An Alternative Approach to Search Frictions

Ricardo Lagos

London School of Economics and New York University

This paper illustrates an alternative approach to modeling search frictions. Frictions are not assumed to exist, but are shown to arise endogenously as a distinctive feature of the set of equilibria that correspond to a particular range of parameter values. The model's spatial structure and the agents' moving decisions are explicitly spelled out, allowing the number of contacts that occur to depend on the way agents choose to locate themselves. An aggregate matching function is shown to exist, and its behavior with respect to changes in parameters such as distances between locations, the agents' payoffs, and the sizes of the populations of searchers on each side of the market is completely characterized.

I. Introduction

A distinctive feature of the search approach is that trades occur bilaterally between agents rather than between an agent and "the market" as in the Walrasian model. This feature makes the process that determines how agents meet a key building block of any equilibrium model of search. The literature typically proceeds by assuming that agents possess limited information, so time and resources have to be spent seeking trading partners. The information structure adopted prevents some potential traders on one side of the market (say buyers) from contacting potential traders on the other side (say sellers), not allowing the market to clear, in the sense that there are both buyers who want

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to buy and sellers who want to sell who are unable to meet. In other words, "frictions" are built in as a feature of the environment.[1]

The "matching function approach" is a way of introducing frictions that has been widely used in labor market applications.[2] This approach proceeds by directly assuming the existence of an aggregate object—the matching function—that gives the number of contacts that occur at any moment in time as a function of the numbers of searchers on both sides of the market. The information imperfections or other features of the environment that must underlie such a function are not made explicit; rather, it is assumed that "their interaction gives rise to a well-behaved function of a small number of variables" (Pissarides 1990, pp. 3–4). "Well-behaved" typically means continuous, differentiable, strictly component-wise increasing, less than the number of searchers on each side of the market, and often also homogeneous of degree one. The thing to note is that the bulk of the search and matching literature adopts a matching function, and in doing so it assumes that meetings are ruled by some exogenous process.

In fact, since adopting a matching function amounts to assuming an exogenous *aggregate* meeting process, it is often unclear what kinds of individual search behavior are consistent with the aggregate structure assumed. All that is known is that matching functions can arise in some environments provided that agents engage in random search.[3] This means that a model in which aggregate meetings are assumed to be ruled by a matching function may be regarded as the reduced form of a model in which individuals are searching randomly. However, for many applications it may be more natural to think that agents possess at least some information that allows them to direct their search in ways that may not be consistent with the random search assumption. Thus the question that arises is whether matching functions necessarily represent a world in which searchers are randomly colliding as particles in space, or whether they can be thought of as reduced forms of environments

[1] At least since Beveridge (1945, p. 409), labor unemployment has been called "frictional" when it coexists with "an unsatisfied demand for labor somewhere."

[2] Bowden (1980), Diamond (1982), Blanchard and Diamond (1989), Pissarides (1990), Aghion and Howitt (1994), Bertola and Caballero (1994), and Mortensen and Pissarides (1994) are some examples. For an early application of a job-matching function, see Phelps (1968).

[3] In the labor literature, for instance, a common story is that workers know where vacancies are but do not know which particular vacancies other workers will visit, allowing for the possibility that some workers are unable to fill vacancies because they were "second in line." This structure often reduces the aggregate meeting process to an "urn-ball" process. Hall (1979), Pissarides (1979), Peters (1991), and Blanchard and Diamond (1994) all derive the number of contacts that will take place in some time interval as a function of the numbers of vacancies and searching workers that is immediately implied by this process. For more on the urn-ball type of structure of these models, see Acemoglu and Shimer (1999).

in which the search is conducted in a more sophisticated way. Here I address this question by investigating the microeconomic foundations of the matching function approach in the absence of the random search assumption.

The treatment of frictions adopted here is in many ways different from the one traditionally followed by the equilibrium search literature. The main difference is perhaps that meeting frictions are not assumed to exist but are shown to arise endogenously as a distinctive feature of the set of equilibria that correspond to a particular range of parameter values. To let agents direct their search and avoid building frictions into the environment, no information imperfections are assumed. In particular, the usual assumption of "nobody knows where anything is" that forces agents to search randomly and guarantees that some potential traders will be unable to meet is suppressed from the analysis. In addition, the model's spatial structure and the agents' moving decisions are spelled out, allowing the number of contacts that occur to depend on the way agents choose to locate themselves.

Being explicit about the meeting process requires making some very specific modeling choices. Here these choices were made so that the model resembles a dynamic market for taxicab rides in which taxicabs seek potential passengers on a grid. At least three features of this market make it an appealing starting point to think about meeting frictions explicitly. First, it is possibly the simplest search environment one can think of: all taxicabs do is try to position themselves in a location in which they can contact a passenger. Second, it is a market in which meeting frictions are present and quite visible: vacant taxicabs normally spend long periods of time waiting for passengers in some parts of the city (notably the airport);[4] at the same time, passengers often wait for taxicabs in others (usually "downtown"). And finally, the price in this industry is typically regulated, which allows the analysis to focus on the role of meeting probabilities and market tightness, namely the key equilibrating variables of any search model.[5]

In this context, some heterogeneity among locations is shown to be necessary—although not sufficient—for an equilibrium to exhibit frictions. In fact, the conditions under which frictions arise depend crucially

[4] La Croix, Mak, and Miklius (1992, p. 151, table 1) report the "typical" waiting times of taxicab drivers in five major U.S. airports to be between two and five hours.

[5] A comparison with standard equilibrium search models (such as that in, e.g., Pissarides [1990]) is in order here. In most search models, prices are determined ex post (i.e., after agents meet) by some sort of sharing rule. Although the price is typically endogenous as it reacts to changes in the environment (such as variations in the relative numbers of buyers and sellers), it plays no allocational role in the sense that the assumed imperfection in the information structure prevents agents from being able to condition their search on any given price. In a taxicab market, on the other hand, prices are typically fixed, but they play a key allocational role.

on the total numbers of searchers on each side of the market as well as on the heterogeneity among locations. From an aggregate perspective, the equilibria with frictions (in the sense that not all possible bilateral meetings occur) look just like the outcomes obtained from standard equilibrium search models in which meeting frictions result from the fact that agents are assumed uninformed.

A function that expresses the total number of meetings in terms of the aggregate stocks of searchers on both sides of the market is shown to exist. This endogenous matching function is derived, and its behavior with respect to changes in parameters such as distances between locations, the agents' payoffs, and the sizes of the populations of searchers is completely characterized. Since agents can direct their search, changes in parameters affect their search strategies, altering the shape of the matching function. This suggests that the results of policy experiments based on models that assume an exogenous meeting process are likely to be misleading if the random search assumption is not a good characterization of the agents' underlying search behavior. If agents are able to direct their search, then the matching function is an equilibrium object and is sensitive to policy.

The rest of the paper is organized as follows. Section II describes the environment. Section III introduces the notion of equilibrium. Section IV characterizes the full set of equilibria for all possible parameter configurations. The endogenous aggregate matching function is derived in Section V, and its properties are discussed in Section VI. Section VII shows how the matching function reacts to policy experiments and illustrates the potential inconsistencies involved in making predictions based on models built around an exogenous meeting technology. Section VIII concludes with a summary of the main results. The Appendix contains proofs of propositions 1 and 2.

