SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 — Prof. Kasa
Macroeconomic Theory — Fall 2009

MIDTERM EXAM - SOLUTIONS

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (8 points each).

1. With complete markets, everyone's consumption will be the same.

   FALSE. *With complete markets everyone has the same marginal rate of substitution, across all dates and states, but the level of consumption will generally differ across agents, depending on their resources.*

2. The Mortensen-Pissarides model generates an inefficiently high unemployment rate.

   FALSE/UNCERTAIN. *This is true only if worker's bargaining power exceeds the elasticity of the matching function with respect to unemployment. If not, unemployment will be too low. If the two are exactly equal, then the unemployment rate is efficient (i.e., Hosios' condition).*

3. The space, $C[0,1]$, of continuous functions on the closed interval $[0,1]$, along with the metric $d(f,g) = [\int_0^1 (f(t) - g(t))^2 dt]^{1/2}$, is a complete metric space.

   FALSE. *Consider the sequence of functions, $f_n(t) = t^n$. This is a Cauchy sequence, yet it converges to a discontinuous function. This is why the Contraction Mapping Theorem is stated in terms of the sup norm (sometimes called the 'norm of uniform convergence').*

The following questions are short answer. Be sure to explain and interpret your answer. Clarity and conciseness will be rewarded.

4. (16 points). In his 2005 *AER* article, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", Robert Shimer argued that the Mortensen-Pissarides model does not fit the data. On what evidence did he base this conclusion? According to Shimer, what is the problem? Can you think of another explanation?

   Shimer argued that the Mortensen-Pissarides model cannot generate nearly enough volatility in vacancies and the unemployment rate, two of the key variables in the model. According to Shimer, the problem is the Nash bargaining wage setting assumption. He argued that this generates too much wage variability over the cycle. For example, in response to a positive productivity shock wages rise too much, which reduces the incentive for firms to create vacancies. He argued that if wages were 'sticky', then the model would generate more variability in vacancies and unemployment. As discussed in the article by Hornstein, Krusell, and Violante, sticky wages aren't the only possibility for generating greater vacancy and unemployment fluctuations. They discuss a paper by Hagedorn and Manovskii (AER, 2008), which argues that if the value of leisure/unemployment compensation is close to the productivity of labor, then the model generates realistic fluctuations in unemployment and vacancies. Unfortunately, it is difficult to (directly) measure the value of leisure.

5. (30 points). Assume the economy is either in a boom (B) or recession (R), each with probability 1/2. The state of the economy (R or B) is i.i.d. over time. At the beginning of each period, workers know the state of the economy for that period, and they can either choose to work at their previous wage or draw a new wage. If a new wage is drawn the old wage is lost, b is received this period, and then work begins at the new wage next period.

   During recessions, new wages (for jobs that begin next period) are i.i.d. draws from the c.d.f. F, where $F(0) = 0$ and $F(M) = 1$ for $M < \infty$. During booms, the worker can choose to quit and take two i.i.d. draws from F of a possible new wage, with the option of working at the higher wage (again for a job starting next period). Getting multiple offers during booms is one way to formalize what "booms" mean to workers. Workers who are unemployed at the beginning of a period receive b this period, and then either draw one (in recessions) or two (in booms) wage offers next period.

   All workers seek to maximize $E_0 \sum_{t=0}^{\infty} (1-\alpha)^t \beta^t y_t$, where $\alpha < 1$ is the probability that a worker dies at the end of a period, and y_t is the worker's income in period-t (i.e., $y_t = w_t$ when employed, and $y_t = b$ when unemployed.

   (a) Write down two Bellman equations that characterize the optimization problem of employed workers.

   The value function in recessions is

$$V(w,R) = \max_{\text{stay,quit}} \left\{ w + \frac{1}{2}\beta(1-\alpha)\sum_{s'} V(w,s'), b + \frac{1}{2}\beta(1-\alpha)\sum_{s'}\int_0^M V(w',s')dF(w') \right\}$$

   and the value function in booms is

$$V(w,B) = \max_{\text{stay,quit}} \left\{ w + \frac{1}{2}\beta(1-\alpha)\sum_{s'} V(w,s'), b + \frac{1}{2}\beta(1-\alpha)\sum_{s'}\int_0^M V(w',s')d(F^2)(w') \right\}$$

   (b) Characterize a worker's quitting policy. Will an employed worker ever quit? If so, who quits and when? Explain. (Hint: Note that the event $\max\{w_1, w_2\} < w$ is the event $(w_1 < w) \cap (w_2 < w)$. Therefore, $\text{prob}(\max\{w_1, w_2\} < w) = F(w)^2$, and $F(w)^2 \leq F(w)$, i.e., F^2 first-order stochastically dominates F).

