure to distinguish between the immediate/instantaneous effect, and the effect on the long-run growth rate. Is there a trade-off between the short-run and long-run effects of this policy?

7. (25 points). **New Technologies and the Stock Market**. Consider an economy comprised of a large number of households, each with a utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad 0 < \beta < 1$$

where u is twice differentiable, increasing, and strictly concave. (If necessary, assume it satisfies the Inada conditions). Initially, there is only one technology. Label it technology 1. This technology produces a flow of (nondurable) output according to

$$y_{1t} = z_1 F(a_{1t}, n_{1t})$$

where a_{1t} is land used at time-t and n_{1t} is labor input at time-t. Assume F is twice differentiable, homogeneous of degree one, concave, and features strictly decreasing marginal products of both factors. Suppose that the marginal product of labor satisfies the Inada condition,

$$\lim_{n \to 0} z_1 F_n(a, n) = \infty$$

Suppose that land and labor are in fixed supply, and normalize their aggregate quantities to one. Suppose agents are free to trade all the assets they want (e.g., bonds of all maturities and ownership shares in firms). Note that with constant returns to scale we can think of there being a single firm. Assume the firm owns all the land, so that firm shares represent claims to the flow of land rents.

Suppose that at time 0 there is an announcement that at time $T > 0$ a new technology will become available. This technology will be able to produce the same good according to the production function

$$y_{2t} = z_{2t} n_{2t}$$

where z_{2t} is a bounded stochastic process that satisfies the property

$$z_{2t} > z_1 F_n(1, 1) \quad \forall t$$

and where n_{2t} is labor employed in the new industry. Note that the announcement only specifies the <u>distribution</u> of z_{2t} for $t \geq T$, not the actual realizations. (The actual values of z_{2t} do not become known until time-t).

(a) Calculate the equilibrium level of output, land rental rate, wage rate, and interest rate before the announcement. Given the land rental rate, what is the stock market value of the firm.

(b) Describe the effect of the announcement on both short-term bonds (i.e., with maturities less than T) and long-term bonds (with maturities greater than T).

(c) Describe the effect of the announcement on expected wages, $E_0 w_t$ for $t = 0$ and $t = T + k, k > 0$. (Workers are free to work in whichever industry pays the higher wage).

(d) Describe the response of the stock market to the announcement (i.e., what happens to the aggregate price of land).

(e) Does the answer to part (d) sound a note of caution about using the stock market as a signal of the future health of the economy?