II. Environment

Time is discrete and continues forever. A city consists of $n \geq 2$ locations across which the populations of people and taxicabs may position themselves. There is a continuum of people with size normalized to unity and a continuum of cabs with measure v. The numbers of people and cabs in location i are denoted l_i and v_i, respectively.

People's wishes to move between locations are taken to be exogenously given by a Markov chain. Specifically, it is assumed that in each period an agent will wish to remain at the current location with probability $1 - u \in (0, 1)$. For simplicity, I use a large-numbers approximation and let the number of agents wishing to remain in a location be nonstochastic. This means that there are u "movers" in the whole city.[6] The probability that a passenger in i wishes to move to j is given by ua_{ij}, with $\sum_{j\neq i}^{n} a_{ij} = 1$ and $a_{ij} \in (0, 1)$. Therefore, there are $u_i = ul_i$ movers in location i, $a_{ij}u_i$ of whom wish to go to location j.

People cannot walk to their desired destination: they must take a cab to get there. Cabs cannot drive more than one passenger per trip and, when vacant, are free to choose the location in which they will try to find a passenger. Cabs are unable to meet passengers in distant locations: contacts occur only among cabs and passengers in the same location. Within each location, a cab (passenger) may not find a passenger (cab) only if there are not enough passengers (cabs) in that location. In other words, letting m_i be the number of cab-passenger meetings that occur in location i, we have[7]

$$m_i = \min\{u_i, v_i\}. \tag{1}$$

When the m_i contacts are random and θ_i is defined as v_i/u_i, a cab in location i will find a passenger with probability $p_i = \min\{1/\theta_i, 1\}$, whereas a mover in i will find a cab with probability $p_i\theta_i$.

A cab that was unable to contact a passenger in a given period can choose to go to a new location in which, in the following period, it may again try to find a passenger. Hence the value of being unmatched in location i at the end of a period is just the discounted value of being at the best location at the beginning of the next period. With modulo n arithmetic, this value can be written as

$$U_i = \beta \max\{V_i, V_{i+1}, \ldots, V_{i+n-1}\}, \quad \text{for } i = 1, \ldots, n, \tag{2}$$

where $\beta \in (0, 1)$ is the discount factor and V_i the value of being in i before contacts take place.[8] When driving a passenger from i to j, cabs charge a "flag drop" rate $b \geq 0$ and a rate $\pi > 0$ per unit distance, and hence a cab's profit from driving somebody from i to j is $\pi_{ij} = b + \pi\delta_{ij}$, with δ_{ij} being the distance between locations i and j. Without loss of generality, $\delta_{ij} = \delta_{ji}$. Since all trips last a period, the value of giving a

[6] In equilibrium some agents wishing to move may be unable to do so. The term "mover" refers to a person who *wants* to move, regardless of the ability to do so. The assumption that the probability of wishing to stay at the present location is the same across locations is made only so that the citywide number of movers remains independent of the distribution of agents across locations.

[7] Notice that it is implicitly assumed that contacts occur only between cabs and movers. This amounts to assuming that cabs have the ability to identify movers.

[8] It may be convenient to think of the timing of events as follows. At some point in every period there is a "meeting session" in which all the period's meetings take place. A cab that was unable to contact a passenger in location i must wait until the next meeting session (in the following period) for another chance to find a passenger. However, such a cab can choose to relocate before the next meeting session. That is, by driving empty, the cab is able to participate in next period's meeting session at location j. The expression "beginning (end) of a period" means "before (after) the period's contacts have occurred." This formulation assumes no moving costs.

ride from i to j is given by the profit from the trip between i and j plus the value of being located at j at the beginning of the next period:

$$V_{ij} = \pi_{ij} + \beta V_j. \tag{3}$$

Finally, the value of being located in i before a period's meetings occur is

$$V_i = p_i \sum_{j \neq i}^{n} a_{ij} \max\{V_{ij}, U_i\} + (1 - p_i)U_i. \tag{4}$$

III. Steady-State Equilibrium

Since a mover can leave a location only if able to get a cab ride, the number of people who move to their desired destination depends on the number of cabs available at their original location. If u_i persons want to move out of i, (only) $p_i\theta_i u_i$ of them will be able to find a cab to do so. When $\theta_i \geq 1$, there are at least as many cabs as movers in i, and all those passengers who want to leave location i are able to do so. Conversely, when $\theta_i < 1$, there are fewer cabs than movers and some of the movers end up rationed. In this case, only a (randomly chosen) fraction $p_i\theta_i$ of the $a_{ij}u_i$ people wanting to go from i to j are able to find a cab to get there. When $m_{ij} = a_{ij}m_i$ denotes the flow of matches from i to j, stationarity of the distribution of people and cabs across locations obtains if

$$\sum_{j \neq i}^{n} m_{ij} = \sum_{j \neq i}^{n} m_{ji}, \quad \text{for } i = 1, \ldots, n-1.$$

In addition, we require that a cab's expected discounted payoff at the beginning of each period is equal across locations:

$$V_1 = V_2 = \cdots = V_n. \tag{5}$$

This no-arbitrage condition ensures that cabs will have no profitable relocation in equilibrium.[9] Condition (5) can be combined with equations (2), (3), and (4) to show that the flow value of being in i at the beginning of a period is given by $(1 - \beta)V_i = p_i\pi_i$, where $\pi_i = \sum_{j \neq i}^{n} a_{ij}\pi_{ij}$ is a cab's expected profit conditional on having contacted a passenger in location i.

A steady-state equilibrium is a time-invariant distribution of cabs and movers across locations such that, given this distribution, cabs maximize

[9] Note that in equilibrium a cab will never find it optimal to turn a passenger down. To see this, let $V = V_i$ for all i, and notice that $V_{ij} = \pi_{ij} + \beta V > U_i = \beta V$ for all i.

profits by optimally choosing where to locate themselves. Formally, a steady-state equilibrium is an allocation $\{(u_i, v_i)\}_{i=1}^{n}$ such that

$$p_i\pi_i = p_n\pi_n, \quad i = 1, \dots, n-1, \tag{E1}$$

$$\sum_{j\neq i}^{n} m_{ij} = \sum_{j\neq i}^{n} m_{ji}, \quad i = 1, \dots, n-1, \tag{E2}$$

and

$$\sum_{i=1}^{n} u_i = u, \quad \sum_{i=1}^{n} v_i = v. \tag{E3}$$

By ensuring that they have no profitable way to relocate at the beginning of each period, condition (E1) guarantees that cabs are maximizing expected discounted profits. The distribution of cabs and people across locations remains constant through time when condition (E2) holds. Condition (E3) requires that the equilibrium distribution be consistent with the total numbers of people and cabs in the city.