   The right-hand side of each Bellman equation is independent of w, while the left-hand side is increasing in w. Hence, the optimal policy is a pair of reservation wages, w_R and w_B. They are characterized by the indifference conditions

$$w_s + \frac{1}{2}\beta(1-\alpha)\sum_{s'} V(w_s, s') = b + \frac{1}{2}\beta(1-\alpha)\sum_{s'}\int_0^M V(w',s')dG_s(w')$$

   where $G_R(w) = F(w)$ and $G_B(w) = F^2(w)$. Since $F^2(w) \leq F(w)$ we know

$$w_R + \frac{1}{2}\beta(1-\alpha)\sum_{s'} V(w_R, s') \leq w_B + \frac{1}{2}\beta(1-\alpha)\sum_{s'} V(w_B, s')$$

   Finally, since $V(w,s)$ is (weakly) increasing in w, we have $w_B \geq w_R$. Hence, workers may accept a job during a recession, then quit during a boom. Intuitively, the value of quitting increases during booms since job offers are better.

6. (30 points). Consider an economy where workers are not allowed to quit their jobs. At the same time, jobs are exogenously destroyed each period with probability α. If a worker's job is destroyed, she draws a new wage, and can immediately start working at the new wage. If she rejects the wage offer, she will have to wait until the beginning of next period to draw a new wage. In other words, workers without a job receive one wage offer per period, and can start working in the same period the wage offer is received (i.e., all accepted jobs last for at least one period). New wages are i.i.d. draws from the c.d.f. F, where $F(0) = 0$ and $F(M) = 1$ for $M < \infty$.

While unemployed, a worker receives unemployment benefits of b. Workers seek to maximize $E_0 \sum_{t=0}^{\infty} \beta^t y_t$, where y_t is the worker's income in period-t (i.e., $y_t = w_t$ when employed, and $y_t = b$ when unemployed.

(a) Write down the Bellman equation for an unemployed worker. Characterize the reservation wage. (Discuss the special case when $\alpha = 1$).

Let $V^u(w)$ be the value function for an unemployed worker with wage offer w. Let $V^e(w)$ be the value function for an employed worker with wage, w, at the beginning of the period (i.e., before learning whether her job is destroyed). These value functions are characterized by the following Bellman equations

$$V^u(w) = \max_{\text{accept,reject}} \{w + \beta V^e(w), b + \beta Q\} \tag{1}$$

$$V^e(w) = \alpha Q + (1-\alpha)(w + \beta V^e(w)) \tag{2}$$

where

$$Q = \int_0^M V^u(w')dF(w')$$

Equation (2) can be solved for $V^e(w)$ as follows

$$V^e(w) = \frac{(1-\alpha)w + \alpha Q}{1 - \beta(1-\alpha)} \tag{3}$$

As expected, $V^e(w)$ is increasing in w. Substituting this into equation (1) reveals that the worker's optimal policy is a reservation wage, w^u. Using eq. (3), we get

$$w^u = (1-\beta)(b + \beta Q) + \beta\alpha(b + \beta Q - Q) \tag{4}$$

Notice that when $\alpha = 1$ we get $w^u = b$, which makes sense, since if $\alpha = 1$, a job lasts only one period, so only current payoffs matter.

(b) Compute the economy's stationary aggregate unemployment rate. (Again, discuss the special case when $\alpha = 1$).

Let U_t be the unemployment rate at the beginning of the period. Each period a fraction, $1 - F(w^u)$, of unemployed workers accept a job offer. At the same time, a fraction, α, of employed workers are fired and draw a new wage. Of these, a fraction $F(w^u)$, reject their offer. Thus, we have the following law of motion for U_t,

$$\begin{aligned} U_{t+1} &= U_t + \alpha(1-U_t)F(w^u) - U_t(1 - F(w^u)) \\ &= \alpha(1-U_t)F(w^u) + U_t F(w^e) \end{aligned}$$

Setting $U_{t+1} = U_t = \bar{U}$ gives the following expression for the steady state unemployment rate

$$\bar{U} = \frac{\alpha F(w^u)}{1 - (1-\alpha)F(w^u)}$$

Not surprisingly, when $\alpha = 1$ we just get $\bar{U} = F(w^u)$.

