SIMON FRASER UNIVERSITY
Department of Economics

Econ 808
Prof. Kasa
Macroeconomic Theory
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## MIDTERM EXAM - October 27

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (5 points each).

1. Contraction mappings have unique fixed points.
2. The Mortensen-Pissarides model generates an efficient unemployment rate when the Nash bargaining power of workers is equal to the elasticity of the matching function with respect to vacancies.
3. Dynamic programming is not applicable if current returns depend on future control variables.

The following questions are short answer. Be sure to explain and interpret your answer. Clarity and conciseness will be rewarded.
4. (20 points). Assume a worker chooses career-job $(\theta, \epsilon)$ pairs subject to the following conditions: There is no unemployment. The worker's earnings at time- $t$ are $\theta_{t}+\epsilon_{t}$. The worker is risk-neutral, and maximizes $E \sum_{t=0}^{\infty} \beta^{t}\left(\theta_{t}+\epsilon_{t}\right)$. A career is a draw of $\theta$ from c.d.f. $F$; a job is a draw of $\epsilon$ from c.d.f. $G$. Successive draws are independent, and $G(0)=F(0)=0, G\left(B_{\epsilon}\right)=F\left(B_{\theta}\right)=1$.
Assume the worker can draw a new career only if he also draws a new job. However, the worker is free to retain his existing career $(\theta)$, and draw a new job $\left(\epsilon^{\prime}\right)$. The worker decides at the beginning of each period whether to stay in the current career-job pair, stay in his current career but draw a new job, or draw a new career-job pair. Assume there is no recall of past jobs or careers.
(a) Let $v(\theta, \epsilon)$ be the beginning-of-period value function for a worker with career-job pair $(\theta, \epsilon)$ who is about to decide whether to draw a new career and/or job. Write down the Bellman equation characterizing $v(\theta, \epsilon)$.
(b) Characterize the optimal policy function. (Hint 1: Partition the $(\theta, \epsilon)$ plane into 3 regions, one characterizing the decision to change both jobs and careers, one characterizing the decision to keep your career but change jobs, and one characterizing the decision to stay put. Hint 2: To characterize these regions define a function $C(\theta)$, which gives the value of changing jobs but keeping your career. Use this function to show that there is a 'reservation career' such that a worker stays in this career for all jobs. Hint 3: Show that the option to change jobs implies that some workers will accept careers below the 'reservation career' if they get an especially good job draw).
5. (20 points). Consider the following 1-period economy. There are two types of workers. Fraction $\lambda$ are risk averse, with concave utility function $u(c)$. Fraction $1-\lambda$ are risk-neutral, with linear utility function $v(c)=c$. Both types of workers produce output $f(k)$ when matched with a firm employing capital $k$, where $f$ satisfies the usual assumptions. There is a large number of potential firms which can enter, buy capital (at price 1 per unit), and post vacancies.
Matching takes place as follows: First, firms decide whether to enter, irreversibly buy some capital, and post wages. Then, workers observe all advertised wages, and decide which jobs to apply for. If employed, a worker receives the promised wage (i.e., ex post renegotiation is not permitted), otherwise he receives some unemployment benefit, $z$. Workers are assumed to make their application decisions without coordination, which leads to the usual matching frictions. In particular, if $q N$ workers apply to $N$ firms (i.e., $N$ firms offer some wage $w^{\prime}$, and $q N$ workers seek wage $w^{\prime}$ ), then the firm gets a worker with probability $1-e^{-q}$, and each worker is employed with probability $\left(1-e^{-q}\right) / q$. (In case you're interested, this is the limit of a standard urn-ball process as $N \rightarrow \infty$ ). Note that workers have to trade-off wages and employment probabilities when formulating their application decisions. Also, note that because firms must choose their capital before matching, if a firm does not get a worker, its capital is sunk.
(a) Define an equilibrium. Show that the equilibrium can be characterized by a pair of constrained maximization problems. (Hint: Think "max expected utility subject to zero expected profits").
(b) Characterize the equilibrium, and show that (generically) there will be an observed wage distribution with just two wages, $w^{h}$ and $w^{l}$ (where $w^{h}>w^{l}$ ), with equilibrium fractions $1-\mu$ and $\mu$, respectively.
(c) Illustrate this equilibrium graphically. (Hint 1: Draw the workers' indifference curves in ( $q, w$ ) space. Plug the firms' first-order condition for $k$ into the zero expected profit condition to get a zero-profit locus of $(q, w)$ combinations. Now combine the indifference curves with the zero profit locus. Hint 2: Remember Moen (JPE, 1997)! ).
6. (10 points). Consider a world with just two 'states' - Good or Bad. There are two firms in the economy - Firm A and Firm B. The share price of Firm A is $\$ 10$ and the share price of Firm B is $\$ 12$. Assume their state-contingent profits are as follows:

|  | Good | Bad |
| :---: | :---: | :---: |
| Firm A | 2 | 1 |
| Firm B | 3 | 0 |

(a) What are the Arrow-Debreu state-contingent claims prices?
(b) Given your answer to part (a), what must be the price of a risk-free asset?