IV. Characterization of Equilibria

In the search literature, "frictions" are certain features of the environment that prevent some bilateral meetings from taking place. Within the present framework, no feature of the environment rules out the possibility that all bilateral meetings occur. In particular, equation (1) guarantees that the failure of some cabs and passengers to contact each other can occur only as a result of the way in which cabs choose to locate. So in this sense "frictions" are a property of the equilibrium allocation and are not necessarily implied by the type of environment assumed.[10] An equilibrium will be said to exhibit frictions if the corresponding allocation simultaneously exhibits vacant cabs and unserved passengers. So with m denoting the aggregate (i.e., citywide) number of meetings, an equilibrium exhibits frictions if $m < \min\{u, v\}$ and is frictionless if $m = \min\{u, v\}$, that is, if all possible bilateral contacts take place. This is the operational definition of frictions that will be adopted hereafter. In order to find the conditions under which frictions arise, it is convenient to consider two cases. In the first case, all locations look

[10] In fact, notice that the notion of equilibrium adopted in Sec. III rules out the two sources of frictions most commonly used in the search literature. When choosing locations, cabs know the distribution of passengers across locations, so no meetings fail to occur because "nobody knows where anything is." And since in equilibrium cabs must have no profitable relocation, there are no frictions due to "coordination problems" as in Hall (1979), Pissarides (1979), Montgomery (1991), Peters (1991), Blanchard and Diamond (1994), and Acemoglu and Shimer (1999).

identical from a cab's perspective, whereas in the second, some locations are "better" than others.

Let $\Pi = \max\{\pi_1, \ldots, \pi_n\} - \min\{\pi_1, \ldots, \pi_n\}$, and suppose that people's wishes to move and distances between locations are such that $\Pi = 0$. That is, a cab's expected profit conditional on having contacted a passenger is the same in any location. Cabs in this type of city maximize expected profits by maximizing the probability of picking up a passenger, so in equilibrium contact rates must be equalized across locations. Indeed, if $\Pi = 0$, condition (E1) becomes

$$\min\left\{\frac{1}{\theta_i}, 1\right\} = \min\left\{\frac{1}{\theta_n}, 1\right\}, \quad i = 1, \ldots, n-1. \tag{6}$$

Hence in equilibrium, either all locations exhibit an excess supply of cabs or none of them does.[11] Depending on the value of the aggregate market tightness v/u, which we denote θ, there are potentially three types of equilibria when $\Pi = 0$: one with excess supply in all locations, another with market clearing in all locations, and a third in which there is excess demand in at least one location but none of the others exhibit excess supply. These results are summarized in the following proposition.

PROPOSITION 1. Assume $\Pi = 0$. (*a*) If $\theta > 1$, then there exists a unique equilibrium: all locations exhibit an excess supply of cabs. (*b*) If $\theta = 1$, then there exists a unique equilibrium: there is market clearing in all locations. (*c*) If $\theta < 1$, then there is a continuum of equilibria in which at least one location exhibits excess demand and none of the others exhibit excess supply.

Proof. See the Appendix.

In any equilibrium in which no location exhibits excess demand (parts *a* and *b* of proposition 1), all movers reach their desired destinations every period. Hence the steady-state distribution of movers across locations is given by the unique invariant distribution of the Markov matrix that rules passengers' wishes to move, which I denote μ. Therefore, $\mu_i u$ is the (unconstrained) steady-state number of movers in location *i*. Given this distribution, there is a unique way for cabs to position themselves so that (6) holds. The equilibrium allocations for this case are reported in column 1 of table 1.

Given that $\Pi = 0$, a cab is indifferent between looking for a passenger in any location in an equilibrium with excess demand in at least one location and no excess supply anywhere (part *c* of proposition 1). The equilibrium distribution of cabs is uniquely determined by condition (E2) and given by μv. The distribution of movers, on the other hand, is indeterminate, as can be seen from the first entry in column 2 of

[11] Hereafter, "excess supply (demand)" is used to mean "excess supply (demand) *of cabs*."

TABLE 1
EQUILIBRIUM ALLOCATIONS WHEN $\Pi = 0$

	$\theta \geq 1$ (1)	$\theta < 1$ (2)
u_i	$\mu_i u$	$\mu_i v + \epsilon_i(u - v)$
v_i	$\mu_i v$	$\mu_i v$
m_i	$\mu_i u$	$\mu_i v$

table 1, where $\epsilon = (\epsilon_1, \ldots, \epsilon_n)$ denotes a vector in the nth-dimensional unit simplex.[12]

Notice that when $\theta > 1$, each market i has an excess supply of cabs equal to $\mu_i(v - u)$. As θ falls, the excess supply shrinks in all markets, and they all clear when $\theta = 1$. As the aggregate number of movers comes to exceed the aggregate number of cabs, we enter a parameter range with multiple equilibria all with the property that no market is in excess supply whereas at least one exhibits excess demand. Finally, note how the number of meetings varies continuously from u to v as the aggregate degree of market tightness θ crosses one from above.

Now suppose the parameters are such that $\Pi > 0$ and label locations so that larger subindexes correspond to locations with smaller conditional expected profit:

$$\pi_1 \geq \cdots \geq \pi_{k-1} > \pi_k = \cdots = \pi_n.$$

Since there could be more than one location with the smallest level of conditional expected profit in the city, the notation allows for $n - k + 1$ such locations, with $2 \leq k \leq n$. This "ranking" together with condition (E1) implies

$$\min\left\{\frac{1}{\theta_1}, 1\right\} \leq \cdots \leq \min\left\{\frac{1}{\theta_{k-1}}, 1\right\}$$
$$< \min\left\{\frac{1}{\theta_k}, 1\right\} = \cdots = \min\left\{\frac{1}{\theta_n}, 1\right\}.$$

So in equilibrium there must be excess supply in locations $1, \ldots, k - 1$, whereas either all locations $k, \ldots, n$ have excess supply or none of them does. Hence, once again, there can potentially be three types of equilibria: locations $k, \ldots, n$ are in excess supply in the first and exhibit

[12] That is, $\sum_{i=1}^{n} \epsilon_i = 1$ and $\epsilon_i \geq 0$, for $i = 1, \ldots, n$. The indeterminacy arises from the fact that when there is more than one location with excess demand, the equilibrium conditions do not pin down the length of the queue of unserved passengers in these locations. Condition (E1) is not affected by the actual number of unserved passengers in any location with excess demand since cabs already meet passengers with certainty in those locations. Condition (E2) is not affected either since flows are always ruled by the short side of the market.

TABLE 2
Equilibrium Allocations When $\Pi > 0$

	$\theta \geq 1/\phi$: $i = 1, \ldots, n$ (1)	$\theta < 1/\phi$: $i = 1, \ldots, k-1$ (2)	$\theta < 1/\phi$: $i = k, \ldots, n$ (3)
u_i	$\mu_i u$	$\mu_i \phi v$	$\mu_i \phi v + \epsilon_j (u - \phi v)$
v_i	$\mu_i \phi_i v$	$\mu_i \phi_i v$	$\mu_i \phi v$
m_i	$\mu_i u$	$\mu_i \phi v$	$\mu_i \phi v$

market clearing in the second and excess demand in the third. Locations $1, \ldots, k-1$ are in excess supply in all three types. The following proposition summarizes the full set of equilibria corresponding to different values of the aggregate market tightness θ when parameters are such that $\Pi > 0$.