(c) Now suppose there are two types of jobs: short-lasting jobs with destruction probability α_s, and long-lasting jobs with destruction probability α_ℓ, where $\alpha_s > \alpha_\ell$. When a worker draws a new wage from F, the job is now randomly designated as either short-lasting (with probability π_s) or long-lasting (with probability π_ℓ), where $\pi_s + \pi_\ell = 1$. Assume the worker observes the characteristics of a job offer, (w, α). Does an unemployed worker's reservation wage depend on whether a job is short-lasting or long-lasting? Explain.

Now α becomes a state variable. The two Bellman equations become

$$V^u(w,\alpha) = \max_{\text{accept,reject}} \{w + \beta V^e(w,\alpha), b + \beta Q\} \tag{5}$$

$$V^e(w,\alpha) = \alpha Q + (1-\alpha)(w + \beta V^e(w,\alpha)) \tag{6}$$

where now

$$Q = \pi_s \int_0^M V^u(w', \alpha_s) dF(w') + \pi_\ell \int_0^M V^u(w', \alpha_\ell) dF(w')$$

Clearly, there will now be a <u>pair</u> of reservation wages, for each of the two different job types, $\alpha_i, i = s, \ell$. Applying the same solution strategy as in part (a) gives us the following expression for the reservation wages

$$w^u(\alpha_i) = (1-\beta)(b + \beta Q) + \beta\alpha_i(b + \beta Q - Q) \tag{7}$$

Notice that $b + \beta Q \leq Q$. (Since Q is the value of being unemployed before drawing a wage, it must be at least a good as rejecting the wage forever and simply consuming the unemployment benefit, i.e., $Q \geq b/(1-\beta)$). From eq. (7), we then see that $w^u(\alpha)$ is a (weakly) decreasing function of α. Therefore, $w^u(\alpha_\ell) \geq w^u(w_s)$. The intuition is clear - since by assumption you can't quit, you will be especially choosey when taking a long-lasting job.

(d) Finally, now suppose workers can quit their jobs. Specifically, at the beginning of each period (before knowing whether her job will be destroyed) an employed worker can quit her job, draw a new wage offer, and immediately start work at the new wage. Characterize the worker's optimal policy. How does your answer to part *c* change?

When quitting is allowed, an employed worker has a nontrivial Bellman equation. We have,

$$V^u(w,\alpha) = \max_{\text{accept,reject}} \{w + \beta V^e(w,\alpha), b + \beta Q\} \tag{8}$$

$$V^e(w,\alpha) = \max_{\text{stay,quit}} \{\alpha Q + (1-\alpha)(w + \beta V^e(w,\alpha)), Q\} \tag{9}$$

where again

$$Q = \pi_s \int_0^M V^u(w', \alpha_s) dF(w') + \pi_\ell \int_0^M V^u(w', \alpha_\ell) dF(w')$$

The reservation wage, $w^e(\alpha)$, for an employed worker satisfies the usual indifference condition:

$$\alpha Q + (1-\alpha)(w^e(\alpha) + \beta V^e(w^e(\alpha), \alpha)) = Q$$

Since by definition $V^e(w^e(\alpha)) = Q$, we then get

$$w^e(\alpha) = (1-\beta)Q \qquad \Rightarrow \qquad \frac{w^e(\alpha)}{1-\beta} = Q$$

Thus, the reservation wage for an employed worker is independent of job type. At the reservation wage, an employed worker is indifferent between staying, quitting, and being fired (because she can immediately draw a new wage). Thus, at the reservation wage, the value of staying forever at the current job is equal to the value of quitting.

To calculate $w^u(\alpha)$, first note that, as before, $b + \beta Q \leq Q$. Thus, the right-side of the max operator in (8) is smaller than the right-side of the max operator in (9). Since at the reservation wage the left-sides of the max operators are the same, we know

$$w^u(\alpha) \leq w^e(\alpha)$$

Therefore, $V^e(w^u(\alpha)) = Q$ (since $V^e(w) = Q$ when $w \leq w^e$). Using this in the indifference condition in (8) implies

$$w^u(\alpha) = b$$

Clearly, when a worker has the option to quit and immediately take a new wage draw without enduring any unemployment, it will be optimal to accept any wage above b. (Remember, in the standard McCall model, it was assumed that if the worker quit, he had to spend one period unemployed before he could draw again).