Proposition 2. Let

$$\phi_j = \frac{\pi_j}{\sum_{i=1}^{n} \mu_i \pi_i},$$

$\phi = \min\{\phi_1, \ldots, \phi_n\}$, and assume $\Pi > 0$. (*a*) If $\theta > 1/\phi$, then there is a unique equilibrium: all locations exhibit excess supply. (*b*) If $\theta = 1/\phi$, then there is a unique equilibrium: locations $1, \ldots, k-1$ exhibit excess supply whereas the market clears in locations $k, \ldots, n$. (*c*) If $\theta < 1/\phi$ and $k < n$, then there is a continuum of equilibria with excess supply only in locations $1, \ldots, k-1$ and excess demand in at least one of the remaining $n - k + 1$ locations. If $k = n$, then the equilibrium is unique: location n exhibits excess demand but all others are in excess supply.

Proof. See the Appendix.

With heterogeneous locations, the equilibrium distribution of cabs reflects the relative attractiveness of each location. When table 1 and table 2 are compared, it is clear that the distribution of cabs in table 2 may be obtained from table 1 by multiplying the number of cabs in each location i by ϕ_i, which measures the relative attractiveness of a location in terms of its conditional expected profit. Since in the equilibria described in parts a and b of proposition 2 no location exhibits excess demand, the equilibrium distribution of movers is the unconstrained steady-state distribution μu. The equilibrium allocations corresponding to parts a and b of proposition 2 are reported in column 1 of table 2.

The equilibrium allocations for locations $1, \ldots, k-1$ and $k, \ldots, n$ corresponding to part c of proposition 2 are reported in columns 2 and 3, respectively, of table 2. The fraction of cabs in each location is equal to the unconstrained steady-state fraction of movers in that location, μ_i, adjusted by the relative attractiveness of the location, as measured by

ϕ_i. Since locations $k, \ldots, n$ are now in excess demand, each period some of the movers in those locations are unable to find a cab to reach their desired destinations. Consequently, the number of passengers flowing out of locations $k, \ldots, n$ is smaller than in the unconstrained steady state, implying a steady-state equilibrium distribution with more movers in locations $k, \ldots, n$ and fewer in the first $k - 1$ locations relative to the unconstrained invariant distribution μu.

Finally, notice how, starting with an aggregate degree of market tightness θ that lies above $1/\phi$, the pattern of excess supply in all locations changes as we vary θ. As the aggregate market tightness decreases, the excess supply in all locations falls. Locations $k, \ldots, n$, for instance, always exhibit the smallest extent of excess supply as measured by θ_i. In fact, they exhibit excess supply when $\theta > 1/\phi$ and move toward market clearing as $\theta \rightarrow 1/\phi$, and the markets in those locations clear precisely when $\theta = 1/\phi$. As θ falls below $1/\phi$, at least one of the last $n - k + 1$ locations moves into excess demand whereas only locations $1, \ldots, k - 1$ remain with excess supply. The last row of table 2 reveals that as θ crosses $1/\phi$ from above, the number of meetings varies continuously from u to ϕv.

Propositions 1 and 2 give a set of conditions that are necessary and sufficient for frictions to arise in equilibrium. Proposition 1 asserts that if all locations are identical in the conditional expected profit sense, then all possible bilateral trades occur in equilibrium regardless of how tight the aggregate market is. Proposition 2 states that although having identical locations is sufficient to guarantee no frictions, it is not necessary: for any given degree of heterogeneity among locations (as measured by ϕ, the relative attractiveness of the worst location), there is a level of market tightness $1/\phi > 1$ such that frictions arise if and only if $\theta < 1/\phi$. In particular, notice that if $\theta = 1$ (i.e., when everyone could potentially find a match), unserved passengers and vacant cabs coexist in equilibrium.

V. The Aggregate Matching Function

This section explores whether—as is often assumed in the equilibrium search literature—the aggregate number of meetings can be expressed as a "well-behaved" function of the aggregate stocks of searchers on both sides of the market.

The equilibrium distributions of cabs and movers corresponding to any geographical configuration (represented by the δ_{ij}'s), any set of moving preferences (represented by the a_{ij}'s), and any aggregate market tightness θ were characterized in Section IV. Given these distributions, the equilibrium number of meetings that take place in each location is reported in the last row of tables 1 and 2. I now show that an aggregate matching function can be generated by aggregating the equilibrium number of meetings across locations.[13]

PROPOSITION 3. There always exists a unique aggregate matching function $M(u, v)$. Moreover, $M(u, v) = \min\{u, \phi v\}$.

Proof. Assume $\Pi = 0$. Then the aggregate number of meetings is $\sum_{i=1}^{n} u_i = u$ if $\theta \geq 1$ and $\sum_{i=1}^{n} v_i = v$ if $\theta < 1$. Thus the aggregate matching function is $\min\{u, v\}$ when parameters are such that $\Pi = 0$. Now suppose $\Pi > 0$. Then the aggregate number of meetings is $\sum_{i=1}^{n} u_i = u$ if $\theta \geq 1/\phi$ and

$$\sum_{i=1}^{k-1} u_i + \sum_{i=k}^{n} v_i = \phi v$$

if $\theta < 1/\phi$. Hence the aggregate number of meetings can be expressed as a function $M(u, v) = \min\{u, \phi v\}$ when parameters are such that $\Pi > 0$. Notice that $\phi = 1$ if $\Pi = 0$, so $\min\{u, \phi v\}$ is the aggregate matching function for any Π. Q.E.D.

In line with the operational definition of frictions introduced in Section IV, we can verify that there are no frictions if $\Pi = 0$ since $M(u, v) = \min\{u, v\}$ in this case. Alternatively, if $\Pi > 0$, then the equilibrium exhibits frictions whenever $M(u, v) < \min\{u, v\}$, namely whenever $\theta < 1/\phi$.

VI. Properties of the Aggregate Matching Function

As mentioned earlier, most equilibrium search models are built around an exogenous aggregate matching function with some convenient properties. Since model predictions critically hinge on these properties, a significant amount of effort has been devoted to establishing their empirical validity.[14] From a theoretical standpoint, studying the foundations

[13] Using aggregation to obtain an expression for the total number of trades as a function of the numbers of traders on each side of the market is not a novel idea. There is a literature that constructs an "aggregate transaction curve" formally equivalent to a matching function by aggregating across a large number of micro markets. This approach goes back at least to Hansen (1970) and has been used more recently by a number of economists working on disequilibrium macroeconomic models (see Lambert [1988] and the references therein). Although similar in spirit, that approach differs from the one in this paper in that the distribution of traders across markets is exogenous in disequilibrium models but endogenously determined here.

[14] For the theoretical implications of different assumptions on returns to matching, e.g., see Diamond (1982), who shows that a matching technology with increasing returns to scale makes multiple Pareto-rankable equilibria possible, or Pissarides (1990, pp. 76–80). Using U.S. manufacturing labor market data, Blanchard and Diamond (1989, pp. 29–30) conclude that "the evidence suggests constant or mildly increasing returns to scale in matching" but that "some downward bias may remain [in their estimates] so that the proponents of strongly increasing returns may still have hope." For more on the empirics of matching functions, see Coles and Smith (1996) and the references therein.

of the relevant meeting technology seems a good way of understanding how fundamentals interact in determining the shape and other key properties of matching functions.

First of all, notice that the matching function derived in the previous section exhibits constant returns to scale. The number of meetings in location i is $\mu_i u$ if $\theta \geq 1/\phi$ and $\mu_i \phi v$ if $\theta < 1/\phi$. Hence for any parameterization, scaling u and v by the same factor just scales the number of meetings in each location and hence the aggregate number of meetings by the same factor.

Perhaps a more striking feature of the meeting technology derived in Section V is the fact that it allows for no substitutability between u and v. When $\theta > 1/\phi$, the aggregate number of meetings responds only to changes in the stock of movers. Additional cabs just increase the excess supply of cabs in each location and hence have no effect on the aggregate number of meetings. On the other hand, an extra mover generates an extra meeting because in this region of the parameter space an equilibrium necessarily exhibits an excess supply of cabs in all locations. Hence, wherever they may end up located in the steady-state equilibrium, any additional number of movers generates that same number of additional meetings.

Alternatively, when $\theta < 1/\phi$, increasing the citywide number of movers just increases the steady-state number of movers in the location(s) with an excess demand for cabs and hence has no effect on the steady-state number of contacts.[15] On the other hand, additional cabs generate additional meetings, but at rate ϕ. If all the additional cabs placed themselves in the locations with excess demand, then each additional cab would generate an additional meeting. But in equilibrium the additional cabs spread themselves across all three locations (the extra number of cabs that go to each location is always proportional to the location's relative attractiveness indexed by $\mu_i \phi_i$), and since some end up in locations with excess supply, the increase in the number of contacts is smaller than the increase in the number of cabs. With more cabs in the locations with excess demand, some of the movers who in each period were previously unable to reach the locations with excess supply will now be able to do so. Thus even if the aggregate number of movers has not changed, their steady-state distribution across locations is affected by the increase in the number of cabs. The new steady-state equilibrium exhibits more movers in the locations with excess supply and fewer in (some of) the locations with excess demand.

The empirical evidence available on the matching functions generated by aggregate labor markets suggests isomatching curves that exhibit some degree of substitutability between unemployment and vacancies.

[15] Notice that u enters the expressions for u_i only in locations $k, \ldots, n$ in table 2.

As Blanchard and Diamond (1989, p. 4) put it, "somewhat to our surprise, even when the unemployment becomes very large, its marginal effect on new hires does not disappear." This stands in contrast with the fact that the isomatching curves implied by the model presented here look like right angles. There are two reasons why increases in the number of movers u have no effect on meetings once it has become "too large" in the steady-state equilibrium of the market modeled here.

The first reason is our focus on steady-state outcomes. The reason why additional movers generate no meetings once the aggregate number of movers exceeds ϕv is that *in the steady state* they all end up in the locations with excess demand for cabs. Outside the steady state, however, the number of meetings will in general depend on how the new movers are distributed across locations. For example, suppose that u is increased and some of the additional movers are placed in the locations with an excess supply of cabs. Then each of these movers will generate an extra meeting, even if $\theta < 1/\phi$.

The second reason is the fact that prices are fixed. As discussed above, if $\theta < 1/\phi$, an increase in u adds only to the number of movers waiting for cabs in the locations with excess demand in the steady-state equilibrium. Since cabs already met passengers with certainty in those locations and the π_i's are fixed, the no-arbitrage condition implies that their equilibrium distribution remains unchanged by the increase in the aggregate number of movers. However, if prices (the π_{ij}'s) were allowed to respond to market conditions, then, in principle, the increase in the number of unserved passengers in the location with excess demand may cause the price of a ride out of that location to rise, inducing a relocation of cabs from the locations with excess supply into the one with excess demand, resulting in an increase in the aggregate number of meetings.

VII. Policy Experiments and the Matching Function

Suppose that we try to study the effects of a fare increase on the number of meetings using the standard search and matching model with an exogenous meeting technology. To make the formulation as close as possible to the environment described in Section II, let us assume that time is discrete, that all rides last exactly one period, and that the populations of cabs and movers are fixed and denoted by v and u, respectively. Let $\bar{\pi}$ be the profit to a cab from selling a ride and let s be a measure of search effort.[16] Assume that the exogenous matching func-

[16] Since the spatial structure is "black-boxed" inside the matching function in this formulation, $\bar{\pi}$ replaces the π_{ij}'s.

tion $f(s, u, v)$ is "well behaved,"[17] as well as that $f_1 > 0$ and $f_{11} < 0$. Letting $\beta \in (0, 1)$ be the discount factor, letting J denote the cab's value of search, and assuming random matching, we can write

$$J = \max_s \left\{ -c(s) + \left[\frac{f(s, u, v)}{v}\right]\bar{\pi} + \beta J \right\},$$

where $c(s)$ is the cost associated with exerting search effort s, with $c' > 0$ and $c'' > 0$. So the standard model predicts an increase in search effort and hence an increase in the aggregate number of meetings in response to the increase in the fare structure $\bar{\pi}$. Notice that within this framework, search effort is the only variable that can make the number of meetings react to changes in parameters.[18]

In contrast, the endogenous aggregate matching function derived in proposition 3 depends on the flag drop rate b, the per mileage charge π, the full set of pairwise distances, and the matrix of wishes to move. In short, it depends on all the parameters that determine a cab's expected profit. When n is used to label the worst possible location, namely, $\pi_n = \min\{\pi_1, \ldots, \pi_n\}$, the expression for the aggregate matching function in proposition 3 can be written as an explicit function of parameters just by noticing that

$$\phi = \frac{(b/\pi) + \sum_{j=1}^{n-1} a_{nj}\delta_{nj}}{(b/\pi) + \sum_{i=1}^{n} \mu_i \sum_{j \neq i} a_{ij}\delta_{ij}}.$$

So anything that makes location n more attractive relative to other locations will shift the corner of the isomatching curve and will reduce the extent of the meeting frictions provided that $\theta < 1/\phi$. Consider the effects that changes in the fare structure (i.e., changes in b or π or both) have on the endogenous meeting process. It is clear from the expression above for ϕ that the behavior of $\alpha \equiv b/\pi$ is all that matters to predict the impact that changes in the fare structure have on aggregate meetings. It is easy to show that $\partial\phi/\partial\alpha > 0$ as long as $\Pi > 0$. Thus, in an equilibrium with frictions (i.e., provided that $\theta < 1/\phi$), the aggregate number of meetings increases with α. But if the increase in $\bar{\pi}$ is associated with a fall in α, then ϕ and the aggregate number of meetings fall. Intuitively, a decrease in the fixed component b relative to the per mileage charge π makes shorter trips relatively less attractive, inducing some cabs in locations with excess demand to look for passengers in locations with excess supply.

In summary, if parameters are such that $\theta < 1/\phi$ and the increase in

[17] That is, f is differentiable, with $f_2 > 0$, $f_3 > 0$, and $f(s, u, v) \leq \min\{u, v\}$.

[18] The optimal choice of search effort is characterized by $c'(s) = [f_1(s, u, v)/v]\bar{\pi}$. In a model without search effort (i.e., if $c(s) = f_1 = 0$ for all s) and fixed populations, the increase in $\bar{\pi}$ would have no effect on the number of meetings.

$\bar{\pi}$ is associated with a fall in α, then the model built around an exogenous matching function predicts an increase in the number of trades whereas the endogenous meeting technology predicts a reduction. So in general, policies change the equilibrium distribution of cabs across locations, which in turn affects the *shape* of the matching function. In other words, this example suggests that unless agents are completely unable to choose how to conduct their search, the meeting process is endogenous and black-boxing it in a fixed aggregate matching function could be misleading.

VIII. Concluding Remarks

The model presented here illustrated an alternative way of modeling search frictions. By letting searchers distribute themselves optimally across spatially distinct meeting points (or locations), I have shown that some equilibria may exhibit frictions provided that the meeting points are not all identical from the searching agents' perspective. When at least one location is "better" than another, the possibility that cabs may "overcrowd" that location arises, leaving another location with unserved passengers. So although all possible contacts occur within each location, cabs may distribute themselves in a way such that some of them are unable to find passengers and some passengers are unable to find cabs. From an aggregate perspective (i.e., when one looks at the total numbers of movers and cabs, disregarding their distribution across locations), this situation looks just like the environments with meeting frictions typically assumed in search-theoretic models, although these frictions are of a different nature.

The description of the technology for the coordination of trade is the main feature of an equilibrium search model. Traditionally, search models have relied on exogenous specifications of this meeting technology. The alternative way of thinking about search frictions proposed here shows that it is possible to generate an endogenous matching function through equilibrium aggregation. In general, this matching function will depend on the agents' incentives, and hence its *shape* will be sensitive to policy. In the light of the widespread use of traditional search and matching models in much of modern macroeconomics, this observation raises a fundamental concern. Conducting policy experiments in models built around exogenous meeting processes is a meaningful exercise only if the random search assumption is a good characterization of the underlying search behavior of the agents being modeled. Sometimes—as in the taxicab application presented here—the "nobody knows where anything is" assumption may not be appropriate, and hence tacitly adopting it by studying the problem with an exogenous matching function may turn out to be misleading.

Appendix

Proof of Proposition 1

If $\Pi = 0$, the first $n-1$ equations in condition (E1) become

$$\min\left\{\frac{1}{\theta_i}, 1\right\} = \min\left\{\frac{1}{\theta_n}, 1\right\} \quad \text{for } i = 1, \ldots, n-1,$$

so in equilibrium, either all locations have an excess supply of cabs or no location does. Potentially, there are three types of equilibria. In the first, all locations exhibit an excess supply of cabs. I deal with this case in part *a*. In the second type, which is analyzed in part *b*, there is market clearing in all locations. Finally, part *c* characterizes the set of equilibria for the case in which at least one location exhibits excess demand and none of the others exhibit excess supply.

Part a.—In an equilibrium with an excess supply of cabs in all locations, conditions (E1)–(E3) become

$$\theta_i = \theta_n, \quad i = 1, \ldots, n, \tag{A1}$$

$$v = \sum_{i=1}^{n} v_i, \tag{A2}$$

$$u_i = \sum_{j \neq i}^{n} a_{ji} u_j, \quad i = 1, \ldots, n-1, \tag{A3}$$

and

$$u = \sum_{i=1}^{n} u_i. \tag{A4}$$

Since there is an excess supply of cabs in all locations, the flows of movers between locations are driven by the Markov process that rules people's wishes to move. The vector $\bar{\mathbf{u}}$ solves the $n-1$ flow equations (A3) if and only if it solves

$$\bar{\mathbf{u}} \cdot \mathbf{A} = \bar{\mathbf{u}}, \tag{A5}$$

where

$$\mathbf{A} = \begin{bmatrix} 1-u & ua_{12} & \cdots & ua_{1n} \\ ua_{21} & 1-u & \cdots & ua_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ua_{n1} & ua_{n2} & \cdots & 1-u \end{bmatrix}$$

is the Markov matrix of people's wishes to move. Although there are n equations in the system given by (A5) and only $n-1$ in (A3), both systems are identical since the first $n-1$ equations are the same whereas the nth equation in (A5) is just a linear combination of the previous $n-1$.[19] The matrix $\mathbf{A}$ is a strictly positive Markov matrix, so it has a unique stationary distribution; namely, there is a unique vector

$$\boldsymbol{\mu} = \left(\mu_1, \mu_2, \ldots, 1 - \sum_{i=1}^{n-1} \mu_i\right)$$

such that

$$\boldsymbol{\mu}\mathbf{A} = \boldsymbol{\mu}. \tag{A6}$$

Furthermore, $\boldsymbol{\mu} > 0$ because $\mathbf{A} > 0$.[20] Hence $\bar{\mathbf{u}} = \boldsymbol{\mu}u$ is the unique solution to the system of equations given by (A3) and (A4). Since the system (A5) is independent of u, $\boldsymbol{\mu}$ does not depend on u and hence $\bar{\mathbf{u}}$ is linear in u. So we can write

$$\bar{u}_i = \mu_i u, \quad i = 1, \ldots, n, \tag{A7}$$

where $\mu_i > 0$ is a function of the elements of the Markov matrix $\mathbf{A}$ only. When $\bar{\mathbf{u}}$ is solved for, the n equations given by (A1) and (A2) can be solved for the equilibrium allocation of taxicabs (denoted $\bar{v}_i$):

$$\bar{v}_i = \mu_i v, \quad i = 1, \ldots, n. \tag{A8}$$

Finally, verify that the conjecture $\bar{v}_i > \bar{u}_i$, for $i = 1, \ldots, n$, is verified if and only if $\theta > 1$. Thus the unique solution to (A1)–(A4) described in (A7) and (A8) constitutes an equilibrium only if $\theta > 1$.

Part b.—With market clearing in all locations, the equilibrium is characterized by (A2)–(A4) and

$$\theta_i = 1, \quad i = 1, \ldots, n. \tag{A9}$$

As in part *a,* (A3) and (A4) can be solved for (A7). The distribution of cabs is then obtained from (A9), and it is seen to satisfy (A2) only if $\theta = 1$. Thus the allocation $u_i = v_i = u\mu_i$ for $i = 1, \ldots, n$ is an equilibrium only if $\theta = 1$.

[19] To see this, rearrange the first $n-1$ equations in (A5) to get

$$a_{nj}u_n = u_j - \sum_{i \neq j}^{n-1} a_{ij}u_i,$$

for $j = 1, \ldots, n-1$. Adding up these $n-1$ conditions yields

$$u_n \sum_{j=1}^{n-1} a_{nj} = \sum_{j=1}^{n-1}\left(1 - \sum_{i \neq j}^{n-1} a_{ji}\right) u_j,$$

which is the same as the last equation in (A5), namely,

$$u_n = \sum_{j=1}^{n-1} a_{jn}u_j,$$

since $\sum_{j=1}^{n-1} a_{nj} = 1$ and $1 - \sum_{i \neq j}^{n-1} a_{ji} = a_{jn}$.

[20] If $\mathbf{z}$ is a vector, $\mathbf{z} > 0$ means that $z_i > 0$ for all i, whereas $\mathbf{z} \geq 0$ means that $z_i \geq 0$ for all i and $z_i > 0$ for some i (i.e., $\mathbf{z} \neq 0$). Similarly, if $\mathbf{Q}$ is a matrix, $\mathbf{Q} > 0$ means that $q_{ij} > 0$ for all i and j. The fact that $\mathbf{A}$ is a Markov matrix implies that 1 is an eigenvalue (hence we know that a $\boldsymbol{\mu}$ satisfying [A6] exists). Additionally, $\mathbf{A} > 0$ implies that 1 is $\mathbf{A}$'s largest eigenvalue. Since $\boldsymbol{\mu}$ is the eigenvector associated with the largest nonnegative eigenvalue of a nonnegative matrix, by Frobenius's theorem we know that $\boldsymbol{\mu} \geq 0$ (see Nikaido 1970; Takayama 1985). Finally, since $\mathbf{A} > 0$ and $\boldsymbol{\mu} \geq 0$, it is obvious that (A6) cannot hold if $\mu_i = 0$ for some i, so $\boldsymbol{\mu} > 0$ must be the case.

Part c.—Suppose that there are k locations with excess demand and $n - k$ that clear, with $1 \leq k \leq n$. If $k \leq n - 1$, label locations so that $i = 1, \ldots, n - k$ denote those with market clearing. In this case, the equilibrium conditions are

$$\theta_i = 1, \quad i = 1, \ldots, n - k, \tag{A10}$$

and

$$v_i = \sum_{j \neq i}^{n} a_{ji} v_j, \quad i = 1, \ldots, n - 1, \tag{A11}$$

together with (A2) and (A4). If $k = n$, then the equilibrium is characterized by (A2), (A4), and (A11) only. Notice that (A11) is just (A3) but with v_i's replacing the u_i's. Therefore, the equilibrium distribution of cabs is $\bar{v} = \boldsymbol{\mu} v$, with $\boldsymbol{\mu}$ defined by (A6). Given the distribution of cabs, the distribution of movers across the locations with market clearing follows immediately from (A10). Since there are $2n - k + 1$ independent equilibrium conditions and $2n$ unknowns, the system will be underdetermined if there is more than one market with excess demand (i.e., if $k \geq 2$). In this case there is an infinite number of solutions to (A2), (A4), (A10), and (A11) since the distribution of movers across the locations with excess demand is indeterminate.[21] Any distribution $\{\hat{u}_i, \hat{v}_i\}_{i=1}^{n}$ with $\hat{v}_i = v\mu_i$ for $i = 1, \ldots, n$, $\hat{u}_i = \hat{v}_i$ for $i = 1, \ldots, n - k$, and $\{\hat{u}_j\}_{j=n-k+1}^{n}$ satisfying

$$\hat{u}_j > \hat{v}_j \tag{A12}$$

and

$$\sum_{j=n-k+1}^{n} \hat{u}_j = u - v \sum_{i=1}^{n-k} \mu_i \tag{A13}$$

solves (A2), (A4), (A10), and (A11). So equilibria of this kind exist if and only if[22] $\theta < 1$. If the equilibrium is not unique, then there is a continuum. Uniqueness obtains if and only if $k = 1$. Q.E.D.

Proof of Proposition 2

If $\Pi > 0$, label locations so that bigger subindexes correspond to locations with a smaller conditional expected profit:

$$\pi_1 \geq \cdots \geq \pi_{k-1} > \pi_k = \cdots = \pi_n. \tag{A14}$$

Since there could be more than one location with the smallest level of conditional expected profit in the city, (A14) allows for $n - k + 1$ such locations, with

[21] As shown in the body of the paper, this multiplicity is irrelevant for the purposes of characterizing the aggregate matching function.

[22] To show sufficiency, notice that (A12) and (A13) imply

$$\sum_{i=n-k+1}^{n} \hat{v}_i < u - \sum_{i=1}^{n-k} \hat{v}_i,$$

so $v < u$ is necessary for both conditions to hold. For necessity, assume that $v < u$ and construct equilibria as follows. Let $\hat{v}_i = \mu_i v$, for $i = 1, \ldots, n$, and let $\hat{u}_i = \hat{v}_i$, for $i = 1, \ldots, n - k$. For $j = n - k + 1, \ldots, n$, let $\hat{u}_j = \hat{v}_j + \epsilon_j(u - v)$, with $\boldsymbol{\epsilon} = (\epsilon_{n-k+1}, \ldots, \epsilon_n)$ being a vector in the k-dimensional unit simplex; i.e., $\epsilon_j \geq 0$ and $\sum_{j=n-k+1}^{n} \epsilon_j = 1$.

$2 \leq k \leq n$. The ranking in (A14) together with the first $n-1$ equations in condition (E1) imply

$$\min\left\{\frac{1}{\theta_1}, 1\right\} \leq \cdots \leq \min\left\{\frac{1}{\theta_{k-1}}, 1\right\}$$
$$< \min\left\{\frac{1}{\theta_k}, 1\right\} = \cdots = \min\left\{\frac{1}{\theta_n}, 1\right\}.$$

So in equilibrium there must be excess supply in locations $1, \ldots, k-1$, whereas either all locations $k, \ldots, n$ have excess supply or none of them does. Hence, there can potentially be three types of equilibria: locations $k, \ldots, n$ are in excess supply in the first and exhibit market clearing in the second and excess demand in the third. Locations $1, \ldots, k-1$ are in excess supply in all three types.

Part a.—With excess supply in all locations, the equilibrium is characterized by $2n$ equations, namely,

$$\left(\frac{1}{\theta_i}\right)\pi_i = \left(\frac{1}{\theta_n}\right)\pi_n, \quad i = 1, \ldots, n-1, \tag{A15}$$

(A2), (A3), and (A4). As in part *a* of proposition 1, (A3) and (A4) can be solved for the unique distribution of movers across locations in (A7). Given the distribution of movers, (A2) and (A15) can be solved for the distribution of taxicabs, which is given by

$$v_i = \mu_i \phi_i v, \quad i = 1, \ldots, n, \tag{A16}$$

with $\phi_i = \pi_i(\sum_{i=1}^{n}\mu_i\pi_i)^{-1}$. The distributions of movers and cabs in (A7) and (A16) constitute an equilibrium with excess supply in all locations if and only if $\mu_i\phi_i v > \mu_i u$ for all i, namely, if $\theta > \max\{1/\phi_1, \ldots, 1/\phi_n\}$. Since $\pi_n = \min\{\pi_1, \ldots, \pi_n\}$, the equilibrium with excess supply in all locations exists if and only if $\theta > \phi^{-1}$, where $\phi = \pi_n(\sum_{i=1}^{n}\mu_i\pi_i)^{-1}$.

Part b.—The conditions that characterize an equilibrium with excess supply in the first $k-1$ locations and market clearing in the remaining $n-k+1$ are

$$\left(\frac{1}{\theta_i}\right)\pi_i = \pi_n, \quad i = 1, \ldots, k-1, \tag{A17}$$

and

$$\theta_i = 1, \quad i = k, \ldots, n, \tag{A18}$$

together with (A2), (A3), and (A4). As in proposition 1, (A3) and (A4) can be solved for the unique distribution of movers across locations given in (A7). Then (A17) and (A18) can be solved for the unique distribution of cabs:

$$v_i = \left(\frac{1}{\pi_n}\right)\mu_i\pi_i u, \quad i = 1, \ldots, k-1, \tag{A19}$$

and

$$v_j = \mu_j u, \quad j = k, \ldots, n. \tag{A20}$$

Finally, for the unique distributions of movers and cabs given in (A7), (A19), and (A20) to be an equilibrium, condition (A2) must hold, so we must verify that

$$\sum_{i=1}^{k-1}\left(\frac{1}{\pi_n}\right)\mu_i\pi_i u+\sum_{i=k}^{n}\mu_i u = v. \tag{A21}$$

Because $\pi_i = \pi_n$ for $i = k, \ldots, n$, the left-hand side of (A21) can be written as

$$u\left(\frac{1}{\pi_n}\right)\sum_{i=1}^{n}\mu_i\pi_i,$$

and therefore this equilibrium exists if and only if $\theta = \phi^{-1}$.

Part c.—I now focus on an equilibrium with excess supply in locations $1, \ldots, k-1$ only, market clearing in the first h of the remaining $n-k+1$ locations (with $0 \leq h \leq n-k$), and excess demand in the rest. In this case, the equilibrium conditions are given by a set of $n+k+h$ equations, namely,

$$\theta_i = \frac{\pi_i}{\pi_n}, \quad i = 1, \ldots, k-1, \tag{A22}$$

$$\theta_i = 1, \quad i = k, \ldots, k+h-1, \tag{A23}$$

$$u_i = \sum_{j=1,j\neq i}^{k-1} a_{ji}u_j + \sum_{j=k,j\neq i}^{n} a_{ji}v_j, \quad i = 1, \ldots, k-1, \tag{A24}$$

and

$$v_i = \sum_{j=1,j\neq i}^{k-1} a_{ji}u_j + \sum_{j=k,j\neq i}^{n} a_{ji}v_j, \quad i = k, \ldots, n-1, \tag{A25}$$

together with (A2) and (A4). Let

$$\sigma = \sum_{i=1}^{k-1} u_i + \sum_{i=k}^{n} v_i,$$

and notice that the system of $n-1$ equations labeled (A24) and (A25) can be written as[23]

$$\bar{\mathbf{s}}\mathbf{A} = \bar{\mathbf{s}}, \tag{A26}$$

with

$$\bar{\mathbf{s}} = \left(\frac{u_1}{\sigma}, \ldots, \frac{u_{k-1}}{\sigma}, \frac{v_k}{\sigma}, \ldots, 1-\sum_{i=1}^{k-1}\frac{u_i}{\sigma}-\sum_{i=k}^{n-1}\frac{v_i}{\sigma}\right).$$

Since (A26) is identical to (A6), it follows that $\bar{\mathbf{s}} = \boldsymbol{\mu}$. Thus the distributions of movers across the first $k-1$ locations and of cabs across locations $k, \ldots, n$ that satisfy (A24) and (A25) are

$$u_i = \mu_i\sigma, \quad i = 1, \ldots, k-1, \tag{A27}$$

and

$$v_i = \mu_i\sigma, \quad i = k, \ldots, n. \tag{A28}$$

Equations (A27) and (A22) imply that

[23] The nth equation in (A26) is implied by the $n-1$ equations in (A24) and (A25).

$$v_i = \left(\frac{1}{\pi_n}\right)\mu_i\pi_i\sigma, \quad i = 1, \ldots, k-1. \tag{A29}$$

The distribution of cabs in (A28) and (A29) satisfies (A2) if and only if

$$\sigma\left[\sum_{i=1}^{k-1}\left(\frac{1}{\pi_n}\right)\mu_i\pi_i + \sum_{i=k}^{n}\mu_i\right] = v,$$

or, equivalently (since $\pi_k = \cdots = \pi_n$), if and only if $\sigma = \phi v$. This allows us to rewrite the distribution of movers in (A23) and (A27) and the distribution of cabs in (A28) and (A29) as

$$u_i = \mu_i\phi v, \quad i = 1, \ldots, k+h-1, \tag{A30}$$

$$v_i = \mu_i\phi_i v, \quad i = 1, \ldots, k-1, \tag{A31}$$

and

$$v_i = \mu_i\phi v, \quad i = k, \ldots, n. \tag{A32}$$

Since there are $n+k+h$ equilibrium conditions (namely, $n+k+h-2$ equations in [A22]–[A25] plus the two "adding-up" conditions) and $2n$ unknowns, there are $n-k-h$ undetermined variables. If $h = n-k$, then the allocation in (A30)–(A32) together with $u_n = u - (1-\mu_n)\phi v$ uniquely solves (A22)–(A25). If $h \leq n-k-1$, then there are at least two locations with excess demand. In this case, the distribution described in (A30), (A31), and (A32) together with any distribution $\{\tilde{u}_j\}_{j=k+h}^{n}$ satisfying

$$\tilde{u}_j > v_j, \quad j = k+h, \ldots, n, \tag{A33}$$

and

$$\sum_{j=k+h}^{n}\tilde{u}_j = u - \phi v\sum_{i=1}^{k+h-1}\mu_i \tag{A34}$$

solves (A22)–(A25). Hence equilibria with excess supply only in locations $1, \ldots, k-1$ and excess demand in at least one of the remaining $n-k+1$ locations exist if and only if[24] $\theta < \phi^{-1}$. If the equilibrium is not unique, then there is a continuum. Uniqueness obtains if and only if $k = n$. Q.E.D.

[24] To show necessity, notice that (A33) and (A34) imply

$$\sum_{j=k+h}^{n} v_j < u - \phi v\sum_{i=1}^{k+h-1}\mu_i$$

or, equivalently (from [A32]), $\phi v < u$. To show sufficiency, assume $\phi v < u$, and construct equilibria as follows. Let v_i, for $i = 1, \ldots, n$, be given by (A31) and (A32), and let u_i, for $i = 1, \ldots, k+h-1$, be given by (A30). For $j = k+h, \ldots, n$, let $u_j = v_j + \epsilon_j(u - \phi v)$, with $\epsilon = (\epsilon_{k+h}, \ldots, \epsilon_n)$ being a vector in the $(n-h-k+1)$–dimensional unit simplex, namely $\epsilon_j \geq 0$ and $\sum_{j=k+h}^{n}\epsilon_j = 1$.

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